1 Basics (definitions)

Evaluate the following

(a) $\log_{10} 1000$

(b) $\log_4 1$

(c) $\log_2 \frac{1}{4}$

(d) $\log_a a^x$

Solve for x

(a) $\log_4 x = 2$

(b) $\log_{\frac{1}{2}} x = 2$

(c) $\log_{10}(2x+1) = 2$

(d) $\log_2 64 = x$

(e) $\log_x 81 = 4$

2 Using logarithm rules

(a) Solve $\log_3 x = \log_3 7 + \log_3 3$

(b) Find $2\log_{10} 5 + \log_{10} 8 - \log_{10} 2$

(c) If $x = \log_{10} 8$ and $y = \log_{10} 3$,
express the following in terms of x and y without using logs:

- $\log_{10} 24$
- $\log_{10} \frac{9}{8}$
- $\log_{10} 720$
- $\log_3 8$
- $\log_9 4$

3 Algebraic expressions

Simplify the following into a single log

(a) $\log_2 xy - 2\log_2 x$

(b) $\log_2 \frac{8x^2}{y} + \log_2 2xy$

(c) $\log_3 9xy^2 - \log_3 xy - 3$

(d) $\log_3 9x^4 - 2\log_3 3x$

4 Logarithmic equations

Solve for x

(a) $2\log_b 4 + \log_b 5 - \log_b 10 = \log_b x$

(b) $\log_b 8 + \log_b x^2 = \log_b x$

(c) $\log_b (x+2) - \log_b 4 = \log_b 3x$

(d) $\log_b (x-1) + \log_b 3 = \log_b x$

5 Harder logarithmic equations

Solve for x

(a) $\log_2 (x^2 - 2x) = 3$

(b) $\frac{\log_b x}{\log_b (5x-3)} = 1$

(c) $\frac{2+\log_b x}{3-\log_b x} = 5$

(d) $\log_b (x^3 - 1) - \log_b (x-1) = \log_b \frac{5}{4}$

(e) $\log_2(\log_3(\log_4 x)) = 0$

6 Level 7-8 questions (probably)

(a) Solve for x

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = 1$$

(b) A circle has radius of $\log_{10} a^2$ and a circumference of $\log_{10} b^4$. What is $\log_a b$?

(c) Evaluate

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \cdots + \frac{1}{\log_{100} 100!}$$

7 Formulas (Important)

- $b^{\log_b a} = a$ (Definition)
- $\log_b b^a = a$ (Definition)
- $\log_b 1 = 0$ (Definition)
- $\log_a bc = \log_a b + \log_a c$ (Multiplication)
- $\log_a \frac{b}{c} = \log_a b - \log_a c$ (Division)
- $\log_a b^c = c\log_a b$ (Exponent in argument)
- $\log_{a^c} b = \frac{1}{c}\log_a b$ (Exponent in base)
- $\log_a b = \frac{1}{\log_b a}$ (Inverse/Reciprocal)
- $\log_a b = \frac{\log_c b}{\log_c a}$ (Change base)
- $\log_{a^b} a^c = \frac{c}{b}$ (Simplify)
- $\log_a b \log_b c = \log_a c$ (Cancellation)