Discrete Mathematics and Algorithms (MA3201)

Assignment Set 2

1. Using the principle of mathematical induction, show that for any integer $n > 1$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \quad > \quad \sqrt{n}$$

2. Let $\alpha = \frac{1+\sqrt{5}}{2}$. Using mathematical induction, prove that $F_n < \alpha^{n-1}$, where $n \geq 3$. Here F_n is the $n-th$ Fibonacci number defined by the recurrence relation: $F_{k+1} = F_k + F_{k-1}$, with $F_1 = F_2 = 1$.

3. Show that $(x^n - 1)$ is divisible by $(x - 1)$ for a positive integers $n \geq 2$ using the principle of mathematical induction.

4. Given a set of sixteen natural numbers, none having a prime factor > 7. Using the pigeonhole principle, show that either some number is a perfect square or, the product of some two distinct numbers is a perfect square.

   [Hint: Use the fundamental theorem of arithmetic. Any number n (> 1) can be expressed uniquely as $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$, where $p_1, p_2, \ldots, p_k$ are distinct prime factors and $k_i \geq 0$ be an integer.]

5. Prove that among 100,000 people there are two who were born in exactly the same time (hour, minute and second).

6. Find $(1\,3\,2) \cdot (1\,7\,4\,2\,3\,8\,6) \cdot (1\,3\,2)$.

7. Let f, g be two permutations on the set $\{1, 2, 3, 4, 5, 6\}$. Find fg, gf, f^{-1}, g^{-1} when

   (i) $f =$ (1 2 4 5 6), $g =$ (2 6 3 4 5).

   (ii) $f =$ (2 3 5 4), $g =$ (4 5 3 2).

8. Show that, if p is an arbitrary permutation and q is the cycle (1 2 $\cdots$ i), then the permutation $q^{-1}pq$ has the same cycle structure as p.