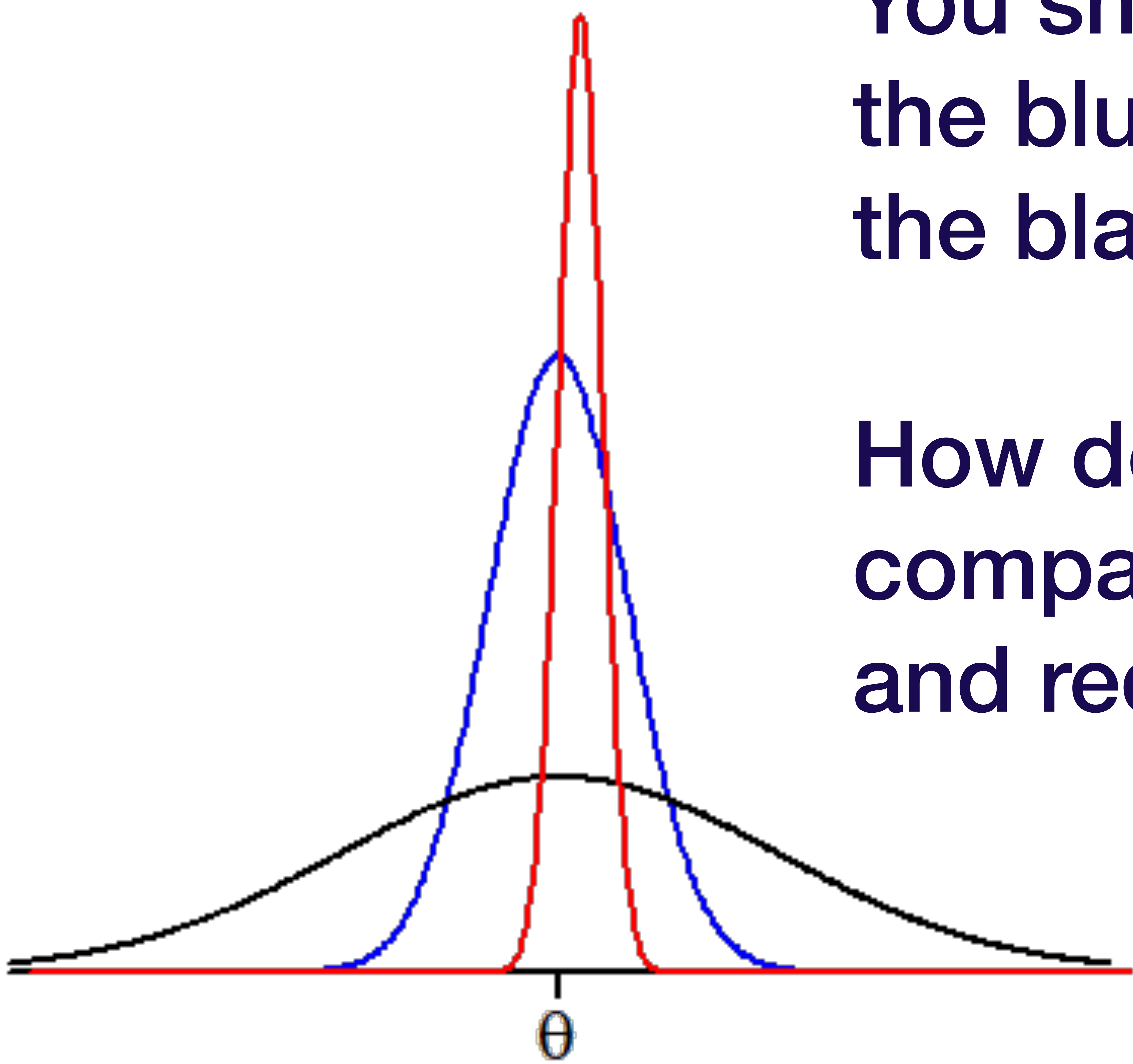
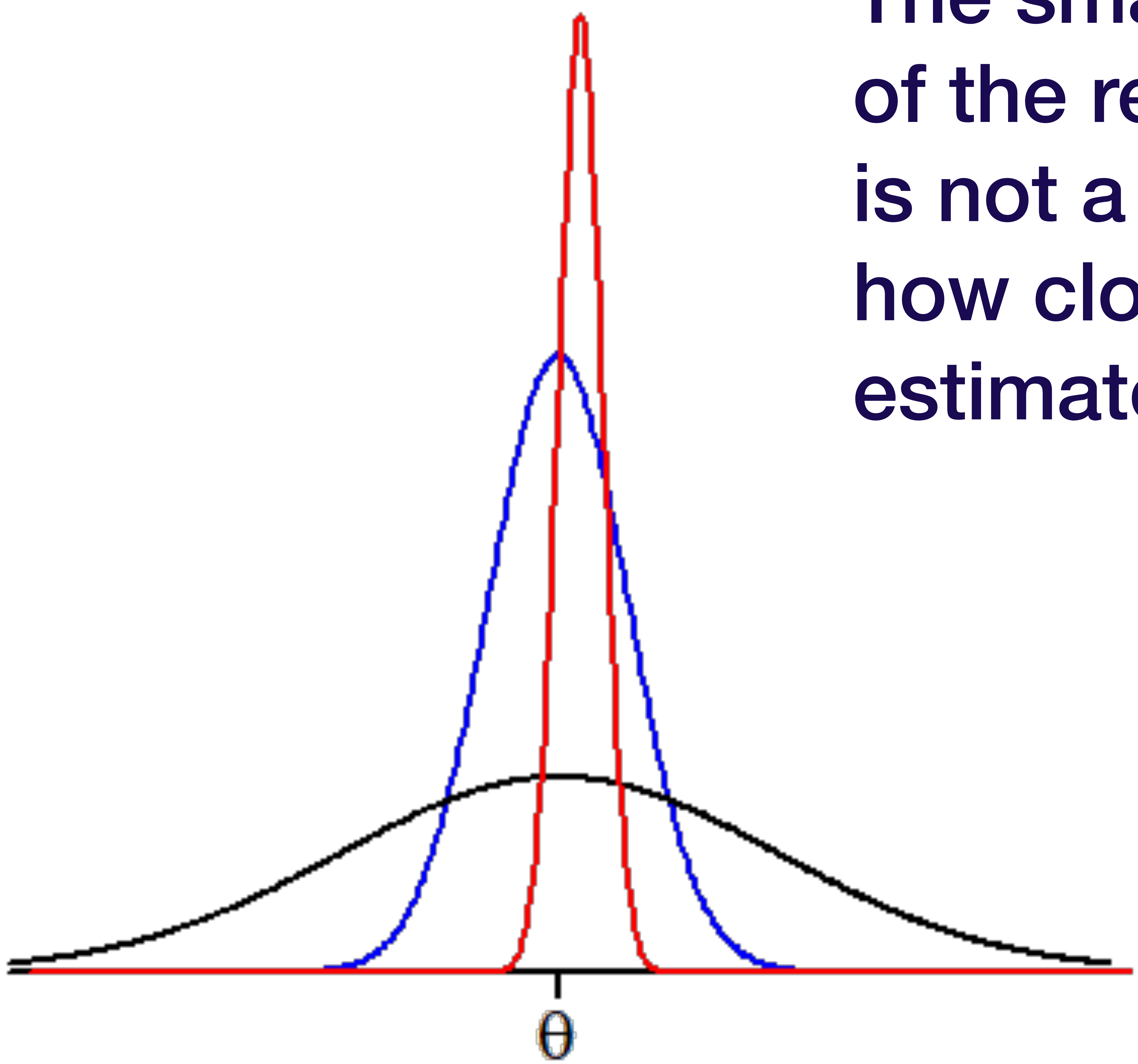
Three estimators of θ



You should prefer the blue curve over the black curve.

How do we compare the blue and red curves?

Three estimators of θ



The smaller variance of the red distribution is not a measure of how close the estimator is to θ.

Definition

Let $\hat{\theta}$ be an estimator of a parameter θ.

The mean squared error of $\hat{\theta}$ is denoted and defined by

$$\mathrm{MSE}(\hat{\theta}) = E[(\underbrace{\overset{\text{r.v.}}{\hat{\theta}} - \overset{\text{its mean}}{\theta}}_{\text{error}})^2]$$

Note: If $\hat{\theta}$ is an unbiased estimator of θ, its mean squared error is simply the variance of θ.

Definition

Let $\hat{\theta}$ be an estimator of a parameter θ.

The bias of $\hat{\theta}$ is denoted and defined by

$$B(\hat{\theta}) = E[\hat{\theta}] - \theta$$

An unbiased estimator has a bias of zero.

Variance, MSE, and Bias

Let $\hat{\theta}$ be an estimator of a parameter θ.

$$\mathrm{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$$

$$= E[(\hat{\theta} - E[\hat{\theta}] + E[\hat{\theta}] - \theta)^2]$$

$$= E[\left((\hat{\theta} - E[\hat{\theta}]) + B[\hat{\theta}]\right)^2]$$

- $E[(\hat{\theta} - E[\hat{\theta}])^2] = \mathrm{Var}[\hat{\theta}]$

Variance, MSE, and Bias

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- $2E\left[(\hat{\theta} - E[\hat{\theta}])\underbrace{B[\hat{\theta}]}\right]$

$$E[\hat{\theta}] - E[E[\hat{\theta}]] = E[\hat{\theta}] - E[\hat{\theta}] = 0$$

Variance, MSE, and Bias

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$$= E[(\hat{\theta} - E[\hat{\theta}] + E[\hat{\theta}] - \theta)^2]$$

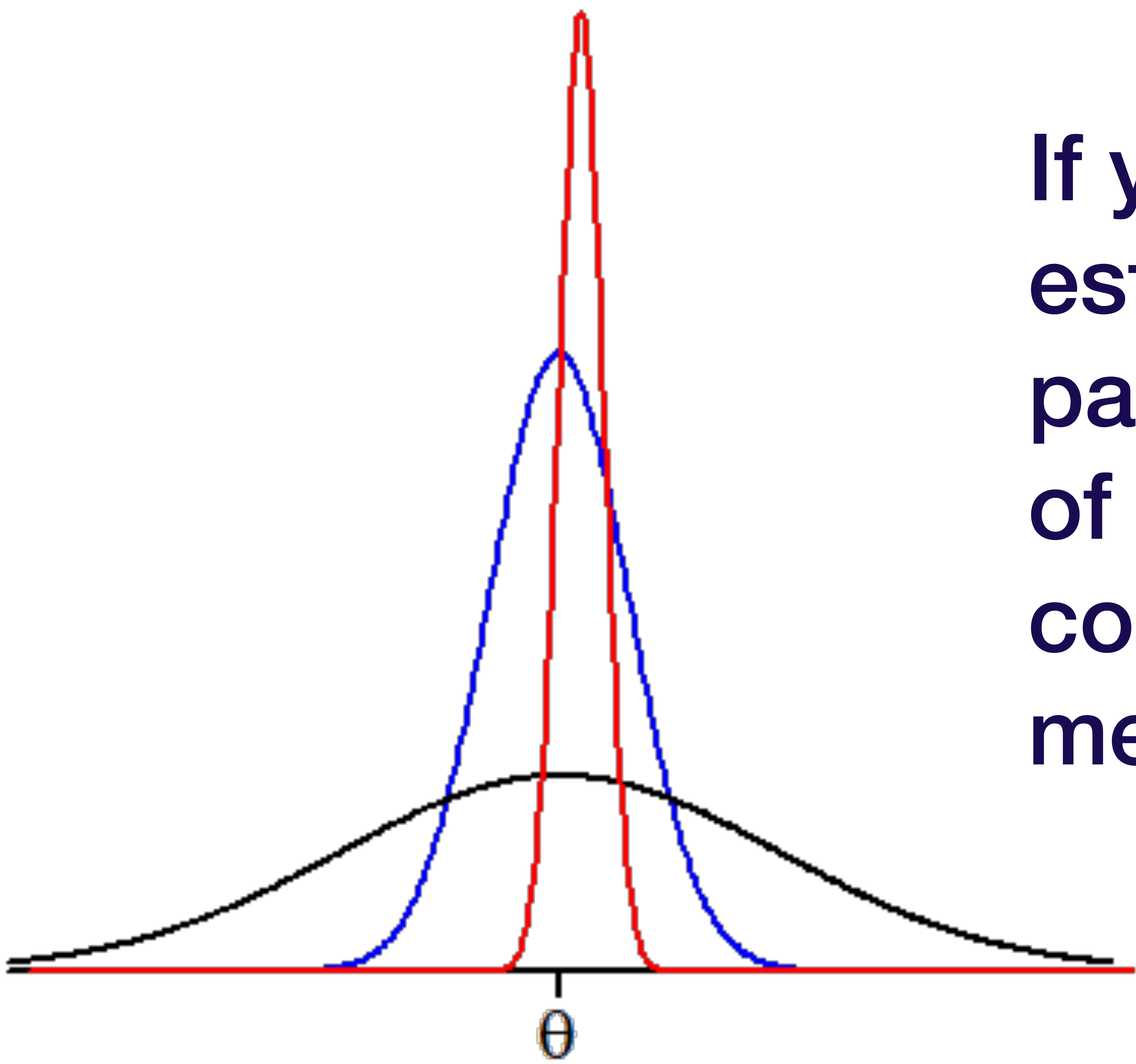
$$= E\left[\left((\hat{\theta} - E[\hat{\theta}]) + B[\hat{\theta}]\right)^2\right]$$

- $E[(B[\hat{\theta}])^2] = (B[\hat{\theta}])^2$

Variance, MSE, and Bias

Let $\hat{\theta}$ be an estimator of a parameter θ.

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}[\hat{\theta}] + (B[\hat{\theta}])^2$$



If you have multiple estimators of a parameter, at least one of which is biased, compare them using mean squared error!

Relative Efficiency

Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two unbiased estimators of a parameter θ.

$\hat{\theta}_1$ is more efficient than $\hat{\theta}_2$ if

$$\mathrm{Var}[\hat{\theta}_1] < \mathrm{Var}[\hat{\theta}_2]$$

The relative efficiency of $\hat{\theta}_1$, relative to $\hat{\theta}_2$ is denoted/defined as

$$\mathrm{Eff}(\hat{\theta}_1, \hat{\theta}_2) = \frac{\mathrm{Var}[\hat{\theta}_2]}{\mathrm{Var}[\hat{\theta}_1]}$$

Relative Efficiency

The relative efficiency of $\hat{\theta}_1$, relative to $\hat{\theta}_2$ is denoted/defined as

$$\mathrm{Eff}(\hat{\theta}_1, \hat{\theta}_2) = \frac{\mathrm{Var}[\hat{\theta}_2]}{\mathrm{Var}[\hat{\theta}_1]}$$

- $\mathrm{Eff}(\hat{\theta}_1, \hat{\theta}_2) > 1$

  $\Rightarrow$ $\hat{\theta}_1$ is more "efficient" at estimating θ than $\hat{\theta}_2$