



Electric Charges And Field

By- VIP Sir

Version 1.0
May, 2018
Class -
Subject - Physics

Venus Bhatnagar Science Institute
Janta Quarter
Indore, 452011 MP
India
www.bhatnagarinstitute.in

Table of Contents

Electric Charges And Field	3
1.1. Frictional Electricity	3
1.2 Electric Charge	3
1.3 Electrostatics	4
Applications of electrostatics	4
1.4 Two kinds of Charges -	4
1.5 Electric theory of Frictional electricity	5
1.6 Conductors and insulators	6
1.7 Electrostatic Induction	7
1.8 Basic properties of electric charge	8
1.9 Additivity of electric charge	9
1.10 Quantization of electric charge	9
1.11. Conservation of charges	10
1.12 Electric charge vs Mass	10
1.12. Coulomb's law in vector form	11
1.13. Dielectric constant: Relative Permittivity	13
1.14. Comparing Electrostatic and Gravitational forces	14
1.15. Forces Between multiple charges : The superposition Principle	15
1.16. Electric Field	15
1.17. Electric Field due to a point charge	16
1.18 Electric field due to a system of point charged	17
1.19 Continuous charge distribution	25
1.7 Numerical example (Type A)	26

Electric Charges And Field

1.1. Frictional Electricity

Q1. What is frictional electricity? When is a body said to be electrified or charged?

“ That the energy due to which matter gets property of attracting the things is known as Electricity”

The property of rubbed substances due to which they attract light object is called electricity. The electricity developed by the rubbing or friction called **friction** or **static electricity**. The rubbed substances which show this property of attraction are said to have become **electrified** or **electrically charged**.

Q.2 Give a historical view of frictional electricity. From where did the term electricity get its origin?

Historical view of frictional electricity. In 600 B.C., Thales of Miletus, one of the founders of Greek science, first noticed that if a piece of amber is rubbed with a woollen cloth, it then acquires the property of attracting light feathers, dust, lint, pieces of leaves, etc.

In 1600 A.D., William Gilbert, the personal doctor to Queen Elizabeth -I of England, made a systematic study of the substances that behave like amber. The Greek name for amber is electron which is the origin of all such words : electricity, electric force, electric charge and electron.

1.2 Electric Charge

Q.3. What is electric charge? Is it a scalar or vector quantity? Name its SI unit.

Electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.

Electric charge is a scalar quantity. Its SI unit is coulomb (C). A proton has a positive charge (+e) and an electron has a negative charge (-e), where large scale matter that consists of equal number of electrons and protons is electrically neutral. If there is an excess of electrons, the body has a negative charge and an excess of protons results in a positive charge.

1.3 Electrostatics

Q.4. What is electrostatics? Mention some of its important applications.

Electrostatic is the study of electric charges at rest. Here we study the forces, fields and potentials associated with static charges.

Applications of electrostatics

The attraction and repulsion between charged bodies have many industrial applications. Some of these are as follows:

1. In electrostatics loudspeaker.
2. In electrostatic spraying of paints and powder coating.
3. In flyash collection in chimneys.
4. In a xerox copying machine.
5. In the design of a cathode-ray tube used in television and radar.

1.4 Two kinds of Charges -

Q.5. How will you show experimentally that (i) there are only two kinds of electric charges and (ii) like charges repel and unlike charges attract each other?

Experiment 1

(i) Rub a glass rod with silk and suspend it from a rigid support by means of a silk thread. Bring another similarly charged rod near it. The two rods repel each other.

Diagram - 1.1

(ii) Bring a plastic rod rubbed with wool near the charged glass rod. The two rods attract each other [fig.1.1(b)].

(iii) Now rub a plastic rod with wool and suspend it from a rigid support. Bring another similarly charged plastic rod near it. There will be a repulsion between the two rods [Fig 1.1 (c)].

Experiment 2

If a glass rod, rubbed with silk, is made to touch two small pith balls which are suspended by silk threads, then the two balls repel each other, as shown in fig .1.2 (a). Similarly two pith balls touched with plastic rod rubbed with fur are found to repel each other. But it is seen that a pith ball touched with glass rod attracts another pith ball touched with a plastic rod.

Diagram - 1.2

From the above experiments, we note that the charge produced on a glass rod is different from the charge produced on a plastic rod. We can conclude that :

1. There are only two kinds of electric charges - positive and negative.
2. Like charges repel and unlike charges attract each other.

The statement 2 is known as the fundamental law of electrostatics

Q.6. what are vitreous and resinous charges? What was wrong with this nomenclature?

Charles Du fay used the terms vitreous and resinous for the two kinds of charges.

1. The charge developed on glass rod when rubbed with silk was called **vitreous charge**.
2. The charge developed on amber when rubbed with wool was called **resinous charge**.

But later on, these terms were found to be misleading. For example, a ground glass rod develops resinous electricity while a highly polished ebonite rod develops vitreous electricity.

Q.7. What are positive and negative charges? What is the nature of charge on an electron in this convention?

‘**Benjamin Franklin (1706-1790)**’ introduced the concept of positives and negative charges. In order to distinguish the two kinds of charges and resinous as negative charge.

1. Positive Charge - When glass rod is rubbed with silk, charge produced on glass to the loss of negative charge is known as positive charge.
2. Negative Charge - When an ebonite rod is rubbed on woolen cloth or cat-skin, charge produce on an ebonite rod due to the loss of positive charge is known as negative charge.
 1. Like charges repel each other.
 2. Unlike charges attract each other.

1.5 Electric theory of Frictional electricity

Q.8. Describe the electronic theory of frictional electricity. Are the frictional forces electric in origin?

Electronic theory of frictional electricity

All Matter is made of atoms. An atom consists of a small central nucleus containing protons and neutrons, around which revolve a number of electrons. In any piece of matter, the positive proton charges and the negative electron charges cancel each other and so the matter in bulk is electrically neutral.

The electrons of the outer shell of an atom are loosely bound to the nucleus. The energy required to remove an electron from the surface of a material is called its work function. When two different bodies are rubbed against each other electrons are transferred from glass rod to silk. The glass rod develops a positive charge due to deficiency of electrons while the silk cloth develops an equal negative charge due to excess of electrons. The combined total charge of the glass rod and silk cloth is still zero, as it was before rubbing i.e., electric charge is conserved during rubbing.

Electric origin of frictional forces

The only way by which an electron can be pulled away from an atom is to exert a strong electric force on it. As electrons are actually transferred from one body to another during rubbing, so frictional forces must have an electric origin.

1.6 Conductors and insulators

Q.9. How do the conductors differ from the insulators? Why can't we electrify a metal rod by rubbing it while holding it in our hand? How can we charge it?

Conductors

The substances through which electric charges can flow easily are called conductors. They contain a large number of free electrons which make them good conductor of electricity. Metals, human and animal bodies, graphite, acids, alkalies, etc. are conductors.

Insulators

The substances through which electric charges cannot flow easily are called insulators. In the atoms of such substances, electrons of the outer shell are tightly bound to the nucleus. Due to the absence of free charge carriers, these substances offer high resistance to the flow of electricity through them. Most of the non metals like glass, diamond, plastic, nylon, wood are insulators.

An important difference between conductors and insulators is that when some charge is transferred to a conductor, it readily gets distributed over its entire surface. On the other hand, if some charge is put on an insulator it stay at the same place.

Q.10. What is meant by earthing or grounding in household circuits? What is its importance?

When a charged body is brought in contact with earth its entire charge passes to the ground in the form of a momentary current. This process in which a body shares its charge with the earth is called **grounding or Earthing**.

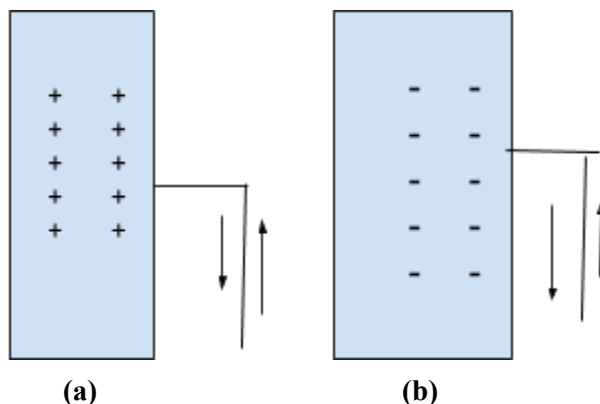


Fig. 1.3 (a) Positively charge (b) Negatively charged, earthed body.

The electric from the mains is supplied to our houses using a three core wiring : live, neutral and earth wires. The live wire red in colour brings in the current. The black neutral wire is the return wire. The green earth wire is connected to a thick metal plate buried deep into the earth. The metallic bodies of the electric appliances such as electric iron, refrigerator, TV, etc. are connected to the earth wire. When any fault occurs or live wire touches the metallic body, the charge flows to the earth and the person who happens to touch the body of the appliance does not receive any shock.

1.7 Electrostatic Induction

Q.11 What is meant by electrostatic induction?

Electrostatic induction is the phenomenon of temporary electrification of a conductor in which opposite charge appear at its closer end and similar charges appear at its farther end in the presence of a nearby charged body.

The positive and negative charges produced at the ends of the conducting rod are called induced charge and the charge on the glass rod which induces these charges on conducting rod is called inducing charge.

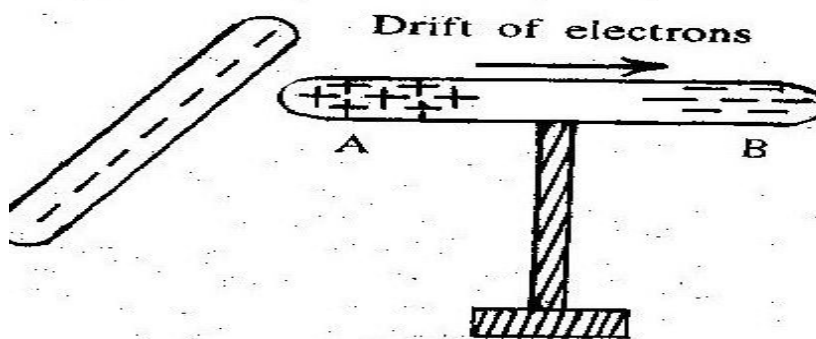
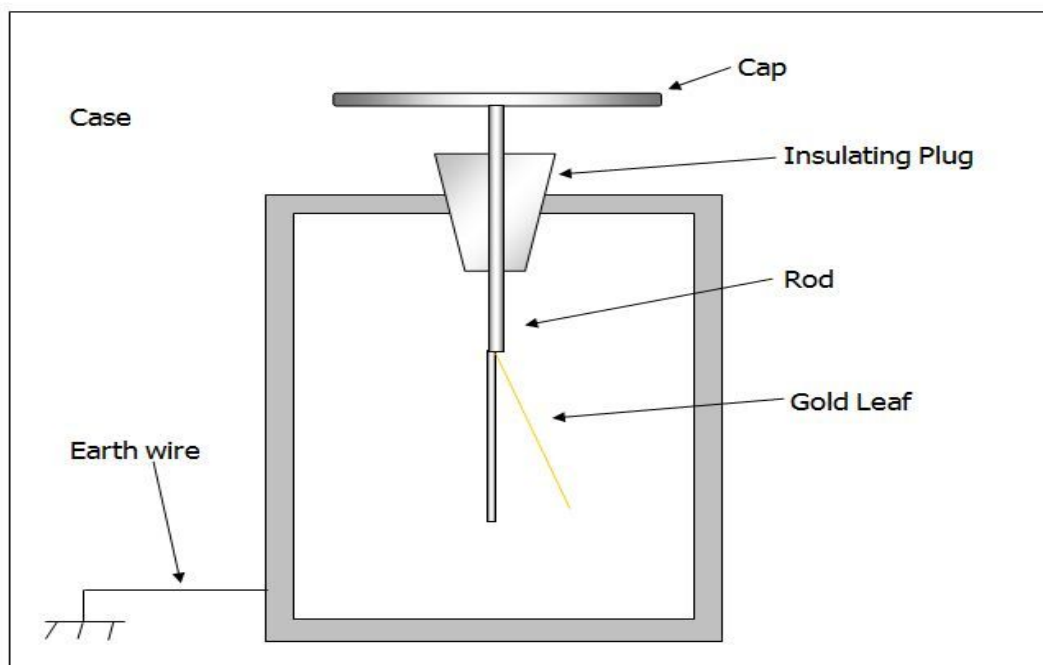


Fig.1.4 Electrostatic Induction

For Your Knowledge

- Gold-leaf electroscope.** It is a device used for detecting an electric charge and identifying its polarity. It consists of a vertical conducting rod passing through a rubber stopper fitted in the mouth of a glass vessel. Two thin gold leaves are attached to lower end of the rod. When a charged object touches the metal knob at the outer end of the rod, the charge flows down to the leaves. The leaves diverge due to repulsion of the like charges they have received. The degree of divergence of the leaves gives a measure of the amount of charge.



1.8 Basic properties of electric charge

It is observed from experiments that electric charge has following three basic properties:

1. Additivity
2. Quantization
3. Conservation

1.9 Additivity of electric charge

Additivity of electric charge means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system

If a system contains charges q_1, q_2, \dots, q_n then its charge is

$$q = q_1 + q_2 + \dots + q_n$$

1.10 Quantization of electric charge

Q.12 What is meant by quantization of a physical quantity?

The Quantization of a physical quantity means that it cannot vary continuously to have any arbitrary value but it can change discontinuously to take any one of only a discrete set of values. The minimum amount by which a physical quantity can change is called its quantum.

Q.13 What is meant by quantization of electric charge? What is the cause of quantization of electric charge?

The experimental fact that electric charges occur in discrete amounts instead of continuous amounts is called quantization of electric charge. The quantization of electric charge means that the total charge q of a body is always an integral multiple of a basic quantum of charge (e), i.e.,

$$q = ne, \text{ where } n = 0, \pm 1, \pm 2, \dots, e = 1.6 \times 10^{-19} \text{ C.}$$

Hence the charge on a charged body is always multiple of e i.e., $2e, 3e, 4e, \dots$.

Thus, It is clear that the electric charge cannot be divided in uncertain manner. The property of electric charge is known as quantization of charge.

‘ e is the smallest unit of charge. It is known as elementary charge is fundamental charge.’

“The cause of quantization of electric charge is that the transfer of electron is always in the form of integer multiple.”

Q.14 Can we ignore the quantization of electric charge? If yes, under what conditions?

While dealing with macroscopic charged $q = ne$ we can ignore the quantization of electric charge this is because the basic charge e is very small and n is very large in most practical situations, so q behaves as if it were continuous i.e., as if a large amount of charge were flowing. For example, when we switch on a 60W bulb near 2×10^{18} electrons pass through its filament per second, here the graininess or structure of charge does not show up i.e., the bulb does not flicker with the entry of each electron. Quantization of charge becomes important at the microscopic level, where the charges involved are of the order of a few tens or hundreds of e .

1.11. Conservation of charges

Q.15. State the law of conservation of charge. Give some examples of illustrate this law.

According to principle of conservation of charges- “The net charge present in an isolated system remains always conserved (or constant).”

or

“in a isolated system the algebraic sum of positive and negative charge remains always constant.”

In other words charge can neither be created nor be destroyed It can transferred from an substance to other.

Law of conservation of charge which states:

1. The total charge of an isolated system remains constant.
2. The electric charges can neither be created nor destroyed, they can only be transferred from one body to another.

The law of conservation of charge is obeyed both in large scale and microscopic processes. In fact, charge conservation is a global phenomenon i.e., total charge of the entire remains constant.

Examples :

1. When a glass rod is rubbed with a silk cloth it develops a positive charge. But at the same time, the silk cloth develops an equal negative charge. Thus the net charge of the glass rod and the silk cloth is zero as it was before rubbing.

1.12 Electric charge vs Mass

Q.16. Compare the properties of electric charge with those of mass of a body.

	Electric Charge	Mass
1.	Electric charge may be positive, negative or zero.	Mass of a body is always positive.
2.	Electric charge is always quantized : $q = ne$	Quantization of mass is ot yet established.
3.	Charge on a body does not depend on its speed.	Mass of body increases with its speed.
4.	Charge is strictly conserved.	Mass is not conserved by itself as some of

		the mass may get changes into energy or vice versa.
5.	Electric forces between two charges may be attractive or repulsive.	Gravitational forces between two masses are always attractive.
6.	Electrostatic forces between different charges may cancel out.	Gravitational forces between different bodies never cancel out.

Q.17. how does the speed of an electrically charged particle affect its 1.mass and 2. Charge?

According to the special theory of relativity, the mass of a body increases with the its speed in accordance with the relation:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where, m_0 = Rest mass of the body, c = speed of light, and m = mass of the body when moving with speed v .

As $v < c$, therefore, $m > m_0$

In contrast to mass, the charge on a body remains constant and does not change as the speed of the body changes.

1.12. Coulomb's law in vector form

Q.18. Write coulomb's law in vector form. What is the importance of expressing it in vector form?

Consider two positive point charges q_1 and q_2 placed in vacuum at distance r from each other. They repel each other.

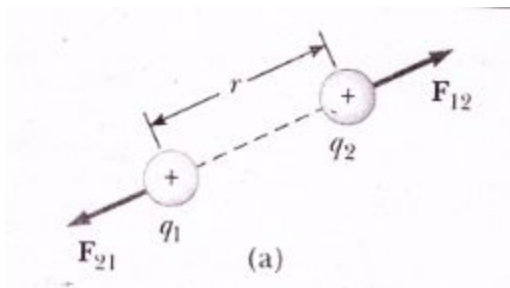


Fig.1.5 Repulsive coulombic forces for $q_1 q_2 > 0$.

In vector form, coulomb's law may be expressed as

$$F_{21} = \text{Force on charge } q_2 \text{ due to } q_1$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Where $\hat{r}_{12} = r_{12}/r$ is a unit vector in the direction from q_1 to q_2 .

Similarly, $F_{12} = \text{Force of charge } q_1 \text{ to } q_2$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

Where $\hat{r}_{12} = r_{21}/r$ is a unit vector in the direction from q_2 to q_1

The coulombian forces between unlike charges are attractive

Importance of vector form

The vector form of coulomb's law gives the following additional information:

1. As $\hat{r}_{21} = -\hat{r}_{12}$, therefore $F_{21} = F_{12}$

This means that the two charges exert equal and opposite forces on each other. So coulombian forces obey Newton's third law of motion.

2. As the coulombian forces act along F_{12} or F_{21} , i.e., along the line joining the centers of two charges, so they are central forces.

Q.19. What is the range over which coulombian forces can act? State the limitations of coulomb's law in electrostatics?

Coulombian forces act over an enormous range of separations (r) from nuclear dimension ($r = 10^{-15}m$) to macroscopic distances as large as $10^{18}m$. inverse square is valid over this range of separation to a high degree of accuracy.

Limitations of Coulomb's Law

Coulomb's law is not applicable in all situations. It is valid only under the following conditions;

1. The electric charges must be at rest.
2. The electric charges must be point charges i.e., the extension of charge must be much smaller than the separation between the charges.
3. The separation between the charges must be greater than the nuclear size $10^{-15}m$, because for distance $< 10^{-15}m$, the strong nuclear force dominates over the electrostatic force.

1.13. Dielectric constant: Relative Permittivity

Q.20. What do you mean by permittivity of a medium? Define dielectric constant in terms of forces between two charges?

Permittivity is a property of the medium which determines the electric force between two charges situated in that medium.

Dielectric constant or relative permittivity

According to Coulomb's law, the force between two point charges q_1 to q_2 , placed in vacuum at distance r from each other is given by

$$F_{vac} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} r_{12} \dots\dots(1)$$

When the same two charges are placed same distance apart in any medium other than vacuum, the force between them becomes

$$F_{med} = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} r_{12} \dots\dots(2)$$

The quantity ϵ is called absolute permittivity or just permittivity of the intervening medium.

Dividing equation (1) and (2), we get

$$\frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} r_{12}}{\frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} r_{12}} = \frac{\epsilon}{\epsilon_0}$$

The ratio $\frac{\epsilon}{\epsilon_0}$ of the permittivity ϵ of the medium to the permittivity ϵ_0 of free space is called relative permittivity ϵ_r of dielectric constant (k) of given medium. Thus

$$\epsilon_r \text{ or } k = \frac{\epsilon}{\epsilon_0} = \frac{F_{vac}}{F_{med}}$$

So one can define dielectric constant in terms of forces between charges as follows:

The dielectric constant or relative permittivity of a medium may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same charges when they are placed the same distance apart in the given medium.

Clearly, When a material medium of dielectric constant k is placed between the charges, the force between them becomes $\frac{1}{k}$ times the original force in vacuum. That is,

$$F_{med} = \frac{f_{vac}}{k}$$

Hence the coulomb's law for any material medium may be written as

$$F_{med} = \frac{1}{4\pi\epsilon_0 k} \cdot \frac{q_1 q_2}{r^2}$$

$$K(\text{vacuum}) = 1$$

$$k(\text{air}) = 1.00054$$

$$k(\text{water}) = 80.$$

1.14. Comparing Electrostatic and Gravitational forces

Q.21. Give a comparison of the electrostatic and gravitational forces.

Electrostatic force is the force of attraction or repulsion between two charges at rest while the gravitational force is the force of attraction between two bodies by virtue of their masses.

Similarities

1. Both forces obey inverse square law i.e., $F \propto \frac{1}{r^2}$
2. Both forces are proportional to product of masses or charges.
3. Both are central forces i.e., they act along the line joining the centres of the two bodies.
4. Both are conservative forces that is the work done against these forces does not depend upon the path followed.
5. Both forces can operate in vacuum.

Dissimilarities

1. Gravitational force is attractive while electrostatic force may be attractive or repulsive.
2. Gravitational force does not depend on the nature of the medium while electrostatic force depends on the nature of the medium between the two charges.
3. Electrostatic forces are much stronger than gravitational forces.

How much is the electrostatic force stronger than the gravitational force?

- (a) (i) From Coulomb's law, the electrostatic force between an electron and a proton separated by distance r is

$$F_e = k \frac{q_1 q_2}{r^2} = \frac{k(-e)(e)}{r^2} = -\frac{ke^2}{r^2}$$

Negative sign indicates that the force is attractive.

From Newton's law of gravitation, the corresponding gravitational attraction is

$$F_G = -G \frac{m_p m_e}{r^2}$$

Where m_p and m_e are the masses of the proton and electron.

Hence $\left| \frac{F_e}{F_G} \right| = \frac{ke^2}{Gm_p m_e}$, but

$$k = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}, e = 1.6 \times 10^{-19} \text{ C}, m_p = 1.67 \times 10^{-27} \text{ kg}, m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$$

$$\left| \frac{F_e}{F_G} \right| = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 9.1 \times 10^{-31}} = 2.27 \times 10^{39}$$

- (b) (ii) Similar to that in part I, the ratio of the magnitudes of electric force to the gravitational force between two protons at a distance r is given by

$$\left| \frac{F_e}{F_G} \right| = \frac{ke^2}{Gm_p m_e} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2} = 1.24 \times 10^{36}$$

Thus the large value of the ratio of the two forces indicates that the electrostatics forces are enormously stronger than the gravitational forces.

- b) the magnitude of the electric field exerted by a proton on an electron is equal to the magnitude of the force exerted by an electron on a proton. The magnitude of this force is

$$F = \frac{ke^2}{r^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(10^{-10})^2} = 2.3 \times 10^{-8} \text{ N}$$

Acceleration of the electron due to the mutual attraction with the proton,

$$a_e = \frac{F}{m_e} = \frac{2.3 \times 10^{-8} N}{9.1 \times 10^{-31} kg} = 2.5 \times 10^{22} ms^{-2}$$

Acceleration of the proton due to the mutual attraction with the electron,

$$a_p = \frac{F}{m_p} = \frac{2.3 \times 10^{-8} N}{1.67 \times 10^{-27} kg} = 1.3 \times 10^{19} ms^{-2}$$

Clearly, the acceleration of an electron or a proton due to electric force is much larger than the acceleration due to gravity. So, we can neglect the effect of gravitational field on the motion of the electron or the proton.

Q.22 Give two examples which illustrate that the electrical forces are enormously stronger than the gravitational forces.

1. A plastic comb passed through hair can easily lift a piece of paper upwards. The electrostatic attraction between the comb and the piece of paper overcomes the force of gravity exerted by the entire earth on the paper.
2. When we hold a book in our hand, the electric forces between the palm of our hand and the book easily overcomes the gravitational force on the book due to the entire earth.

1.15. Forces Between multiple charges : The superposition Principle

Q.23 State the principle of superposition of electrostatic forces. Hence write an expression for the force on a point charge due to a distribution of N-1 point charges in terms of their position vectors.

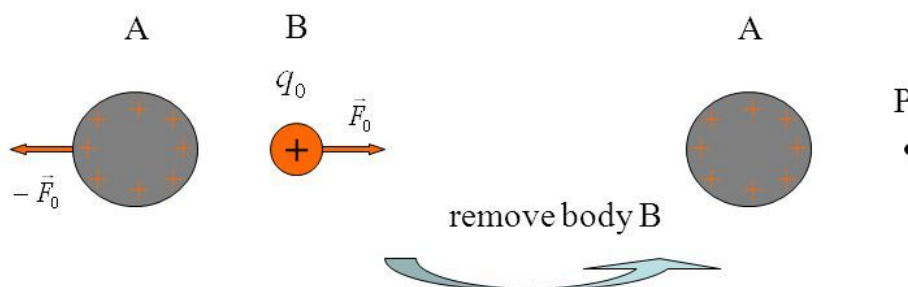
The principle of superposition states that when a number of charges are interacting, the total force on a given charge is the vector sum of the forces exerted on it due to all other charges. The force between two charges is not affected by the presence of other charges.

Numerical - page 1.19

1.16. Electric Field

Q.24 Briefly develop the concept of electric field.

The Electrostatic force acts between two charged bodies even without any direct contact between them. The nature of this action at distance force can be understood by introducing the concept of electric field.



Consider a charged body carrying a positive charge q placed at point O . It is assumed that the charge q produces an electrical environment in the surrounding space, called electric field.

To test the existence of electric field at any point P , we simply place a small positive charge q_0 , called the test charge at the point P . If a force F is exerted on the test charge, then we say that an electric field E exists at the point P . The charge q is called the source charges as it produces the field E .

Q25. Define electric field at a point. Give its units and dimensions.

An electric field is said to exist at a point if a force of electrical origin is exerted on a stationary charged body placed at that point. Quantitatively, the electric field or the electric intensity or the electric field strength E at a point is defined at that point, without disturbing the position of source charge.

Suppose a test charge q_0 experiences a force F at the point P . Then the electric field at that point will be

$$E = \frac{F}{q_0}$$

The electric field at a point is defined as the electrostatic force per unit test charge acting on a vanishingly small positive test charge placed at that point. Hence

$$E = \lim_{q_0 \rightarrow 0} \frac{F}{q_0}$$

The electric field F is a vector quantity whose direction is same as that of force F exerted on a positive test charge.

Units and dimensions of electric field - as the electric field is force per unit charge, so its SI unit is newton per coulomb (NC^{-1}). It is equivalent to volt per metre (Vm^{-1}).

The dimensions for E can be determined as follows:

$$\begin{aligned} [E] &= \frac{\text{force}}{\text{charge}} = \frac{MLT^{-2}}{C} \quad [\because 1A = \frac{1C}{1s}] \\ &= \frac{MLT^{-2}}{A \cdot T} = [MLT^{-3}A^{-1}] \end{aligned}$$

Q26. Give the physical significance of electric field.

The force experienced by the test charge q_0 is different at different point. So E also varied from point to point. In general, E is not a single vector but a set of infinite vectors. Each point r is associated with a unique vector $E(r)$.

By knowing electric field at any point, we can determine the force on a charge placed at that point. The coulomb force on a charge q_0 due to a source charge q may be treated as two stage process:

- (i) The source charge q produces a definite field $E(r)$ at every point r .
- (ii) The value of $E(r)$ at any point r determine the force on charge q_0 at that point. This force is

$$F = q_0 E(r)$$

$$\text{Electrostatic force} = \text{Charge} \times \text{Electric Field}$$

Thus an electric field plays an intermediary role in the forces between two charges:

$$\text{Charge} \Leftrightarrow \text{Electric field} \Leftrightarrow \text{charge}$$

It is in this sense that the concept of electric field is useful. Electric field is a characteristic of the system of charges and is independent of the test. Charge that we place at a point to determine the field.

1.17. Electric Field due to a point charge

Q26. Obtain an expression for the electric field intensity at a point at a distance r from a charge q . What is the nature of the field?

A single point charge has the simplest electric field. Consider a point charge q placed at the origin O . we wish to determine its electric field at a point p at a distance r from it. For this, imagine a test charge q_0 placed at point P . according to coulomb's law, the force on charge q_0 is

$$F = \frac{1}{4\pi\epsilon} \cdot \frac{qq_0}{r^2} \hat{r}$$

Where \hat{r} is a unit vector in the direction from q to q_0 .

Electric field at point p is

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r}$$

The magnitude of the field E is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

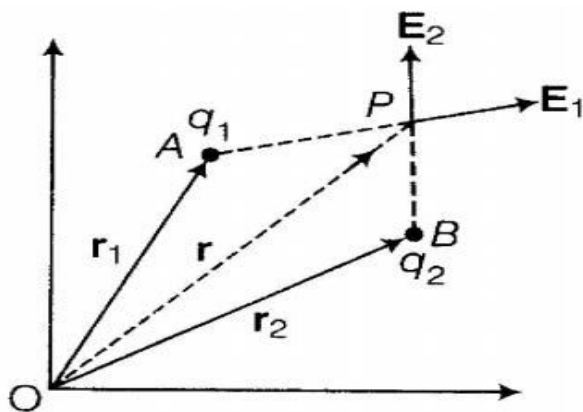
Clearly, $E \propto \frac{1}{r^2}$ this means that at all points on the spherical surface drawn around the point charge, the magnitude of E is same and does not depend on the direction of r . Such a field is called spherically symmetric or radial field, i.e., a field which looks the same in all directions when seen from the point charge.

1.18 Electric field due to a system of point charged

Q27. Deduce an expression for the electric field at a point due to a system of N point charges.

Consider a system of N point charges q_1, q_2, \dots, q_n having position vectors r_1, r_2, \dots, r_n with respect to the origin O. we wish to determine the electric field at point p whose position vector is r . According to Coulomb's law, the force on charge test q_0 due to charge q_1 is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{r_{1p}^2} \hat{r}_{1p}$$



Where \hat{r}_{1p} is a unit vector in the direction from q_1 to p and r_{1p} is the distance between q_1 and p. Hence the electric field at point P due to charge q_1 is

$$E_1 = \frac{F_1}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1p}^2} \hat{r}_{1p}$$

Similarly, electric field at point p due to charge q_2 is

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2p}^2} \hat{r}_{2p}$$

According to principle of superposition of electric fields, the electric field at any point due to a group of charged each charge individually at that point, when all other charges are assumed to be absent.

Hence, the electric field at point P due to the system of N charges is

$$E = E_1 + E_2 + \dots + E_n$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_{1p}^2} \hat{r}_{1p} + \frac{q_2}{r_{2p}^2} \hat{r}_{2p} + \dots + \frac{q_n}{r_{np}^2} \hat{r}_{np} \right]$$

Or

$$E = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_{ip}^2} \hat{r}_{ip}$$

1.19 Continuous charge distribution

Q28. What is a continuous charge distribution? How can we calculate the force on a point charge q due to a continuous charge distribution?

In practice we deal with charges much greater in magnitude than the charge on an electron, so we can ignore the quantum nature of charges and imagine that the charge is spread in a region in a continuous manner. Such a charge distribution is known as a continuous distribution.

Calculation of the force on a charge due to a continuous charge distribution

According to coulomb's law the force on point charge q_0 due to small charge dq is

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r^2} \hat{r}$$

Where $\hat{r} = r/r$, is a unit vector pointing from the small charge dq towards the point charge q_0 .

By the principle of superposition, the total force on charge q_0 will be the vector sum of forces exerted by all such small charges and is given by

$$F = \int dF = \int \frac{dq}{r^2} \hat{r}$$

$$F = \frac{q_0}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

Q29. Name the different types of continuous charge distributions. Define their respective charge densities. Write expression for the electric field produced by each type of charge distribution.

Different types of continuous charge distributions - there are 3 types of continuous charge distribution:

1. Volume charge distribution - it is a charge distribution spread over a three dimensional volume or region V of space. We define the volume charge density at any point in this volume as the charge contained per unit volume at that point i.e., $\rho = \frac{dq}{dV}$. The SI unit for ρ is coulomb per cubic metre (Cm^{-3}).

If a charge q is distributed over the entire volume of a sphere of radius R , then its volume charge density is $\rho = \frac{dq}{\frac{4}{3}\pi R^3} Cm^{-3}$, The charge contained in small volume

$$dV \text{ is } dq = \rho dV, \rho = \frac{dq}{dV}$$

Total electrostatic force exerted on charge q_0 due to the entire volume V is given by

$$F_v = \frac{q_0}{4\pi\epsilon_0} \int_v \frac{dq}{r^2} \cdot \hat{r} = \frac{q_0}{4\pi\epsilon_0} \int_v \frac{\rho}{r^2} dV \hat{r}$$

Electric field due to the volume charge distribution at the location of charge q_0 is

$$E_v = \frac{F_v}{q_0} = \frac{1}{4\pi\epsilon_0} \int_v \frac{\rho}{r^2} dV \hat{r}$$

2. Surface charge distribution - It is a charge distribution spread over a two dimensional surface S in space we define the surface charge density at any point on this surface as the charge per unit area at that point $\sigma = \frac{dq}{dS}$ the SI unit for σ is Cm^{-2} .

For example, if a charge q is uniformly distributed over the surface of a spherical conductor of radius R , then its surface charge density is $\sigma = \frac{q}{4\pi R^2} Cm^{-2}$

The charge contained in small area dS is $dq = \sigma dS$

Total electrostatic force exerted on charge q_0 due to the entire surface S is given by

$$F_s = \frac{q_0}{4\pi\epsilon_0} \int_s \frac{\sigma}{r^2} dS \hat{r}$$

Electric field due to the surface charge distribution at the location of charge q_0 is

$$E_s = \frac{F_s}{q_0} = \frac{1}{4\pi\epsilon_0} \int_s \frac{\sigma}{r^2} dS \hat{r}$$

3. Line charge distribution - It is a charge distribution along a one dimensional curve or line L in space. We define the line charge density at any point on this line as the charge per unit length of the line at that point, i.e., $\lambda = \frac{dq}{dL}$, the SI unit for λ is Cm^{-1} .

For example, if a charge q is uniformly distributed over a ring of radius R then its linear charge density is $\lambda = \frac{q}{2\pi R} Cm^{-1}$.

The charge contained in small length dL is

$$dq = \lambda dL$$

Total electrostatic force exerted on charge distribution q_0 is due to the entire length L is given by

$$F_l = \frac{q_0}{4\epsilon_0\pi} \int_l \frac{\lambda}{r^2} dL \hat{r}$$

Electric field due to the line charge distribution at the location of charge q_0 is

$$E_l = \frac{F_l}{q_0} = \frac{1}{4\epsilon_0\pi} \int_l \frac{\lambda}{r^2} dL \hat{r}$$

The total electric field due to a continuous charge distribution is given by

$$E_{count} = E_v + E_s + E_l$$

$$E_{count} = \frac{1}{4\epsilon_0\pi} \left[\int_v \frac{\rho}{r^2} dV \hat{r} + \int_s \frac{\sigma}{r^2} dS \hat{r} + \int_l \frac{\lambda}{r^2} dL \hat{r} \right]$$

General charge distribution- A general charge distribution consists of continuous as well as discrete charges. Hence total electric field due to a general charge distribution at the location of charge q_0 is given by

$$E_{total} = E_{discrete} + E_{count}$$

$$E_{total} = \frac{1}{4\epsilon_0\pi} \left[\sum_{i=1}^N \frac{q_i}{r_i^2} + \int_v \frac{\rho}{r^2} dV \hat{r} + \int_s \frac{\sigma}{r^2} dS \hat{r} + \int_l \frac{\lambda}{r^2} dL \hat{r} \right]$$

In all the above cases, $\hat{r} = r/r$ is a variable unit vector directed from each point of the volume, surface or line charge distribution towards the location of the point charge q_0 .

1.7 Numerical example (Type A)

Based on Quantization of charge

1. can we give 2.2×10^{-19} C charge to substance.

Solution -

$$q = 2.2 \times 10^{-19}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{formula} = q = \pm ne$$

$$n = \frac{q}{e}$$

$$n = \frac{2.2 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$n = 1.375$$

So it is not possible to transfer because we can transfer electron is an integer.

2) A conductor has 14.4×10^{-19} C positively charged. How many electron are lacking or in excess.

Solution -

$$q = 14.4 \times 10^{-19}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{formula} = q = \pm ne$$

$$n = \frac{q}{e}$$

$$n = \frac{14.4 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$n = 9$$

so that conductor has taking of 9 electron.

3) calculate the charge in coulomb in the nucleus of ${}^7\text{N}14$?

Solution-

There is 7 proton in nucleus of nitrogen

\therefore charge in nucleus of ${}^7\text{N}14$ is

$$q = ne$$

$$q = 7 * 1.6 \times 10^{-19} \text{ C}$$

$$q = 11.2 * 10^{-19} \text{ C}$$

Suppose a proton and electron are placed at a distance r from each other.

The electrostatic force between them can be given by coulomb's law

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \text{ N}$$

$$F_e = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2}$$

And for newton gravitational law

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2, m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$F_g = \frac{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-19}}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$\frac{F_e}{F_g} = 2.27 \times 10^{39}$$

$$F_e = (2.27 \times 10^{39}) F_g$$

second part

comparison between electrostatic and gravitational force.

Electrostatic force

1. Similarities

1. It always inverse square law

2. it is a central force which act along the line joining the two point charges.
3. It is a conservative force
4. It acts in vacuum

Disimilarity

1. electrostatic force may be attraction force of repulsion
2. electrostatic force is strong in nature as compare to gravitational force.

Gravitational force

1. it is also obeys inverse square law
2. it is also a central force which act along the line joining the masses.
- 3 It is also a conservative force.
4. it also acts in vacuum

1. it is always attraction force
3. is is weak in nature as compared to electrostatic force.

* electric lines of force or field lines

Hence the lines of force due to an isolated positive charge are radially outward ie the lines of force due to positive charge status from the charges and goes to infinity

Diagram

2. Hence the lines of force due to an isolated negative charge are radially inwards ie. The lines of force due to negative charge starts from infinity and ends at negative charge.

3. It fig c the lines of force for the system of two equal and opposite charges are shown

Diagram

4 fig d electric lines of force starting from a system of two equal and similar charges executes sideway pressure on each other.

Diagram

5. e if two positive charges is not equal in magnitude the neutral point is situated on the line joining the charge but the to the charges whose magnitude is smaller

diagram

6 fig f shows the lines of force due to a positively charged sheet.

Diagram

Properties of electric lines of force

1. electric lines of force starts from positive charge and end into the negative charge or on a conductor.
2. tangent drawn at any point gives the direction of electric field intensity at that point. This property gives the reason why two lines of force never intersect? It is because if two lines intersect at any point then two tangents can be drawn at that point means there may be two directions of electric field intensity which is not possible.
3. lines of force try to contract longitudinally or a tension acts lengthwise. The property can explain why two unlike charges pull each other.
4. the lines of force have a tendency to separate from each other in the direction perpendicular to their length this property gives reason why lines always leave or meet the surface normally
5. we can say that if the lines of force are closer then the electric field is weaker
6. the electric lines of force do not pass through a conductor this shows that electric field inside a conductor is zero.

7. electric lines of force start normally from a positively charged conductor and end normally on the surface of a negatively charged conductor

electric dipole

A system of two equal and opposite charges which are placed very close to each other is called electric dipole

diagram 16

electric Dipole moment

Electric dipole moment is a vector quantity whose magnitude is equal to the product of magnitude of any one charge of dipole and the distance between the two charges.

$$P = |q| \cdot 2l$$

direction of p is always negative to positive.

$$\text{Unit - S.I } p = C \cdot m$$

$$\text{cgs } p = \text{stat C} \cdot \text{cm}$$

$$\begin{aligned} \text{D.f. } \rightarrow p &= It \cdot m \\ &= \text{ampere} \cdot \text{sec} \cdot m \\ &= [m0l^1a^1] \end{aligned}$$

Electric field due to electric dipole-

due to an electric dipole the intensity of electric field at a point can be determined in two standard positions

1. Axial Position

2. Co-axial position

1. axial position – a line passing through the negative and positive charges at the electric dipole is called axial lines of electric dipole. A point on the axial line is called 2nd on position

Diagram pag 17

Let Ab be an electric dipole consist of charges +q and -q which are 2l distance apart and placed in a of dielectric constant ϵ_r .

Field intensity due to +q at charge p is

$$E_1 = \frac{1}{4\pi\epsilon_0} * \frac{q}{r^2} \text{ (along } r \text{ vector)}$$

numerical page no. 18-20