

<https://www.geeksforgeeks.org/dsa/c-program-for-tower-of-hanoi/>

```
void TOH(int n, string src, string dst, string helper)
```

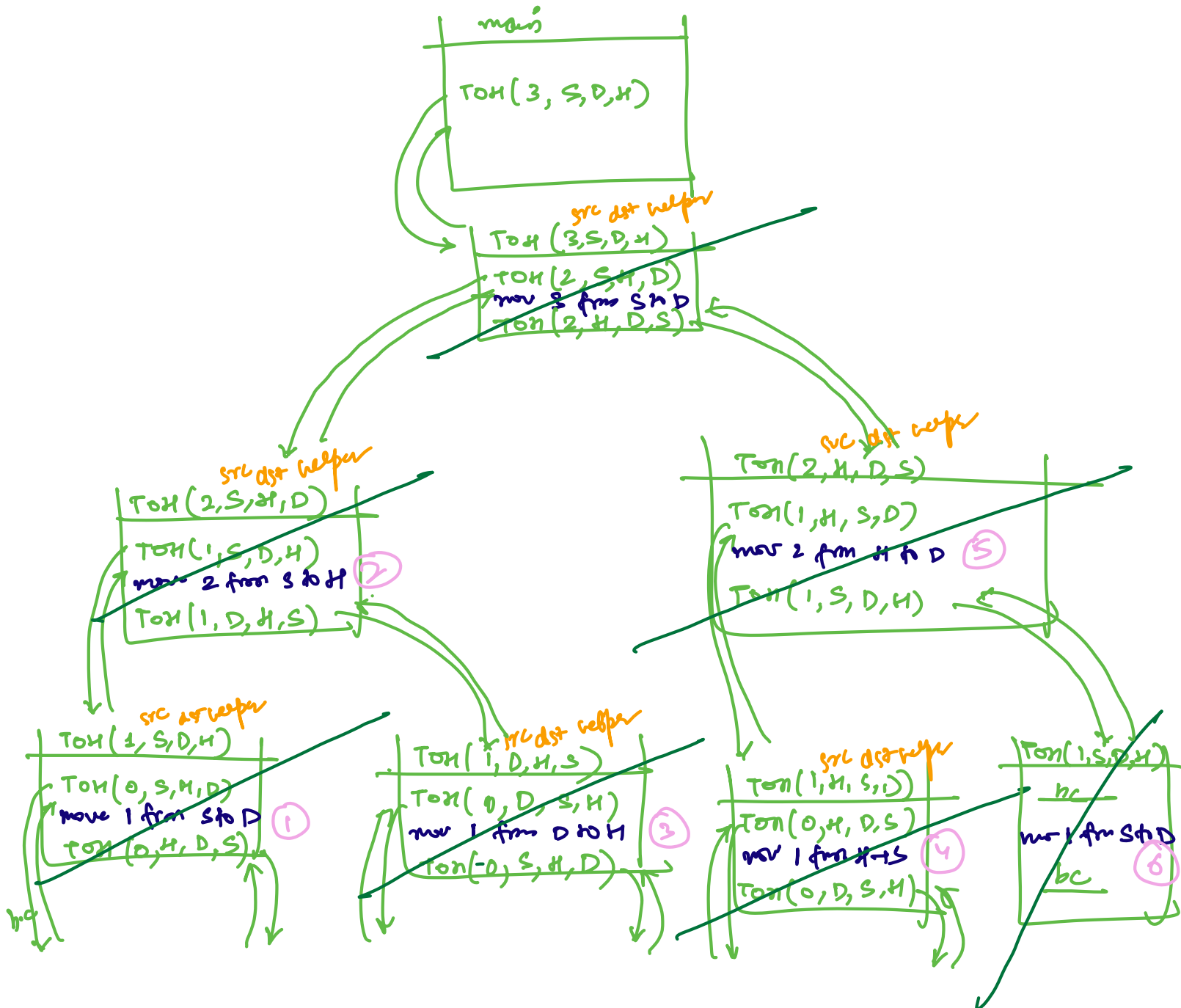
```
{
    if(n == 0)
        return ;
```

```
TOH(n-1, src, helper, dst) ;
```

```
cout << "move " << n << " disc from " << src << " to " << dst << endl ;
```

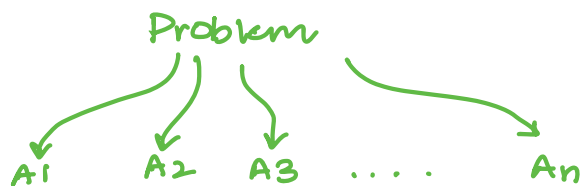
```
TOH(n-1, helper, dst, src) ;
```

}

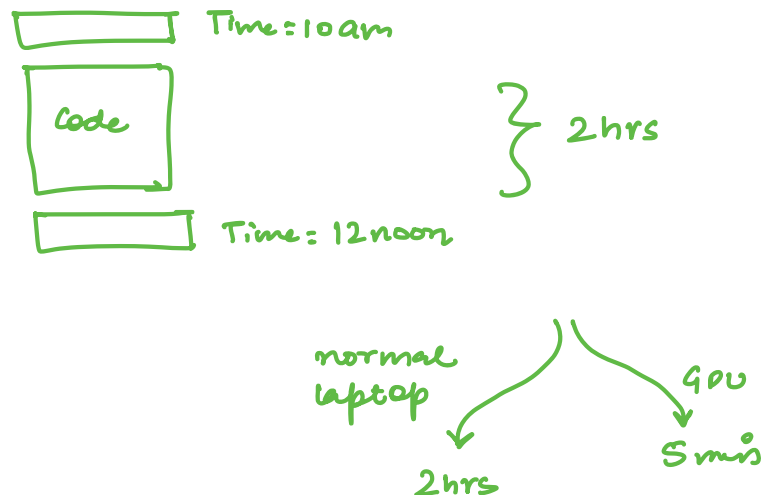


Time complexity

least time
least space



Experimental Approach:

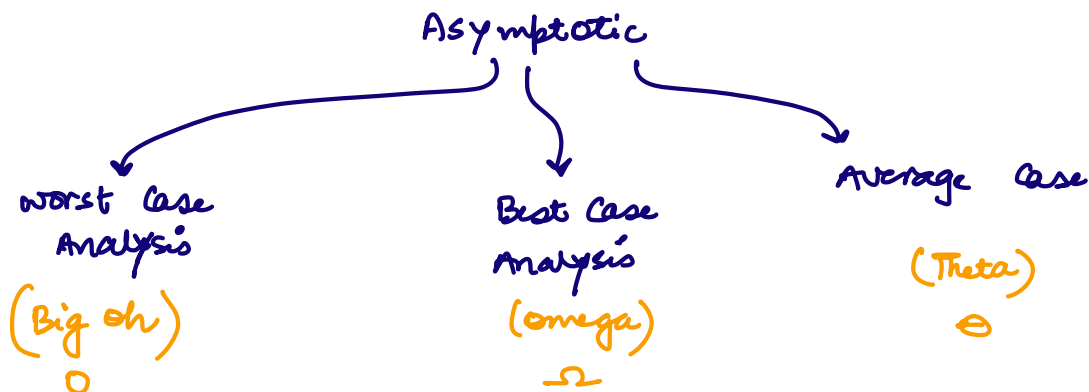


Environmental conditions / Computation Power

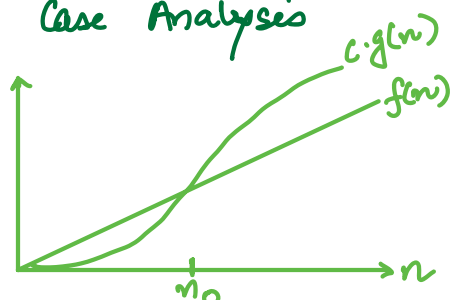
Asymptotic Analysis

how your algo is dependent on the size of input.

n



Worst Case Analysis



$$f(n) \leq c \cdot g(n) \quad \forall n > n_0 \text{ \& } c > 0$$

$$f(n) = O(g(n))$$

eg. $f(n) = n+2$

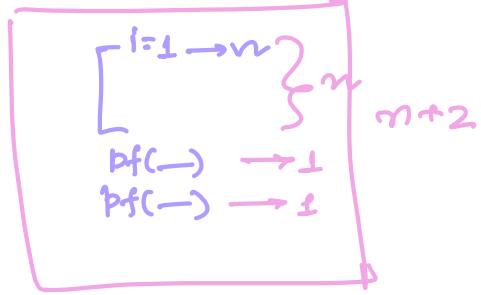
Big-Oh of $f(n)$?

$$f(n) \leq c \cdot g(n)$$

$$n+2 \leq c \cdot n$$

$$n+2 \leq 3 \cdot n$$

$n=1$	3	\leq	3
$n=2$	4	\leq	6
$n=3$	5	\leq	9
	\vdots		\vdots



$$\underbrace{n+2}_{f(n)} \leq \underbrace{3 \cdot n}_{g(n)}$$

$$\forall n \geq 1 \text{ \& } c=3$$

$$n+2 = O(n)$$

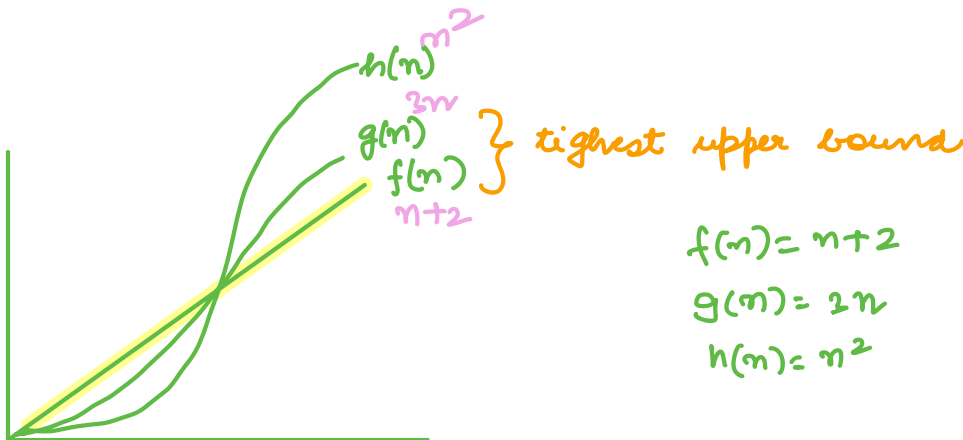
eg: $f(n) = 2n^2 + 3n + 1$

Big Oh of $f(n) = ?$

$$2n^2 + 3n + 1 \leq 6 \cdot n^2$$

$n=1$	6	\leq	6
$n=2$	$8+6+1$	\leq	24
	15	\leq	24

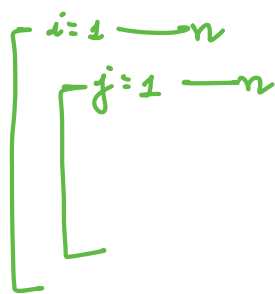
$$2n^2 + 3n + 1 = O(n^2)$$



$$f(n) = n+2$$

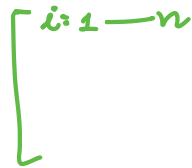
$$g(n) = 2n$$

$$h(n) = n^2$$



Pf (hello)

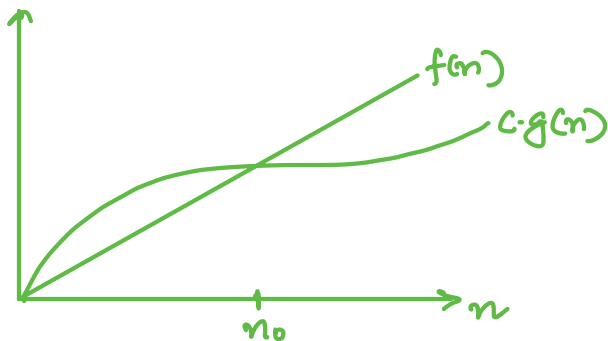
Pf (bye)



$$f(n) = n^2 + 2 + n$$

$$= O(n^2) \checkmark \text{W.C.}$$

Best Case Analysis



$$f(n) \geq c \cdot g(n) \quad \forall n \geq n_0 \text{ \& } c > 0$$

$$f(n) = \Omega(g(n))$$

Linear Search

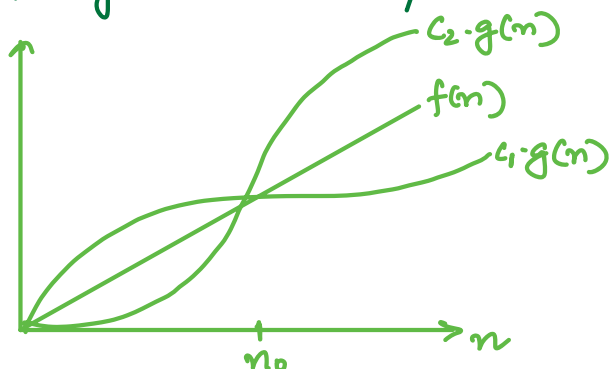
0	1	2	3	4
10	20	5	100	50

$n=5$

$$BC = \Omega(1)$$

$$WC = \underbrace{n-1}_{f(n)} \leq \underbrace{1 \cdot n}_c \underbrace{=}_{g(n)} O(n)$$

Average Case Analysis



WC & BC both are same


$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$f(n) = \Theta(g(n))$$

Time Complexity Q's

1. $i = 1$
 while ($i \leq n$)
 {
 pf(DTU);
 $i++$;
 } $O(n)$

2. $i = 0$
 while ($i \leq n$)
 {
 pf(DTU)
 $i += 2$;
 } $\frac{n}{2}$
 $O(n)$



3. $i = 1$
 while ($i \leq n$)
 {
 pf(DTU)
 $i = i + 2$;
 $i = i + 3$;
 } $\frac{n}{5}$
 $O(n)$

$i = i + 5$

4. $i = 1$
 while ($i \leq n$)
 {
 pf(DTU);
 $i = i * 2$;
 } k times

$i > n$
 $2^k > n$
 $k = \log_2 n$

time	i value
1	2^1
2	2^2
3	2^3
4	2^4
\vdots	
k	2^k

5. $i = 1$
 while ($i \leq n$)
 {
 pf(DTU);
 $i = i * 3$;
 } $\log_3 n$

6. $i = 1$
 while ($i \leq n$)
 {
 pf(DTU);
 $i = i * 2$;
 $i = i * 3$;
 } $\log_6 n$

$i = i * 6$

7. while ($n > 0$)
 {
 pf(DTU)
 $n = n / 2$;
 } $\log_2 n$

8. while ($n > 0$)
 {
 pf(DTU)
 $n = n / 3$;
 } $\log_3 n$

9. while ($n > 0$)
 {
 pf(DTU)
 $n = n / 2$;
 $n = n / 3$;
 } $\log_6 n$

Nested loops

inner loop
outer loop
independent

dependent (unroll)

```
for (i=1 — n) → n
{
  for (j=1 — n) → n } n^2
}
```

```
for (i=1 — n)
{
  for (j=1 — i)
  {
  }
```

inner most
line: no of
times
execute

Q: for (i=1; i ≤ n; i++)
{
 for (j=1; j ≤ n; j=j+i)
 {
 pf(DTO);
 }
}

<u>i=1</u> j=1; j ≤ n; j=j+1 n times	<u>i=2</u> j=1; j ≤ n; j=j+2 $\frac{n}{2}$ times	<u>i=3</u> j=1; j ≤ n; j=j+3 $\frac{n}{3}$...	<u>i=n</u> j=1; j ≤ n; j=j+n $\frac{n}{n} = 1$ time
--	--	--	-----	---

$$n + \frac{n}{2} + \frac{n}{3} + \dots + \frac{n}{n}$$

$$n \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$

$n \log n$

$$\sum_{i=1}^n \frac{1}{i} = \log i \Big|_{i=1}^n$$

$= \log n$

Q: $i=1, S=1$

while ($i \leq n$)

{

pf(OTU);

$S = S + i$;

$i++$;

}

$\left. \begin{array}{l} i=1 \\ i \leq n \\ i++ \end{array} \right\} O(n)$

$i=1, S=0$

while ($S \leq n$)

{

pf(OTU);

$S = S + i$;

$i++$;

}

} k times

$i=1$ 1st time $\rightarrow S=1$

$i=2$ 2nd time $\rightarrow S=1+2$

$i=3$ 3rd $\rightarrow S=1+2+3$

$i=k$ kth $\rightarrow S=1+2+3+\dots+k$

$S > n$

$1+2+3 \dots k > n$

$\frac{k(k+1)}{2} > n$

$k^2 > n$

$k = \sqrt{n}$

Q: for ($i=1; i \leq k; i++$) $\rightarrow k$

{

for ($j=1; j \leq \frac{n}{k}; j++$) $\rightarrow \frac{n}{k}$

{

pf(OTU);

}

}

$k \times \frac{n}{k} = O(n)$

$i++$ \rightarrow 10, 20, 30, 40, 50, 60 $\leftarrow j--$

$O(n)$

Palindrome
Reverse

$i=0$

$j=n-1$

while ($i \leq j$)

{

$i++$

$j--$

} $\frac{n}{2} = O(n)$

Prime

SoE

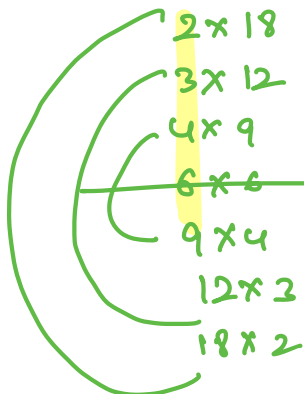
<https://leetcode.com/problems/count-primes/description/>
<https://www.geeksforgeeks.org/dsa/sieve-of-eratosthenes/>

n given Prime? \checkmark / \times

\downarrow
 \sqrt{n}

36

$$\sqrt{36} = \underline{6}$$



$$n = 10^5$$

$$1 \rightarrow 10^5$$

\downarrow
 \sqrt{n}

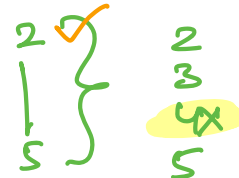
SoE

not at: true
not: false

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

$$1 \rightarrow \textcircled{25}$$

$$\sqrt{25} = 5$$



$$n = 25$$

$$0 \rightarrow 25$$

$$0 \quad 1$$

$\textcircled{2 \times 3}$
 $2 \times 2 \times$
 $3 \times 3 \checkmark$

multiple?
table
 $2 \times 2 = 4$
 $2 \times 3 = 6$
 $2 \times 4 = 8$
 $2 \times 5 = 10$
 $3 \times 2 = 6$
 $3 \times 3 = 9$
 $3 \times 4 = 12$
 $3 \times 5 = 15$
 $4 \times 2 = 8$
 $4 \times 3 = 12$
 $4 \times 4 = 16$
 $4 \times 5 = 20$
 $5 \times 2 = 10$
 $5 \times 3 = 15$
 $5 \times 4 = 20$
 $5 \times 5 = 25$

$\textcircled{25}$