GCD-

B < 16 >
A < 8 >

A divides B completely < remainder = 0 >

then we say that A is the factor (or) divisor of B, and B is the multiple of A

GCD-

Greatest Common Divisor
HCF
(Highest Common Factor)

10, 15

↳ divisors of 10 are → 1, 2, 5, 10

↳ divisors of 15 are → 1, 3, 5, 15

GCD(10, 15) = 5

How to compute the GCD of A and B?

```
i = 1... A

   if((A % i) == 0 &&
      (B % i) == 0) {

          gcd = i;
   }
```

0 → 15
15 → 15

{ 0 % 15 = 0
  15 % 15 = 0 }

Is this efficient?

Optimized solution-

Euclidean Algorithm-

GCD of two numbers doesn't change if the smallest number is subtracted from the bigger number.

A =	24	8	8
B =	16	16	8

Solution-

```
int gcd(int a, int b) {
        // Everything divides 0
        if(a==0) {
                return b;
        }

        if(b==0) {
                return a;
        }

        // base case
        if(a==b) {
                return a;
        }

        if(a> b) {
            return gcd(a-b, b);
        }
    return gcd(a, b-a);
}
```

a	b
35,	14
14,	35
7,	14
0	7

a	b
20	30
10	20
0	10

a	b
12	18
6	12
6	6
0	6

→ Euclid said that if we subtract a smaller number from a bigger number, its gcd doesn't change

70, 8

62, 8

54, 8

46, 8

38, 8

30, 8

22, 8

14, 8

6, 8

70

⇒ 70 % 8 = 6

8 % 6

6, 8

6, 2

6 % 2

6, 2

4, 2

2, 2

0, 2

A more optimized algorithm with a modulo operation-

```
static int gcd ( int a, int b) {
        if(a == 0) {
            return b;
        }

        return gcd(b % a, a );
}
```

LCM-

Lowest Common Multiple

A=10
B=15

Multiples of A => 10, 20, $\boxed{30}$, 40, 50, . . .
Multiples of B => 15, $\boxed{30}$, 45, 60, 75, . . .

LCM(10,15) = 30

** $\boxed{LCM(A,B) \times HCF(A,B) = A \times B}$

① If it is true,

$$LCM(A,B) = \frac{A \times B}{HCF(A,B)}$$

Relation between GCD and LCM

Numbers–

$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$

$LCM(20, 30) = 2 \times 5 \times 2 \times 3$

$HCF(20, 30) = 2 \times 5$

$$2\times5\times2\times3 \quad 2\times5 \quad = \quad \underset{2\times2\times5 \qquad 2\times3\times5}{20\times30}$$

2(3)		2(3)
3(1)	$\Longleftrightarrow$	3(1)
5(2)		5(2)

$$\boxed{LCM(20,30) \times HCF(20,30) = 20\times30}$$

Fundamental theorem of arithmetic–

- ~~Unique factorization theorem~~
- Unique prime factorization theorem

- Every integer greater than 1 is a prime number (or) can be represented as a product of prime numbers.
  <Can be many prime numbers>

$43 \rightarrow$ Prime $= 43$

$36 \rightarrow$ Not a prime $= 2\times 18$

$18 \rightarrow 2\times 9$

$9 \rightarrow 3\times3$

$= 2 \times 2 \times 3 \times 3$

$LCM(A,B) \times HCF(A,B) = A \times B$

$\Rightarrow \quad A = P_1 \times P2 \times C_1 \times C_2 \times P_3$ Prime numbers

$B = q_1 \times q_2 \times C_1 \times C_2 \times q_3$

$LCM(A,B) = C_1 \times C_2 \times P_1 \times P_2 \times P_3 \times q_1 \times q_2 \times q_3$

$HCF(A,B) = C_1 \times C_2$

$LCM(A,B) \cancel{\times} HCF(A,B)$	$A \times B$
$1(C_1)$	$1(P_1)$
$1(C_2)$	$1(P_2)$
$1(P_1)$	$1(C_1)$
$1(P_2)$	$1(C_2)$
$1(P_3)$	$1(P_3)$
$1(q_1)$	
$1(q_2)$	$1(q_1)$
$1(q_3)$	$1(q_2)$
	$1(C_1)$
$1(C_1)$	$1(C_2)$
$1(C_2)$	$1(q_3)$

```
int LCM( int a, int b) {

        return (a*b) / gcd(a,b);

}
```