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Construct a 95% confidence interval for  $\lambda$ .

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Step One: Choose a statistic.

Choose one whose distribution you know and is one that depends on the unknown parameter.

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

- $\sum_{i=1}^n X_i \sim \Gamma(n, \lambda)$

- $X \sim \Gamma(\alpha, \beta) \quad c > 0$

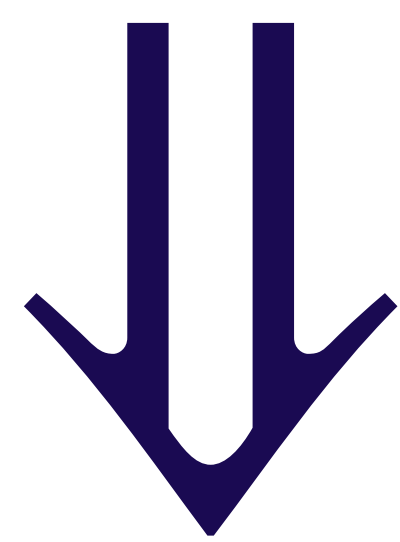
$$\Rightarrow cX \sim \Gamma(\alpha, \beta/c)$$

- So,

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim \Gamma(n, n\lambda)$$

Step Two: Find a function of the statistic and the parameter you are trying to estimate whose distribution is known and parameter free.

$$\bar{X} \sim \Gamma(n, n\lambda)$$



$$\lambda \bar{X} \sim \Gamma(n, n)$$

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**Question: Can we find critical values for the  $\Gamma(n, n)$  distribution?**

**Answer: We can use R.**

`qgamma(0.975,n,n)` and `qgamma(0.025,n,n)`

**Historically, people used tables.**

**Transform gamma into chi-squared.**

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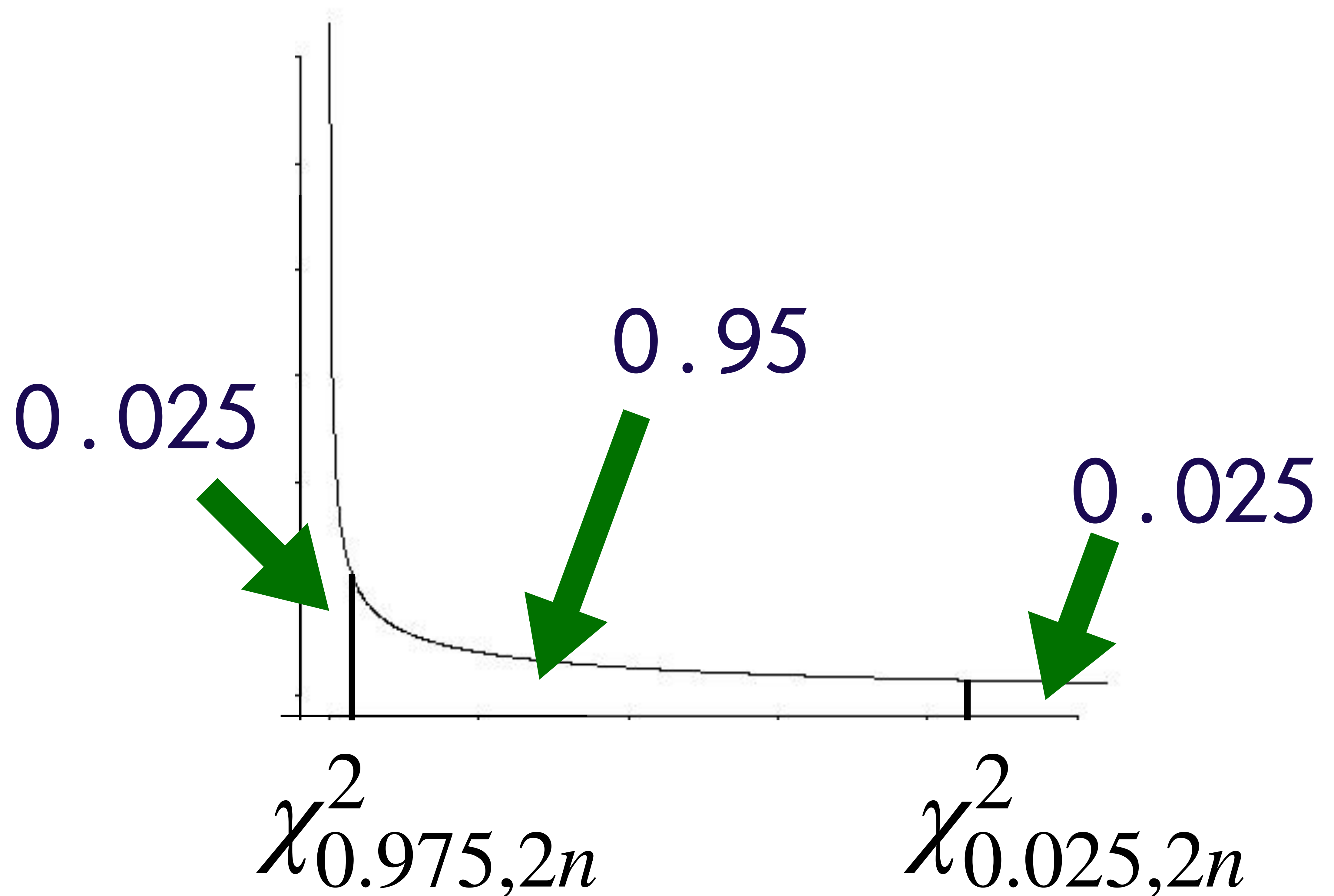
$$\bar{X} \sim \Gamma(n, n\lambda) \Rightarrow \lambda\bar{X} \sim \Gamma(n, n)$$

$$\Rightarrow 2n\lambda\bar{X} \sim \Gamma\left(n, \frac{1}{2}\right)$$

$$= \Gamma\left(\frac{2n}{2}, \frac{1}{2}\right) = \chi^2(2n)$$



Step Three: Find appropriate critical values.



Step Four: Put your statistic from Step Two between the critical values and solve for the unknown parameter “in the middle”.

$$2n\lambda\bar{X} \sim \chi^2(2n)$$

$$\chi_{0.975,2n}^2 < 2n\lambda\bar{X} < \chi_{0.025,2n}^2$$

$$\frac{\chi_{0.975,2n}^2}{2n\bar{X}} < \lambda < \frac{\chi_{0.025,2n}^2}{2n\bar{X}}$$