Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean $\mu$ and known variance $\sigma^2$.

Consider testing the simple versus simple hypotheses

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

$$H_0 : \mu = 3 \qquad H_1 : \mu = 5$$

## Definition/Notation:

Let $\alpha$ = P(Type I Error)

= P(Reject $H_0$ when it's true)

= P(Reject $H_0$ when $\mu = 3$)

$\alpha$ is called the level of significance of the test.

It is also sometimes referred to as the size of the test.

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

## Developing a Test

### Step One:

Choose an estimator for $\mu$.

$$\widehat{\mu} = \overline{X}$$

### Step Two:

Give the “form” of the test.

The form of the test:

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

- We are looking for evidence that $H_1$ is true.
- The $N(3, \sigma^2)$ distribution takes on values from $-\infty$ to $\infty$.
- $\overline{X} \sim N(\mu, \sigma^2/n) \Rightarrow \overline{X}$ also takes on values from $-\infty$ to $\infty$.

The form of the test:

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

- It is entirely possible that $\overline{X}$ is very large even if the mean of its distribution is 3.

- However, if $\overline{X}$ is very large, it will start to seem more likely that $\mu$ is larger than 3.

- Eventually, a population mean of 5 will seem more likely than a population mean of 3.

# Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

## Step One:

Choose an estimator for $\mu$.

$$\hat{\mu} = \overline{X}$$

## Step Two:

Give the "form" of the test.

$$\boxed{\text{Reject } H_0\text{, in favor of } H_1\text{, if } \overline{X} > c \text{ for some } c \text{ to be determined.}}$$

## Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

### Step Three:

Find c.

Reject $H_0$, in favor of $H_1$, if $\overline{X} > c$ .

- If c is too large, we are making it difficult to reject $H_0$.

We are more likely to fail to reject when it should be rejected.

Type II Error

## Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

### Step Three:

Find c.

Reject $H_0$, in favor of $H_1$, if $\overline{X} > c$ .

- If c is too small, we are making it to easy to reject $H_0$.

We are more likely reject when it should not be rejected.

Type I Error

## Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

### Step Three:

Find c.

Reject $H_0$, in favor of $H_1$, if $\overline{X} > c$ .

This is where $\alpha$ comes in.

$$\begin{aligned} \alpha &= \mathrm{P(Type\ I\ Error)} \\ &= \mathrm{P(Reject\ H_0\ when\ true)} \\ &= \mathrm{P}(\overline{X} > c \text{ when } \mu = 3) \end{aligned}$$

## Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

### Step Four:

Give a conclusion!

**Example:** $X_1, X_2, \ldots, X_{10} \overset{\text{iid}}{\sim} N(\mu, 4)$

Find a hypothesis test for

$$H_0 : \mu = 5 \qquad \text{vs} \qquad H_1 : \mu = 3$$

Use level of significance $\alpha = 0.05$.

Find a "test of size 0.05".

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

# Example:

## Step One:

Choose an estimator for $\mu$.

$$\widehat{\mu} = \overline{X}$$

## Step Two:

Give the "form" of the test.

Reject $H_0$, in favor of $H_1$, if $\overline{X} < c$ for some c to be determined.

## Example:

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

### Step Three:

Find c.

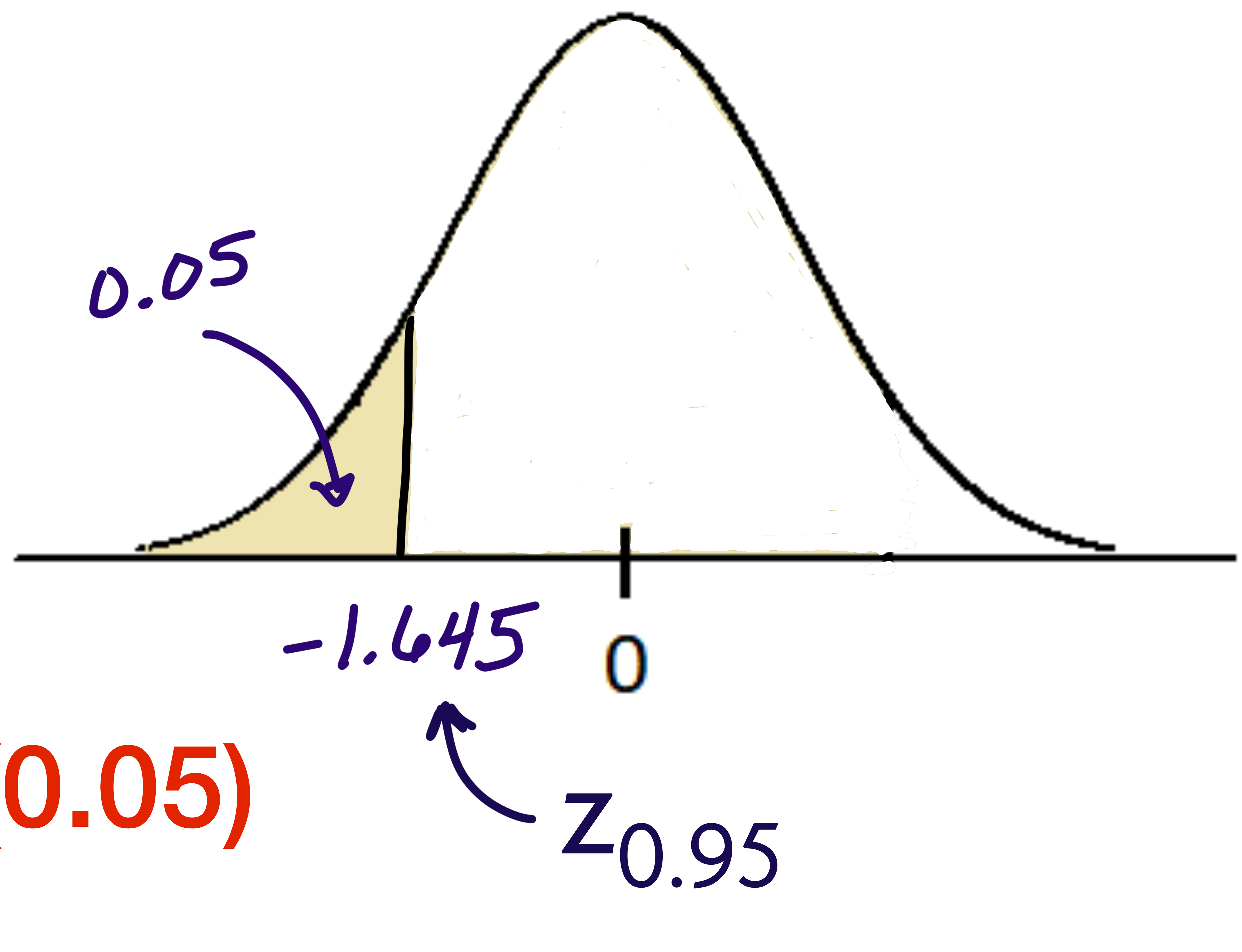
$$0.05 = P(\text{Type I Error})$$
$$= P(\text{Reject } H_0 \text{ when true})$$
$$= P(\overline{X} < c \text{ when } \mu = 5)$$
$$= P\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < \frac{c - 5}{2/\sqrt{10}} \text{ when } \mu = 5\right)$$

## Example:

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

## Step Three:

Find c. $$0.05 = P\left(Z < \frac{c-5}{2/\sqrt{10}}\right)$$




qnorm(0.05)

**Example:**

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

**Step Three:**

**Find c.**

$$0.05 = P\left(Z < \frac{c - 5}{2/\sqrt{10}}\right)$$

$$\Rightarrow \quad \frac{c - 5}{2/\sqrt{10}} = -1.645$$

$$\Rightarrow \quad c = 3.9596$$

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

**Example:**

**Step Four:**

Give a conclusion.

Reject $H_0$, in favor of $H_1$, if

$$\overline{X} < 3.9596$$