Lineær algebra

FE-MAT1001: Obligatorisk 1

Oppgave 1

- Løs vha. Gauss-eliminasjon

$x_1 - 0.25x_2 - 0.25x_3 = 50$

$-0.25x_1 + x_2 - 0.25x_4 = 50$

$-0.25x_1 + x_3 - 0.25x_4 = 25$

$-0.25x_2 - 0.25x_3 + x_4 = 25$

$(x_1, x_2, x_3, x_4) = (x, y, z, w)$

- Multipliserer hele systemet med 4

$$\begin{aligned} 4x - y - z &= 200 \\ -x + 4y - w &= 200 \\ -x + 4z - w &= 100 \\ -y - z + 4w &= 100 \end{aligned}$$

- Fjerner x i $L2$ og $L3$ vha. $L1$.
L2: $4L2 + L1$
$4(-x + 4y - w) + (4x - y - z) = 4(200) + 200$
$-4x + 16y - 4w + 4x - y - z = 1000$
$16y - y - 4w - z = 1000$
$15y - 4w - z = 1000$

L3: $4L3 + L1$
$4(-x + 4z - w) + (4x - y - z) = 4(100) + 200$
$-4x + 16z - 4w + 4x - y - z = 400 + 200$
$-y + 16z - z - 4w = 600$
$-y + 15z - 4w = 600$

$$4x - y - z = 200$$

$$15y - 4w - z = 1000$$

$$-y + 15z - 4w = 600$$

$$-y - z + 4w = 100$$

- Fjerner y i $L3$ og $L4$ vha. $L2$.
L3: $15L3 + L2$
$15(-y + 15z - 4w) + (15y - 4w - z) = 15(600) + 1000$
$-15y + 225z - 60w + 15y - 4w - z = 10000$
$225z - z - 60w - 4w = 10000$
$224z - 64w = 10000$

L4: $15L4 + L2$
$15(-y - z + 4w) + (15y - 4w - z) = 15(100) + 1000$
$-15y - 15z + 60w + 15y - 4w - z = 2500$
$-15z + 60w - 4w - z = 2500$
$-16z + 56w = 2500$

$$4x - y - z = 200$$

$$15y - 4w - z = 1000$$

$$224z - 64w = 10000$$

$$-16z + 56w = 2500$$

- Fjerner z i $L4$ vha. $L3$
L4: $224L4 + 16L3$
$224(-16z + 56w) + 16(224z - 64w) = 224(2500) + 16(10000)$
$-3584z + 12544w + 3584z - 1024w = 720000$
$12544w - 1024w = 720000$
$11520w = 720000$
$w = \frac{720000}{11520} = \frac{125}{2}$

$$4x - y - z = 200$$

$$15y - 4w - z = 1000$$

$$224z - 64w = 10000$$

$$w = \frac{125}{2}$$

- Fjerner w i $L2$ og $L3$ vha. $L4$.
L2: $L2 + 4L4$
$15y - 4w - z + 4(w) = 1000 + 4(\frac{125}{2})$
$15y - 4w - z + 4w = 1250$
$15y - z = 1250$

L3: $L3 + 64L4$
$224z - 64w + 64(w) = 10000 + 64(\frac{125}{2})$
$224z = 10000 + 4000$
$z = \frac{14000}{224} = \frac{125}{2}$

$$4x - y - z = 200$$

$$15y - z = 1250$$

$$z = \frac{125}{2}$$

$$w = \frac{125}{2}$$

- Fjerner så z i $L1$ og $L2$ vha. $L3$
L1: $L1 + L3$
$4x - y - z + (z) = 200 + (\frac{125}{2})$
$4x - y = \frac{525}{2}$

L2: $L2 + L3$
$15y - z + (z) = 1250 + (\frac{125}{2})$
$15y = \frac{2625}{2}$
$y = \frac{\frac{2625}{2}}{15} = \frac{175}{2}$

$$4x - y = \frac{525}{2}$$

$$y = \frac{175}{2}$$

$$z = \frac{125}{2}$$

$$w = \frac{125}{2}$$

- Fjerner y i $L1$ vha. $L2$.
L1: $L1 + L2$
$4x - y + (y) = \frac{525}{2} + (\frac{175}{2})$
$4x = \frac{700}{2}$
$x = \frac{\frac{700}{2}}{4} = \frac{175}{2}$

$$x = \frac{175}{2}$$

$$y = \frac{175}{2}$$

$$z = \frac{125}{2}$$

$$w = \frac{125}{2}$$

$$(x_1, x_2, x_3, x_4) = (x, y, z, w) = (\frac{175}{2}, \frac{175}{2}, \frac{125}{2}, \frac{125}{2})$$

Systemets koeffisientmatrise
$$\begin{bmatrix} 1 & -0.25 & -0.25 & 0 \\ -0.25 & 1 & 0 & -0.25 \\ -0.25 & 0 & 1 & -0.25 \\ 0 & -0.25 & -0.25 & 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & -\frac{1}{4} & 1 \end{bmatrix}$$
Determinanten til matrisen
$$\begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & -\frac{1}{4} & 1 \end{bmatrix}, \begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$
Fortegnsskjemaet forteller oss om fortegnet vi må bruke for å finne determinanten. Men det er også mulig å finne fortegnet ved å finne raden og kolonnen tallet befinner seg i.

Tallet i rad 1 og kolonne $1 = 1$. $Rad + kolonne = 1 + 1 = 2 =$ et partall, og vi vet derfor at det er $+$ tegn foran dette tallet når vi skal finne determinanten.

- Så holder vi over raden og kolonnen som vi valgte og ganger dette tallet med den nye 3×3 matrisen vi finner.
- Vi gjør det samme for alle tallene på denne raden

$$+1\left(\begin{vmatrix} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) - (-\tfrac{1}{4})\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) + (-\tfrac{1}{4})\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) -$$
$$0\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 1 & 0 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \end{vmatrix} \Rightarrow \left(\begin{vmatrix} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) + \frac{1}{4}\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) - \frac{1}{4}\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right)$$
- De nye 3×3 matrisene deler vi så opp igjen til 2×2 matriser og følger fortegnsskjemaet.
$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$
For å gjøre det mer oversiktlig, velger jeg å finne determinanten til hver av matrisene først, for så å summere dem.

1.
$$\left(\begin{vmatrix} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) \Rightarrow$$
$$+1*\left(\begin{vmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) - 0*\left(\begin{vmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) + (-\tfrac{1}{4})*\left(\begin{vmatrix} 0 & 1 \\ -\frac{1}{4} & -\frac{1}{4} \end{vmatrix}\right) = 1(1*1-((-\tfrac{1}{4})*$$
$$(-\tfrac{1}{4}))) - 0 + -\tfrac{1}{4}(0*(-\tfrac{1}{4}) - (1*(-\tfrac{1}{4})) \Rightarrow$$
$$1*(1-\tfrac{1}{16}-\tfrac{1}{16}) = \tfrac{7}{8}$$

2.
$\frac{1}{4}\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) \Rightarrow$
$+\frac{1}{4}*\left(+(-\frac{1}{4})\left(\begin{vmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) - (-\frac{1}{4})\left(\begin{vmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) + 0 * \left(\begin{vmatrix} 0 & 1 \\ -\frac{1}{4} & -\frac{1}{4} \end{vmatrix}\right)\right) = \frac{1}{4}\left(-\frac{1}{4}(1-(\frac{1}{16})) + \frac{1}{4}(0-(\frac{1}{16})) +$
$\frac{1}{4}(-\frac{1}{4} + \frac{1}{64} - \frac{1}{64}) = -\frac{1}{16}$

3.
$-\frac{1}{4}\left(\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} & 0 \\ 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 1 \end{vmatrix}\right) \Rightarrow$
$-\frac{1}{4}\left(+(-\frac{1}{4}) * \left(\begin{vmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) - (-\frac{1}{4})\left(\begin{vmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) + 0\begin{vmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{vmatrix}\right) = -\frac{1}{4}\left(-\frac{1}{4}(0-(\frac{1}{16})) + \frac{1}{4}(1-(\frac{1}{16}))\right) \Rightarrow$
$-\frac{1}{4}(\frac{1}{64} + \frac{1}{4} - \frac{1}{64}) = -\frac{1}{16}$

Summerer: 1, 2 og 3
$\frac{7}{8} - \frac{1}{16} - \frac{1}{16} = 2 * (\frac{7}{8}) - \frac{2}{16} = \frac{14}{16} - \frac{2}{16} = \frac{12}{16} = \frac{3}{4} = \underline{\underline{0.75}}$
$|Matrisen| = \underline{\underline{0.75}}$

Oppgave 2
- Løs vha. Gauss-eliminasjon

$-4T_1 + T_2 + T_4 = -100$
$T_1 - 4T_2 + T_3 + T_5 = -20$
$T_2 - 4T_3 + T_6 = -20$
$T_1 - 4T_4 + T_5 + T_7 = -80$
$T_2 + T_4 - 4T_5 + T_6 + T_8 = 0$
$T_3 + T_5 - 4T_6 + T_9 = 0$
$T_4 - 4T_7 + T_8 = -260$
$T_5 + T_7 - 4T_8 + T_9 = -180$
$T_6 + T_8 - 4T_9 = -180$

Velger å sette dette likningssystemet på matriseform.

$$\begin{array}{c} \\ L1 \\ L2 \\ L3 \\ L4 \\ L5 \\ L6 \\ L7 \\ L8 \\ L9 \end{array} \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -100 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 & -80 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L1 \rightleftarrows L2$
$L3 \rightleftarrows L4$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -100 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 & -80 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L2 : L2 + 4L1$
$L3 : L3 - L1$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & : -20 \\ 0 & -15 & 4 & 1 & 4 & 0 & 0 & 0 & 0 & : -180 \\ 0 & 4 & -1 & -4 & 0 & 0 & 1 & 0 & 0 & : -60 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & : -20 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & : 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & : 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & : -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & : -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & : -180 \end{bmatrix}$$

$L2 \rightleftarrows L4$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 4 & -1 & -4 & 0 & 0 & 1 & 0 & 0 & -60 \\ 0 & -15 & 4 & 1 & 4 & 0 & 0 & 0 & 0 & -180 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L3 : L3 - 4L2$
$L4 : L4 + 15L2$
$L5 : L5 - L2$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 15 & -4 & 0 & -4 & 1 & 0 & 0 & 20 \\ 0 & 0 & -56 & 1 & 4 & 15 & 0 & 0 & 0 & -480 \\ 0 & 0 & 4 & 1 & -4 & 0 & 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L6 \rightleftarrows L3$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & -56 & 1 & 4 & 15 & 0 & 0 & 0 & -480 \\ 0 & 0 & 4 & 1 & -4 & 0 & 0 & 1 & 0 & 20 \\ 0 & 0 & 15 & -4 & 0 & -4 & 1 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L4 : L4 + 56L3$
$L5 : L5 - 4L3$
$L6 : L6 - 15L3$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 1 & -8 & 16 & 0 & 1 & -4 & 20 \\ 0 & 0 & 0 & -4 & -15 & 56 & 1 & 0 & -15 & 20 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & -260 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L5 : L5 - L4$
$L6 : L6 + 4L4$
$L7 : L7 - L4$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & -68 & 225 & 0 & 1 & -60 & 500 \\ 0 & 0 & 0 & 0 & 225 & -780 & 1 & 0 & 209 & -1900 \\ 0 & 0 & 0 & 0 & -60 & 209 & -4 & 1 & -56 & 220 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L8 \rightleftarrows L5$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 225 & -780 & 1 & 0 & 209 & -1900 \\ 0 & 0 & 0 & 0 & -60 & 209 & -4 & 1 & -56 & 220 \\ 0 & 0 & 0 & 0 & -68 & 225 & 0 & 1 & -60 & 500 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L6 : L6 - 225L5$
$L7 : L7 + 60L5$
$L8 : L8 + 68L5$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & -780 & -224 & 900 & -16 & 38600 \\ 0 & 0 & 0 & 0 & 0 & 209 & 56 & -239 & 4 & -10580 \\ 0 & 0 & 0 & 0 & 0 & 225 & 68 & -271 & 8 & -11740 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \end{bmatrix}$$

$L9 \rightleftarrows L6$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 209 & 56 & -239 & 4 & -10580 \\ 0 & 0 & 0 & 0 & 0 & 225 & 68 & -271 & 8 & -11740 \\ 0 & 0 & 0 & 0 & 0 & -780 & -224 & 900 & -16 & 38600 \end{bmatrix}$$

$L7 : L7 - 209L6$
$L8 : L8 - 225L6$
$L9 : L9 + 780L6$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 0 & 56 & -448 & 840 & 27040 \\ 0 & 0 & 0 & 0 & 0 & 0 & 68 & -496 & 908 & 28760 \\ 0 & 0 & 0 & 0 & 0 & 0 & -224 & 1680 & -3136 & -101800 \end{bmatrix}$$

$L7 : \frac{1}{56}L7$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 15 & \frac{3380}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 68 & -496 & 908 & 28760 \\ 0 & 0 & 0 & 0 & 0 & 0 & -224 & 1680 & -3136 & -101800 \end{bmatrix}$$

$L8 : L8 - 68L7$
$L9 : L9 + 224L7$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 15 & \frac{3380}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48 & -112 & -\frac{28520}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -112 & 224 & 6360 \end{bmatrix}$$

$L9 : 48L9 + 112L8$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 15 & \frac{3380}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48 & -112 & -\frac{28520}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1792 & -151040 \end{bmatrix}$$

$L9 : -\frac{1}{1792}L9$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 56 & -480 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & -180 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & -180 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 15 & \frac{3380}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48 & -112 & -\frac{28520}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L8 : L8 + 112L9$
$L7 : L7 - 15L9$
$L6 : L6 + 4L9$
$L5 : L5 - L9$
$L4 : L4 - 56L9$
$L3 : L3 - L9$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 0 & -\frac{590}{7} \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 0 & -5200 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & -\frac{1850}{7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & \frac{1100}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 & -\frac{5470}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48 & 0 & \frac{37560}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L8 : \frac{1}{48}L8$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 0 & -\frac{590}{7} \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 0 & -5200 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & -\frac{1850}{7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & \frac{1100}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 & -\frac{5470}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L7 : L7 + 8L8$
$L6 : L6 - L8$
$L5 : L5 + 4L8$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 0 & -\frac{590}{7} \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 0 & -5200 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & \frac{1280}{7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L5 : L5 - L7$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -20 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 0 & -\frac{590}{7} \\ 0 & 0 & 0 & 1 & 60 & -209 & 0 & 0 & 0 & -5200 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L4 : L4 + 209L6$
$L3 : L3 + 4L6$
$L2 : L2 - L6$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -20 \\ 0 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{915}{14} \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{680}{7} \\ 0 & 0 & 0 & 1 & 60 & 0 & 0 & 0 & 0 & \frac{59915}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L4 : L4 - 60L5$
$L3 : L3 - L5$
$L1 : L1 - L5$

$$\begin{bmatrix} 1 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -90 \\ 0 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{915}{14} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{190}{7} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{1115}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L2 : L2 + 4L3$
$L1 : L1 - L3$

$$\begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{820}{7} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{605}{14} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{190}{7} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{1115}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$L1 : L1 + 4L2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{390}{7} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{605}{14} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{190}{7} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{1115}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{590}{7} \end{bmatrix}$$

$$\begin{vmatrix} T_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{390}{7} \\ 0 & T_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{605}{14} \\ 0 & 0 & T_3 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{190}{7} \\ 0 & 0 & 0 & T_4 & 0 & 0 & 0 & 0 & 0 & \frac{1115}{14} \\ 0 & 0 & 0 & 0 & T_5 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 0 & 0 & 0 & T_6 & 0 & 0 & 0 & \frac{635}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & T_7 & 0 & 0 & \frac{790}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_8 & 0 & \frac{1565}{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_9 & \frac{590}{7} \end{vmatrix}$$

Setter inn verdiene jeg fant for å sjekke at det stemmer likningssystemet.

$-4(\frac{390}{7}) + \frac{605}{14} + \frac{1115}{14} = -100$
$\frac{390}{7} - 4(\frac{605}{14}) + \frac{190}{7} + 70 = -20$
$\frac{605}{14} - 4(\frac{190}{7}) + \frac{635}{14} = -20$
$\frac{390}{7} - 4(\frac{1115}{14}) + 70 + \frac{790}{7} = -80$
$\frac{605}{14} + \frac{1115}{14} - 4(70) + \frac{635}{14} + \frac{1565}{14} = 0$
$\frac{190}{7} + 70 - 4(\frac{635}{14}) + \frac{590}{7} = 0$
$\frac{1115}{14} - 4(\frac{790}{7}) + \frac{1565}{14} = -260$
$70 + \frac{790}{7} - 4(\frac{1565}{14}) + \frac{590}{7} = -180$
$\frac{635}{14} + \frac{1565}{14} - 4(\frac{590}{7}) = -180$
Svarene jeg fikk stemmer med likningssystemet.

$T_1 = \frac{390}{7} = 55.71C$
$T_2 = \frac{605}{14} = 43.21C$
$T_3 = \frac{190}{7} = 27.14C$
$T_4 = \frac{1115}{14} = 79.64C$
$T_5 = 70C$
$T_6 = \frac{635}{14} = 45.35C$
$T_7 = \frac{790}{7} = 112.85C$
$T_8 = \frac{1565}{14} = 111.78C$
$T_9 = \frac{590}{7} = 84.28C$