Revisão entre Fravano e Camargo: 2021

(1) $\Rightarrow$ $\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{v})$, $\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla} p}{\rho}$

(2) $\Rightarrow$ $v_S \equiv \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{dp_1}{d\rho_1}}$

$\rho \rightarrow \rho_0 + \rho_1$ $\quad$ $\vec{v} \rightarrow \vec{v_0} + \vec{v_1}$ $\quad$ $p \rightarrow p_0 + p_1$

e (1) fica $\frac{\partial \rho_1}{\partial t} = -\rho_0 \vec{\nabla}\cdot\vec{v_1}$, $\frac{\partial \vec{v_1}}{\partial t} = -\frac{\vec{\nabla} p_1}{\rho_0}$ ... (3)

(2) $\Rightarrow$ $v_S^2 = \frac{dp_1}{d\rho_1} \Rightarrow dp_1 = v_S^2 \, d\rho_1$

$\frac{dp_1}{dx_i} = v_S^2 \frac{d\rho_1}{dx_i}$

$\frac{dp_1}{dx_i}\vec{e_i} = v_S^2 \frac{d\rho_1}{dx_i}\vec{e_i}$

$\sum_i \frac{dp_1}{dx_i}\vec{e_i} = v_S^2 \cdot \underbrace{\sum_i \frac{d\rho_1}{dx_i}}$

$\boxed{\vec{\nabla} p_1 = v_S^2 \cdot \vec{\nabla}\rho_1}$ (4)

~~o~~ tomando grad (3) em $\frac{\partial \rho_1}{\partial t} = -\rho_0 \vec{\nabla} \cdot \vec{v}$

tem-se: $\vec{\nabla}(3) \Rightarrow \frac{\partial}{\partial t} \vec{\nabla} \rho_1 = -\rho_0 \vec{\nabla}(\vec{\nabla} \cdot \vec{v_1}) \quad \dots (*)$

$$\Downarrow (4)$$

$$\frac{1}{v_s^2} \vec{\nabla} \rho_1$$

$$\Downarrow (3): \vec{\nabla} \rho_1 = -\rho_0 \frac{\partial \vec{v}}{\partial t}$$

$$(*) \Rightarrow \frac{\partial}{\partial t}\left(-\frac{\rho_0}{v_s^2} \frac{\partial \vec{v_1}}{\partial t}\right) = -\rho_0 \vec{\nabla}(\vec{\nabla} \cdot \vec{v_1})$$

$$\frac{1}{v_s^2} \frac{\partial^2 \vec{v_1}}{\partial t^2} = \vec{\nabla}(\vec{\nabla} \cdot \vec{v_1})$$

$$\boxed{\frac{\partial^2 \vec{v_1}}{\partial t^2} = v_s^2 \vec{\nabla}(\vec{\nabla} \cdot \vec{v_1})} \quad \dots (5)$$

Devido ao teorema rot(grad) = 0 podemos

dizer que $\exists\,\Phi : \boxed{\vec{v}_1 = \vec{\nabla}\Phi}$ ... (8)

Retornando a equação (5):

$$\frac{\partial^2 \vec{v}_1}{\partial t^2} = v_S^2 \cdot \vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}_1\right)$$

$$\frac{\partial^2 \vec{\nabla}\Phi}{\partial t^2} = v_S^2 \cdot \vec{\nabla}\left(\underbrace{\vec{\nabla}\cdot\vec{\nabla}\Phi}_{\nabla^2\Phi}\right)$$

$$\vec{\nabla}\left(\frac{\partial^2\Phi}{\partial t^2}\right) = \vec{\nabla}\left(v_S^2 \cdot \nabla^2\Phi\right)$$

~~podemos supor Φ a~~

de onde temos $\frac{\partial^2\Phi}{\partial t^2} = v_S^2\nabla^2\Phi + \kappa(t)$,

podemos supor $\kappa(t) \equiv 0$, obtendo

$$\vec{\nabla} \times (5) \Rightarrow \frac{\partial^2}{\partial t^2}(\vec{\nabla} \times \vec{v}_1) = v_S^2 \underbrace{\vec{\nabla}_x \vec{\nabla}(}_{\text{rot/grad} = 0}$$

$$\frac{\partial^2}{\partial t^2}(\vec{\nabla} \times \vec{v}_1) = 0 \quad \dots (**)$$

Supondo $\vec{\nabla} \times \vec{v}_1 = 0$ pois a dependência em t podemos superharmonizar, considerando uma componente de Fourier por vez; com $\frac{\partial}{\partial t} \to i\omega$ a eq. (**) vai resultar

$$(i\omega)^2(\vec{\nabla} \times \vec{v}_1) = 0$$

e ~~devido~~ p/ $\forall\, \omega \neq 0$ temos então

$$\boxed{\vec{\nabla} \times \vec{v}_1 = 0} \quad \dots (7)$$

$$\boxed{\frac{\partial^2 \Phi}{\partial t^2} = v_s^2 \cdot \nabla^2 \Phi} \;\ldots\ldots (1)$$

Simetria axial $\Rightarrow \nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \Phi}{\partial r}\right)$

$$\Phi = Z(z)\,\phi(r)\,.\,T(t)$$

$$(1) \Rightarrow Z\phi \,.\, \ddot{T} = v_s^2 \,.\, T \,.\, \nabla^2(Z\phi)$$

$$\frac{1}{T}\,.\,\ddot{T} = \frac{v_s^2}{Z\phi}\,.\,\nabla^2(Z\phi) \equiv -\omega^2$$

$$\begin{cases} \ddot{T} + \omega^2 T = 0 \\ \nabla^2(Z\phi) + \underbrace{\frac{\omega^2}{v_s^2}}_{k_s^2} Z\phi = 0 \end{cases}$$

$$\begin{cases} T = T(0)\cdot e^{\pm i\omega t} \\ \nabla^2(Z\phi) + k^2 Z\phi = 0 \quad \dots (II) \end{cases}$$

$$\nabla^2() = \partial_z^2 + \frac{1}{r}\partial_r\left[r\,\partial_r()\right]$$

$$\nabla^2(Z\phi) = \phi\, Z'' + \frac{Z}{r}\frac{d}{dr}\left[r\frac{d\phi}{dr}\right]$$

$$\frac{(II)}{Z\phi} \Rightarrow \frac{1}{Z}Z'' + \frac{1}{r}\frac{1}{\phi}\frac{d}{dr}\left[r\frac{d\phi}{dr}\right] + k_s^2 = 0$$

$$-\frac{1}{Z}Z'' = \frac{1}{r}\frac{1}{\phi}\frac{d}{dr}\left[r\frac{d\phi}{dr}\right] + k_s^2 = k^2$$

avec k issu de (11)

$$\boxed{k = \frac{\omega^2}{v_{ph}^2}}$$

$$\begin{cases} Z'' + k^2 Z = 0 \Rightarrow \boxed{Z = Z(0)\cdot e^{\pm i k z}} \text{ conforme (11)} \\ \frac{d}{dr}\left[r\frac{d\phi}{dr}\right] + (k_s^2 - k^2)\, r\,\phi = 0 \quad \dots (II') \end{cases}$$

$$(\mathrm{II}') \Rightarrow r\frac{d^2\phi}{dr^2} + \frac{d\phi}{dr} + (k_s^2 - k^2) \cdot r\phi = 0$$

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + (k_s^2 - k^2) \cdot \phi = 0 \quad \ldots \quad (\mathrm{II}'')$$

k de grupo: $k_s = \frac{\omega}{v_s}$

k de fase: $k = \frac{\omega}{v_{ph}}$

Essa é a eq. de Bessel com $n=0$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right) \cdot y = 0$$

$$y = J_n(x) \text{ ou } Y_n(x)$$

O detalhe é que precisa transformar $(k_s^2 - k^2)$ em 1

$$\frac{(\mathrm{II}')}{(k_s^2 - k^2)} \Rightarrow \frac{d^2\phi}{d\left[\sqrt{k_s^2 - k^2} \cdot r\right]^2} + \frac{1}{\left(\sqrt{k_s^2 - k^2} \cdot r\right)} \frac{d\phi}{d\left(\sqrt{k_s^2 - k^2} \cdot r\right)} + \phi = 0$$

Assim, $r\sqrt{k_s^2 - k^2} \equiv x$ resulta

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\cdot\frac{d\phi}{dx} + \phi = 0 \qquad (12)$$

e $x = \sqrt{k_s^2 - k^2}\cdot r$, dividindo por k dentro da raiz e multiplicando fora

$$x = \sqrt{\frac{k_s^2}{k^2} - 1}\cdot k r \quad , \quad k_s^2 = \frac{\omega^2}{v_s^2} \quad , \quad k^2 = \frac{\omega^2}{v_{ph}^2}$$

$$\frac{k_s^2}{k^2} = \frac{v_{ph}^2}{v_s^2}$$

$$x = \sqrt{\frac{v_{ph}^2}{v_s^2} - 1}\cdot k r \qquad (13)$$

Definições usuais: $v_s = \frac{d\omega}{dk}$ = velocidade grupo

$v_{ph} = \frac{\bar{\omega}}{\bar{k}}$ = velocidade fase

r. f. Moisés Z p. 111/117

Sondando modos

Se $v_{ph} \simeq v_s$, $x \to \infty$ e $dx \to 0$ então

o último termo de (12) fica desprezível

perto das derivadas:

$$(12) \Rightarrow \underbrace{\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx}}_{\text{muito maior que } \phi} + \phi = 0$$

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} \approx 0$$

voltando a variável r: $x = \sqrt{\ }\cdot kr$, $dx = \sqrt{\ }\cdot dr$

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} \simeq 0$$

$$\frac{d\phi'}{dr} + \frac{1}{r}\phi' = 0$$

$$\frac{d\phi'}{dr} = -\frac{1}{r}\phi'$$

$$\frac{d\phi'}{\phi'} = -\frac{dr}{r}$$ tem de integrar de r_0 a r e não 0 a r

$\boxed{\text{singularidade em ln}}$

$$\int_{\phi'(r_0)}^{\phi'(r)} \frac{d\phi'}{\phi'} = -\int_{r_0}^{r} \frac{dr}{r}$$

$$\ln\left[\frac{\phi'(r)}{\phi'(r_0)}\right] = -(\ln r - \ln r_0)$$

$$= -\ln\left(\frac{r}{r_0}\right) = \ln\left(\frac{r_0}{r}\right)$$

$$\frac{\phi'(r)}{\phi'(r_0)} = \frac{r_0}{r}$$

$$\phi'(r) = \phi'(r_0) . r_0 . \frac{1}{r}$$

$$\frac{d\phi}{dr} = \phi'(r_0) . r_0 \; \frac{1}{r}$$

$$d\phi = \phi'(r_0) r_0 \cdot \frac{dr}{r}$$

$$\int_{\phi(r_0)}^{\phi(r)} d\phi = \phi'(r_0)\cdot r_0 \int_{r_0}^{r} \frac{dr}{r}$$

$$\phi(r) - \phi(r_0) = \phi'(r_0)\cdot r_0 \cdot \ln\left(\frac{r}{r_0}\right)$$

$$\boxed{\phi(r) = \phi(r_0) + \underbrace{\left(\frac{d\phi}{dr}\Big]_{r_0} \cdot r_0\right)}_{\text{"}\equiv \phi_0\text{"}} \cdot \ln\left(\frac{r}{r_0}\right)} \quad \ldots\ldots \quad (15)$$

n devia ser esse o ϕ_0?

Capítulo de acústica do Moyses 2 p. 125: $\left(\frac{\partial p}{\partial \rho}\right)_S = \gamma \cdot \frac{p}{\rho}$

Supomos processo adiabático com $\frac{\partial p_1}{\partial \rho_1} \equiv v_s^2$

$$\underbrace{\left(\frac{\partial p}{\partial \rho_1}\right)_S}_{v_s^2} = \gamma \underbrace{\frac{p_1}{\rho_1}}_{\text{"?}} \cdot v_{ph}^2 \quad \text{para um gás ideal}$$

geralmente $\gamma > 1$ (ar: $\gamma \cong 1{,}4$) $\Rightarrow$ $\boxed{\frac{v_{ph}^2}{v_s^2} = \frac{1}{\gamma} < 1}$

$\Rightarrow$ geralmente é subsônico (em gases)

Compondo entao Φ e $\vec{v}$ a partir de ϕ:

$$\Phi = Z \cdot T \cdot \phi \sim e^{\pm i\omega} \cdot e^{\pm i k z} \left[\phi(r_0) + r_0 \phi_0' \cdot \ln\left(\frac{r}{r_0}\right)\right]$$

$$\vec{\nabla}\Phi \sim e^{\pm i\omega} \vec{\nabla}\left(e^{\pm kz} \cdot \ln \frac{r}{r_0}\right)$$

$$\vec{\nabla} = \hat{r}\partial_r + \hat{z}\partial_z$$ pois $\bar{n}$ tem dependência azimutal

$$\vec{\nabla}\Phi \sim e^{\pm i\omega t}\left[\pm \hat{z} \cdot k e^{\pm kz} \cdot \ln\left(\frac{r}{r_0}\right) + \hat{r} \frac{e^{\pm kz}}{\frac{r}{r_0}} \cdot \frac{1}{r_0}\right]$$

$$\vec{v}_1 \sim e^{\pm i\omega t} \cdot e^{\pm kz} \left[\pm \hat{z} \cdot k \ln\left(\frac{r}{r_0}\right) + \hat{r} \cdot \frac{1}{r}\right] \cdots (✡)$$

Se $\vec{v}_1 = v_{1z} \cdot \hat{z} + v_{1r} \cdot \hat{r} = \dot{z} \cdot \hat{z} + \dot{r}\,\hat{r} \Longrightarrow$

$$(✡) \Longrightarrow \begin{cases} \dot{z} \sim \pm e^{\pm i\omega t} \cdot k\, e^{\pm i k z} \cdot \ln \frac{r}{r_0} \\ \dot{r} \sim e^{\pm i\omega t} e^{\pm i k z} \cdot \frac{1}{r} \end{cases} \Longrightarrow \begin{cases} \dot{z} \equiv \pm A e^{\pm i\omega t} e^{\pm i k z} \cdot \ln \frac{r}{r_0} \\ \dot{r} \equiv B e^{\pm i\omega t} e^{\pm i k z} \cdot \frac{1}{r} \end{cases}$$