NAME: UDAY GARG. ROLL NO: 102103450 CLASS - CO16. (1)

Assign- Parameter Estimation

1) Mean $\to \theta_1$ Variance $\to \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

take log.

$$\log L(\theta_1, \theta_2) = -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

for θ_1 diff $\log(L(\theta_1, \theta_2))$ w.r.t θ_1 & set it to zero.

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0.$$

$$\boxed{\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

M.L.E of θ_1 is simple mean.

for θ_2 diff w.r.t θ_2 & put zero.

$$\boxed{\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

2) $B(m, \theta)$ Binomial distribution.

$m \to$ no. of trails.

$\theta = (0, 1)$ prob. of success

$$L\theta = \prod_{i=1}^{n} f(x_i; n, \theta).$$

PMF,

$$f(x; n, \theta) = {}^{n}C_x \cdot \theta^x \cdot (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x_i}\right) \cdot \left(\theta^{x_i}\right) \cdot (1-\theta)^{m-x_i}$$

take log.

$$\log(L(\theta)) = \sum_{i=1}^{n} \log\left({}^{m}C_{x_i}\right) + \sum_{i=1}^{n} x_i \log(\theta) + \sum_{i=1}^{n} (m - x_i)\log(1-\theta)$$

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{m} (m - x_i).$$

multiply both sides by $\theta(1-\theta)$

$$(1-\theta) \sum_{i=1}^{m} x_i = \theta \sum_{i=1}^{m} (m - x_i)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$

M.L.E of θ for $B(n, \theta)$ is $\bar{x}$ where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$