

# Tests for the mean of a normal distribution.



Let  $X_1, X_2, \dots, X_n$  be a random sample from the normal distribution with mean  $\mu$  and known variance  $\sigma^2$ .

Consider testing the simple versus simple hypotheses

$$H_0 : \mu = \mu_0 \quad H_1 : \mu = \mu_1$$

where  $\mu_0$  and  $\mu_1$  are fixed and known.

## Step One:

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

Choose an estimator for  $\mu$ .

$$\hat{\mu} = \bar{X}$$

---

## Step Two:

Give the “form” of the test.

Suppose that  $\mu_0 < \mu_1$ .

Reject  $H_0$ , in favor of  $H_1$  if  $\bar{X} > c$ ,  
where  $c$  is to be determined.

# Developing a Test

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

$$\mu_0 < \mu_1$$

## Step Three:

Find c.

$$\alpha = P(\text{Type I Error})$$

$$= P(\text{Reject } H_0 \text{ when true})$$

$$= P(\bar{X} > c \text{ when } \mu = \mu_0)$$

$$= P\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > \frac{c - \mu_0}{\sigma/\sqrt{n}} \text{ when } \mu = \mu_0\right)$$

# Developing a Test

## Step Three:

Find c.

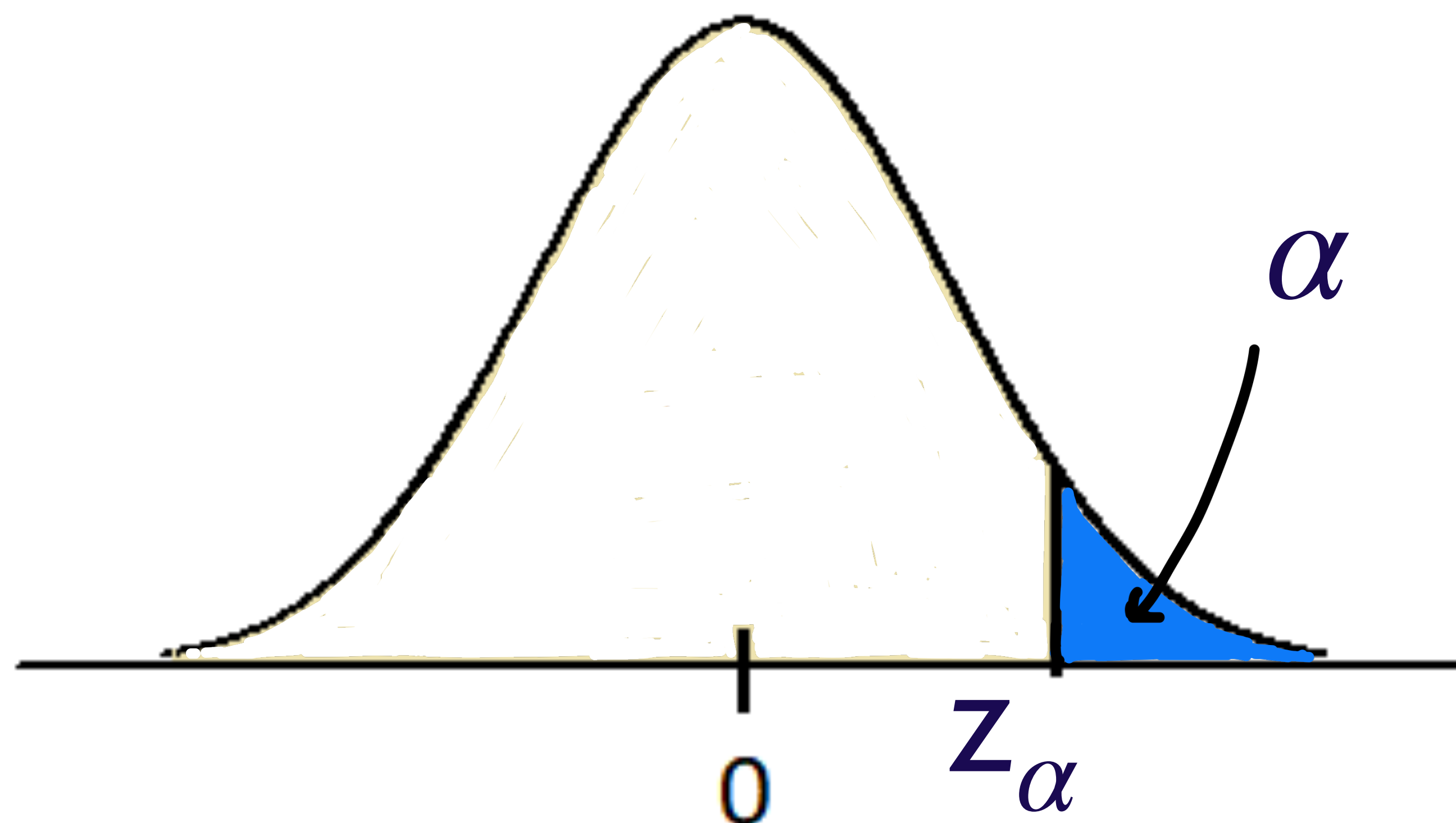
$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

$$\mu_0 < \mu_1$$

$$\alpha = P\left(Z > \frac{c - \mu_0}{\sigma/\sqrt{n}}\right)$$

where  
 $Z \sim N(0, 1)$



# Developing a Test

## Step Three:

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

$$\mu_0 < \mu_1$$

$$\Rightarrow \frac{c - \mu_0}{\sigma/\sqrt{n}} = z_\alpha$$

$$\Rightarrow c = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$$

## Developing a Test

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

$$\mu_0 < \mu_1$$

### Step Four:

Give a conclusion!

Reject  $H_0$ , in favor of  $H_1$  if

$$\bar{X} > \mu_0 + z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

# Developing a Test

## Step Four:

Give a conclusion!

Reject  $H_0$ , in favor of  $H_1$  if

$$\bar{X} < \mu_0 + z_{1-\alpha} \frac{\sigma}{\sqrt{n}}$$

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu = \mu_1$$

~~$$\mu_0 \leq \mu_1$$~~

$$\mu_0 > \mu_1$$



What ever happened to the Type II error? What is going to change if we look at composite hypotheses? What if we don't know  $\sigma^2$ ?

And what the heck is a P-value?

We'll answer all these questions and more in the next module!

