A. DSP

Convolution: $y[n] = x[n] * h[n] = \sum_m x[m]\,h[n-m]$

Cross-correlation: $y[n] = x[n] \otimes h[n] = \sum_m x[m]\,h[n+m]$

Periodic Signals: $x[n] = (1/N)\sum_{k=0}^{N-1} X[k] \sin(2\pi \frac{k}{N} n + \phi[k])$

Fourier Transforms:

$x(t)$ is CP, $x(t) = \sum_k X[k] e^{jk\omega_0 t}$

X_k is DA, $X_k = T^{-1}\int x(t) e^{-jk\omega_0 t} dt$

$x(t)$ is CA, $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

$X(j\omega)$ is CA, $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

DFT: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$, $n = 0, 1, \ldots, N-1$

$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$, $k = 0, 1, \ldots, N-1$

If $x[n]$ is real, $X[-k] = X^*[k]$

STFT: $X[k,m] = \sum_{n=0}^{N-1} x[n] w[n-mH] e^{-j\frac{2\pi}{N}kn}$; $k = 0, 1, \ldots, N-1$

Sampling: $F_s = \frac{1}{T_s}$, Nyquist rate $= 2F_0$, $F_{max} = \frac{F_s}{2}$

Euclidean distance, $d(x_1, x_2) = |x_1 - x_2| = \sqrt{\sum_n (x_1[n] - x_2[n])^2}$

Cosine similarity, $d(x_1, x_2) = x_1^T . x_2 / (|x_1||x_2|) = \cos\theta$

KL divergence, $d(p_1(x), p_2(x)) = -\sum_x p_1(x) \log(p_2(x)/p_1(x))$

B. MACHINE LEARNING

$\sigma(h) = 1/(1+e^{-h})$

Confusion Matrix

		Predicted N	Predicted P
Actual	N	TN	FP
	P	FN	TP

Accuracy $= (TP+TN)/All$

$P = TP/(TP+FP)$

$R = TP/(TP+FN)$

$F = 2PR/(P+R)$

Linear regression: $y = w_0 + w_1 x + w_2 x^2 + \ldots + w_D x^D$, $y = \phi w$

→ Least squares solution: $w = (\phi^T\phi)^{-1}\phi^T t$

→ L2 regularisation: $w = (\lambda I + \phi^T\phi)^{-1}\phi^T t$

→ L2 reg.-min-norm: $w = \phi^T(\phi\phi^T)^{-1} t$

C. DEEP LEARNING

Non-linear model: $y = w_2\sigma(w_1 x)$

Activation functions: $\sigma(x) = \frac{1}{1+e^{-x}}$; $\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$; ReLU $g(x) = \max(0, x)$

$y = w^T x + w_0$, Dist.-from origin $= \hat{w}.\vec{x}$, Dist. from line $= \frac{y(\vec{x})}{\|\vec{w}\|}$

Softmax (h), $y_c = e^{h_c}/[\sum_{c'} e^{h_{c'}}]$

Categorical cross entropy: $E_{xent} = -\sum_c t_c \log(y_c)$

Binary cross entropy: $E_{bin} = -\sum_i (t_i \log y_i + (1-t_i)\log(1-y_i))$

Shifting: $x_{i,s} \leftarrow x_{i,s} - \mu_i$. Scaling: $x_{i,s} \leftarrow x_{i,s}/\sigma_i$

D. PROBABILITY THEORY

Marginalization: $P(w) = \sum_t P(t, w)$

Product rule: $P(w,t) = P(w|t)P(t)$

Bayes theorem: $P(t|w) = \frac{P(w|t)P(t)}{P(w)}$

Independence: $P(w|t) = P(w)$

$P(w,t) = P(w).P(t)$

Transformation: $p_y(y) = \sum_i [p_x(x)/|\frac{dy}{dx}|]_{x_i}$

Expectation: $E(f(x)) = \int f(x)p(x)d(x)$

$V(f(x)) = E[(f(x) - E(f(x)))^2]$

Conditional: $E_x[f(x)|y] = \int f(x)p(x|y)dx$

$Cov[x,y] = E[(x - E(x))(y^T - E(y^T))]$

Correlation: $\rho_{x,y} = corr(x,y) = \frac{cov(x,y)}{\sigma_x\sigma_y}$

$N(x|\mu,\sigma^2) = (2\pi\sigma^2)^{-1/2} \exp\{\frac{-1}{2\sigma^2}(x-\mu)^2\}$

$\hat{\mu}_{ML} = \frac{1}{N}\sum_{i=1}^N S_i$, $\hat{\sigma}^2_{ML} = \frac{1}{N}\sum_{i=1}^N (S_i - \hat{\mu}_{ML})^2$

$N(x|\mu,\Sigma) = \frac{1}{(2\pi)^{D/2}} \cdot \frac{1}{|\Sigma|^{1/2}} \cdot \exp\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\}$

$\mu_{ML} = \frac{1}{N}\sum_{i=1}^N S_i$, $\Sigma_{ML} = \frac{1}{N}\sum_{i=1}^N (S_i - \mu_{ML})(S_i - \mu_{ML})^T$

E. GMM and EM

$p(x) = \sum_k p(Z_k) N(x; \mu_k, \sigma_k)$; $Z_k \in \{0,1\}$, $\sum_k Z_k = 1$

$\ln \mathcal{L} = \sum_n \ln[\sum_k \pi_k N(S_n|\mu_k, \Sigma_k)]$

$\mu_k = \frac{\sum_n \gamma_{nk} S_n}{N_k}$, $\Sigma_k = \frac{\sum_n \gamma_{nk}(S_n - \mu_k)(S_n - \mu_k)^T}{N_k}$, $\pi_k = \frac{N_k}{N}$

Entropy: $H(p(x)) = -\sum_x p(x)\log(p(x))$

Jensen's inequality: $f(\sum_i \alpha_i x_i) \le \sum_i \alpha_i f(x_i)$, $f'' \ge 0$

$f(\sum_i \alpha_i x_i) \ge \sum_i \alpha_i f(x_i)$, $f'' \le 0$

Mutual info. b/w r.v.: $I(X;Y) = H(X) + H(Y) - H(X,Y)$

Auxiliary loss $\mathcal{L}(q(Z), \theta) = \sum_Z q(Z)\log\frac{p(X|Z|\theta)}{q(Z)}$

Gap $= \log p(X|\theta) - \mathcal{L}(q(Z),\theta) = KL[q(Z)||p(Z|X,\theta)]$

E-step: $q \leftarrow \arg\min_q KL[q(Z)||p(Z|X,\theta)]$

M-step: $\theta \leftarrow \arg\max_\theta E_q[\log p(X, Z|\theta)]$

F. PCA

$x = \sum_{i=1}^D (x^T u_i)u_i$, $\tilde{x} = \sum_{i=1}^M (x^T u_i)u_i$, $x \in \mathbb{R}^D$

$MSE = \frac{1}{N}\sum_{n=1}^N \sum_{i=M+1}^D (x_n^T u_i)^2 = \sum_{i=M+1}^D u_i^T S u_i = \sum_{i=M+1}^D \lambda_i$

Normalization: $SU = UL$, $y_n = L^{-1/2} U x_n$

Matrix factorization: $Y = U^T X$ (−, +, 0 values)

G. NMF

$E = |V - WH|^2 = \sum_{i,j} |V_{ij} - \sum_r W_{ir} H_{rj}|^2$, $W_{ir} \ge 0$, $H_{rj} \ge 0\ \forall i,j$

$W_{ir} \leftarrow W_{ir}\frac{(VH^T)_{ir}}{(WHH^T)_{ir}}$ and $H_{rj} \leftarrow H_{rj}\frac{(W^T V)_{rj}}{(W^T W H)_{rj}}$

Reconstruct pitch separately, $V \approx w_1 h_1^T + w_2 h_2^T$, $|\hat{X}_s| = w_1 h_1^T$

Reconstruction with masking, $M_s = \frac{w_1 h_1^T}{\sum_{i=1}^k w_i h_i^T}$, $|\hat{X}_s| = M_s \odot |X|$

H. PLCA

$V_{ft} \sim P_t(f) = \sum_z P(f|z) P_t(z)$

$\log \mathcal{L} = \sum_{f,t} V_{ft} \log[\sum_z P_t(z) P(f|z)]$

$\mathcal{L}(q,\theta) = \sum_{f,t} V_{ft} \sum_z q_t(z|f) \log\left[\frac{P_t(z)P(f|z)}{q_t(z|f)}\right]$

E step: $q_t(z|f) = P_t(z)P(f|z) / \sum_z P_t(z)P(f|z)$

M step: $P_t(z) = \sum_f V_{ft} q_t(z|f) / \sum_z \sum_f V_{ft} q_t(z|f)$

$P(f|z) = \sum_t V_{ft} q_t(z|f) / \sum_f \sum_t V_{ft} q_t(z|f)$