The modified performance measure affects only the boundary conditions at $t = 2$. From entry 2 of Table 5-1 we have

$$\begin{aligned} p_1^*(2) &= x_1^*(2) - 5 \\ p_2^*(2) &= x_2^*(2) - 2. \end{aligned} \tag{5.1-73}$$

c_1 and c_2 are again zero because $\mathbf{x}^*(0) = \mathbf{0}$. Putting $t = 2$ in Eq. (5.1-69) and substituting in (5.1-73), we obtain the linear algebraic equations

$$\begin{bmatrix} 0.627 & -2.762 \\ 9.151 & -11.016 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}. \tag{5.1-74}$$

Solving these equations, we find that $c_3 = -2.697$, $c_4 = -2.422$; hence,

$$\begin{aligned} x_1^*(t) &= 2.697t - 2.422 + 2.560\epsilon^{-t} - 0.137\epsilon^{t} \\ x_2^*(t) &= 2.697 - 2.560\epsilon^{-t} - 0.137\epsilon^{t}. \end{aligned} \tag{5.1-75}$$

c. Next, suppose that the system is to be transferred from $\mathbf{x}(0) = \mathbf{0}$ to the line

$$x_1(t) + 5x_2(t) = 15 \tag{5.1-76}$$

while the original performance measure (5.1-64) is minimized. As before, the solution of the state and costate equations is given by Eq. (5.1-69), and $c_1 = c_2 = 0$. The boundary conditions at $t = 2$ are, from entry 3 of Table 5-1,

$$\begin{aligned} x_1^*(2) + 5x_2^*(2) &= 15 \\ -p_1^*(2) &= d \\ -p_2^*(2) &= 5d. \end{aligned} \tag{5.1-77}$$

Eliminating d and substituting $t = 2$ in (5.1-69), we obtain the equations

$$\begin{bmatrix} 15.437 & -20.897 \\ 11.389 & -7.389 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}, \tag{5.1-78}$$

which have the solution $c_3 = -0.894$, $c_4 = -1.379$. The optimal trajectory is then

$$\begin{aligned} x_1^*(t) &= 0.894t - 1.379 + 1.136\epsilon^{-t} + 0.242\epsilon^{t} \\ x_2^*(t) &= 0.894 - 1.136\epsilon^{-t} + 0.242\epsilon^{t}. \end{aligned} \tag{5.1-79}$$

Example 5.1-2. The space vehicle shown in Fig. 5-4 is in the gravity field of the moon. Assume that the motion is planar, that aerodynamic forces are negligible, and that the thrust magnitude T is constant. The control

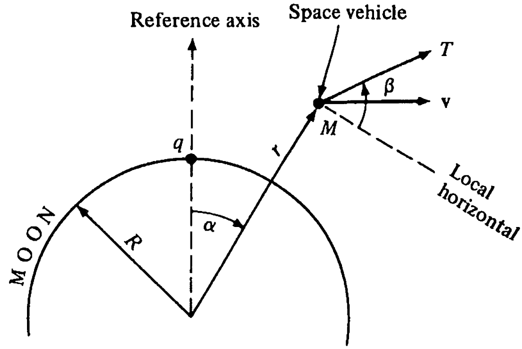

Figure 5-4 A space vehicle in the gravity field of the moon

variable is the thrust direction $\beta(t)$, which is measured from the local horizontal. To simplify the state equations, we shall approximate the vehicle as a particle of mass M. The gravitational force exerted on the vehicle is $F_g(t) = Mg_0R^2/r^2(t)$; g_0 is the gravitational constant at the surface of the moon, R is the radius of the moon, and r is the distance of the spacecraft from the center of the moon. The instantaneous velocity of the vehicle is the vector $\mathbf{v}$, and α is the angular displacement from the reference axis. Selecting $x_1 \triangleq r$, $x_2 \triangleq \alpha$, $x_3 \triangleq \dot{r}$, and $x_4 \triangleq r\dot{\alpha}$ as the states of the system, letting $u \triangleq \beta$, and neglecting the change in mass resulting from fuel consumption, we find that the state equations are

$$\begin{aligned}
\dot{x}_1(t) &= x_3(t) \\
\dot{x}_2(t) &= \frac{x_4(t)}{x_1(t)} \\
\dot{x}_3(t) &= \frac{x_4^2(t)}{x_1(t)} - \frac{g_0R^2}{x_1^2(t)} + \left[\frac{T}{M}\right] \sin u(t) \\
\dot{x}_4(t) &= -\frac{x_3(t)x_4(t)}{x_1(t)} + \left[\frac{T}{M}\right] \cos u(t).
\end{aligned} \tag{5.1-80}$$

Notice that these differential equations are nonlinear in both the states and the control variable. Let us consider several possible missions for the space vehicle.

Mission a. Suppose that the spacecraft is to be launched from the point q on the reference axis at $t = 0$ into a circular orbit of altitude D, as shown in Fig. 5-5(a), in minimum time. $\alpha(t_f)$ is unspecified, and the vehicle starts from rest; thus, the initial conditions are $\mathbf{x}(0) = [R \quad 0 \quad 0 \quad 0]^T$.

From the performance measure

$$J(u) = \int_0^{t_f} dt \tag{5.1-81}$$