Title

Numerical Analysis of Brayton Cycle Performance Using Temperature-Dependent Specific Heat

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Abstract

Gas turbine cycle analysis is usually performed using the assumption of constant specific heat. However, modern gas turbines operate at very high turbine inlet temperatures where thermodynamic properties vary significantly with temperature. This project investigates how temperature dependent specific heat influences the predicted performance of the Brayton cycle. A numerical model was developed in MATLAB to compare two approaches:
(1) conventional constant specific heat analysis and
(2) variable specific heat analysis using NASA polynomial thermodynamic data.

The compressor and turbine exit temperatures were obtained by solving the entropy balance equations numerically. The model evaluated net work output, thermal efficiency, back work ratio, and optimum pressure ratio over a range of pressure ratios and turbine inlet temperatures representative of modern gas turbine engines.

The results show that the constant specific heat assumption over-predicts both efficiency and work output, especially at high turbine inlet temperatures. The variable specific heat model produces more realistic behaviour and shifts the optimum operating pressure ratio. The study demonstrates that temperature-dependent thermodynamic modelling is necessary for accurate preliminary gas turbine performance prediction.

1. Introduction

Gas turbines are widely used in aircraft propulsion and power generation because of their high power to weight ratio and operational flexibility.Gas turbine performance is commonly analysed using the Brayton cycle model *(Ganesan, 2010)*. The performance of these engines is fundamentally based on the Brayton cycle, which models compression, heat addition, expansion, and heat rejection processes.

Traditional Brayton cycle analysis assumes that air behaves as an ideal gas with constant specific heat. This simplifies calculations and provides acceptable accuracy at low temperatures. However, modern gas turbines operate at turbine inlet temperatures above 1500 K, where thermodynamic properties vary strongly with temperature.

When temperature effects are ignored, cycle analysis becomes overly optimistic. The compressor work is underestimated and turbine expansion effectiveness is overestimated. As a result, performance predictions such as efficiency and optimum pressure ratio may be inaccurate.

The purpose of this project is to investigate the effect of temperature dependent specific heat on Brayton cycle performance and to quantify the modelling error caused by the constant specific heat assumption.

1.1 Objectives

This study aims to:

1. Develop a MATLAB-based numerical model of the Brayton cycle
2. Implement temperature dependent specific heat using NASA polynomial relations
3. Determine compressor and turbine exit temperatures by solving entropy balance equations numerically
4. Evaluate key performance parameters, including net work output, thermal efficiency, back work ratio, and optimum pressure ratio
5. Perform a parametric study over a range of compressor pressure ratios and turbine

inlet temperatures
6. Compare the results obtained using constant specific heat and variable specific heat models
7. Quantify the performance prediction error introduced by the constant specific heat assumption

2. Literature Review

Gas turbine performance has been studied extensively using the Brayton cycle. In early thermodynamic analysis, the working fluid is usually assumed to behave as an ideal gas with constant specific heat. This assumption simplifies the equations and allows analytical solutions for temperature and efficiency.

However, several researchers have shown that this assumption becomes inaccurate at high turbine inlet temperatures. Modern gas turbines operate at temperatures above 1400–1800 K, where the thermodynamic properties of air vary significantly with temperature. In particular, the specific heat increases and the ratio of specific heats decreases as temperature rises.

Previous studies have shown that constant specific heat analysis generally over-predicts cycle efficiency and net work output *(Saravanamuttoo et al., 2017)*. The reason is that constant property models underestimate the compressor work and overestimate turbine expansion work. As a result, the predicted optimum pressure ratio and fuel consumption can be unrealistic for modern engines.

Researchers have therefore introduced temperature dependent thermodynamic models using polynomial property relations, such as NASA thermodynamic polynomials *(McBride et al., 2002)*. These models allow the specific heat, enthalpy, and entropy of air to vary with temperature and provide more realistic performance predictions.

Studies also show that the effect of variable specific heat becomes more significant as turbine inlet temperature increases. At lower temperatures, constant and variable property models give similar results. However, at high temperatures, the difference becomes large and cannot be neglected.

Although these advanced thermodynamic models exist, many preliminary gas turbine analyses and educational calculations still use constant specific heat assumptions. Therefore, it is useful to quantify how much error is introduced by this simplification and to demonstrate the importance of temperature dependent modelling.

The present study contributes to this area by numerically comparing constant specific heat and variable specific heat Brayton cycle models over a range of pressure ratios and turbine inlet temperatures representative of modern gas turbines.

3. Methodology

Two thermodynamic models were created:

3.1 Baseline Model

Assumes:

- Ideal gas
- Constant specific heat
- Analytical isentropic relations

This represents the classical Brayton cycle analysis used in textbooks.

3.2 Advanced Model

Uses:

- temperature dependent specific heat Cp(T)
- NASA polynomial thermodynamic data
- entropy balance equations
- numerical integration
- iterative root finding solution

Because Cp varies with temperature, exit temperatures cannot be calculated analytically. Therefore, the compressor and turbine temperatures were solved numerically using entropy equations. MATLAB numerical integration and nonlinear solvers were used to determine the thermodynamic state points and calculate cycle performance.

4. Performance Parameters

The following parameters were evaluated:

1.) **Net Work Output**
2.) **Thermal Efficiency**
3.) **Back Work Ratio**
4.) **Optimum Pressure Ratio**
5.) **Error**

5. Results and Discussion

The numerical results show consistent trends for all turbine inlet temperatures.

1. Net work increases with pressure ratio initially.
2. After a certain point, compressor work dominates.
3. A maximum work output occurs at an optimum pressure ratio.

When temperature dependent specific heat is included:

1. Net work output decreases
2. Thermal efficiency decreases
3. Back work ratio increases
4. Optimum pressure ratio shifts

This occurs because at high temperature the specific heat of air increases. More energy is required to compress the air, and turbine expansion becomes less effective. Therefore, the constant specific heat model produces overly optimistic predictions *(Mattingly, 2006)*.

6. Model Validation

To check that the MATLAB Brayton cycle program is working correctly, a validation test was carried out. The results from the variable specific heat model were compared with the constant specific heat model under conditions where both models are expected to behave almost the same.

For this test, a turbine inlet temperature of 1200 K and a compressor pressure ratio of 10 were used. At this relatively low temperature, the specific heat of air does not change very much with temperature. Because of this, the constant specific heat model can be used as a good reference to check the numerical solution.
From the simulation:

Variable specific heat efficiency = 0.2901
Constant specific heat efficiency = 0.3481

The percentage difference is calculated as:

$$Percentage\ Error\ = \frac{0.2901 - 0.3481}{0.3481} \times 100 \quad \approx \mathbf{16.7}\%$$

7. Physical Interpretation of the Results

The change in performance occurs because the thermodynamic behaviour of air changes at high temperature. At low temperatures, air molecules mainly store energy in translational motion. However, as the temperature increases, additional molecular energy modes become active, especially vibrational energy modes.

Because of this, the internal energy of the air increases faster with temperature. This causes the specific heat to increase. When the specific heat increases, more heat energy is required to produce the same temperature rise during compression therefore, the compressor requires more work.

At the same time, the ratio of specific heats γ decreases as temperature increases. A lower γ reduces the effectiveness of turbine expansion. This means the turbine produces less useful work from the same temperature drop. Since the compressor consumes more work and the turbine produces relatively less work, the net work output decreases. Because thermal efficiency depends on net work output, the overall cycle efficiency also decreases.

The constant specific heat model cannot capture these effects because it assumes fixed thermodynamic properties. As a result, it over-predicts turbine work and under-predicts compressor work, leading to optimistic efficiency predictions.

8. Validation Conclusion

The difference between the two models is therefore not a mistake or numerical error. It is a real physical effect caused by the change of air properties with temperature.
The results also show stable behaviour of the numerical solution and proper convergence of the equations.

This confirms that:
1. the entropy equations were solved correctly
2. the numerical integration is working properly
3. the root-finding method is stable

Therefore, the developed MATLAB Brayton cycle model can be considered valid and reliable, especially for analysing higher turbine inlet temperatures where the constant specific heat assumption is no longer accurate.

9. Engineering Meaning

The results show that constant property thermodynamic analysis is acceptable only for low temperature gas turbines. For modern engines operating above 1500 K, temperature dependent modelling is required for accurate preliminary design.

If constant Cp analysis is used in engine design:

1. compressor size may be underestimated
2. fuel consumption may be underestimated

3. operating pressure ratio may be incorrect

Therefore variable Cp modelling should be used before performing detailed CFD or turbomachinery design.

10. Conclusion

This project investigated the influence of temperature dependent specific heat on Brayton cycle performance using a MATLAB numerical model. Two approaches were compared: the conventional constant specific heat analysis and a variable specific heat model based on NASA polynomial thermodynamic data.

The results showed that the constant specific heat assumption over-predicts both thermal efficiency and net work output, especially at high turbine inlet temperatures. When temperature dependent properties were included, the predicted compressor work increased, turbine expansion effectiveness decreased, and the back work ratio became higher. In addition, the optimum pressure ratio shifted from the value predicted by the constant specific heat model. These findings demonstrate that simplified Brayton cycle analysis can lead to unrealistic performance predictions for modern high temperature gas turbines.

Therefore, temperature dependent thermodynamic modelling is necessary for accurate preliminary gas turbine performance evaluation. The developed numerical model provides a more realistic representation of engine behaviour and can serve as a useful preliminary step before detailed turbomachinery design or computational fluid dynamics analysis.

This project provided a strong foundation in numerical thermodynamic modelling and gas turbine performance analysis. It also encouraged further interest in higher fidelity propulsion modelling methods, such as mean-line turbomachinery analysis and computational fluid dynamics, which are important for modern aircraft engine development.

11. References

Çengel, Y.A. and Boles, M.A. (2019) Thermodynamics: An Engineering Approach. 9th edn. New York: McGraw-Hill Education.

Mattingly, J.D. (2006) Elements of Propulsion: Gas Turbines and Rockets. Reston, VA: AIAA.
Saravanamuttoo, H.I.H., Rogers, G.F.C., Cohen, H. and Straznicky, P. (2017) Gas Turbine Theory. 7th edn. Harlow: Pearson Education.

McBride, B.J., Gordon, S. and Reno, M.A. (2002) NASA Thermodynamic Polynomials and Coefficients for Calculating Thermodynamic Properties of Individual Species. NASA Technical Paper 2002-211556.

Ganesan, V. (2010). Gas Turbines (4th Edition). Tata McGraw-Hill Education, New Delhi.