Here I will derive the **Fast Fourier Transform** from the Discrete Fourier transform equation given by wikipedia using the same symmetry exploited by the **CooleyTukey** algorithm.

Lets begin...
Start with the general Fourier transform equation...

$$X_k = \sum_{n=0}^{N-1} x_n(cos(-2\pi k\frac{n}{N}) + isin(-2\pi k\frac{n}{N})), k \in \mathbb{Z}$$

We will apply euler's identity $e^{ix} = cos(x) + isin(x)$

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi k\frac{n}{N}}, k \in \mathbb{Z}$$

This form makes the equation simpler for optimizatin shown below

$$\begin{aligned}
X_k &= \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi k\frac{n}{N}} \\
&= \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i2\pi k\frac{2m}{N}} + \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i2\pi k\frac{2m+1}{N}} \\
&= (\sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i2\pi k\frac{2m}{N}}) + e^{-i2\pi k\frac{1}{N}} \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i2\pi k\frac{2m}{N}}
\end{aligned}$$