

TM acceptor
 $w \in L$
 $w \notin L$

TM transducer
output

TRANSDUCER

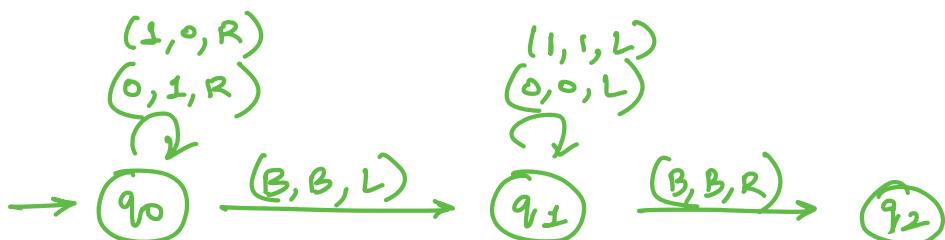
Eg: TM to find 1's complement of a binary no.

Input: 0011

$\dots |B|0|0|1|1|B|\dots$

Output: 1100

$\dots |B|1|1|0|0|B|\dots$



Eg: TM for computing 2's complement

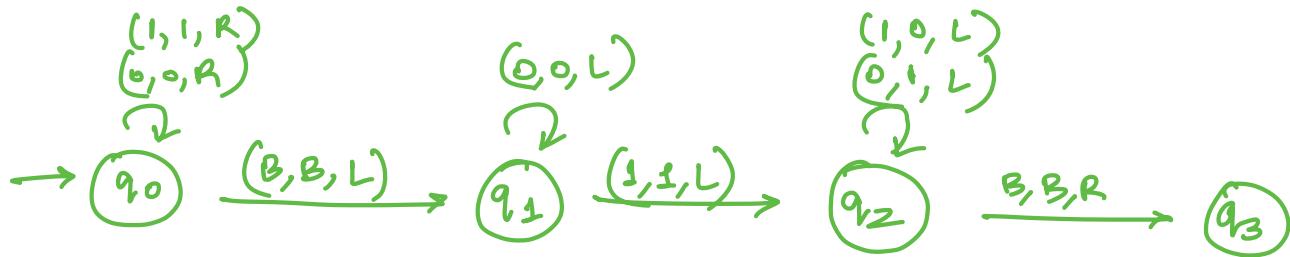
Original: 0111010000

1's Comp: 1000101111

+

$$\begin{array}{r}
 & + \\
 1000101111 & \\
 \hline
 1000110000
 \end{array}$$

1000110000



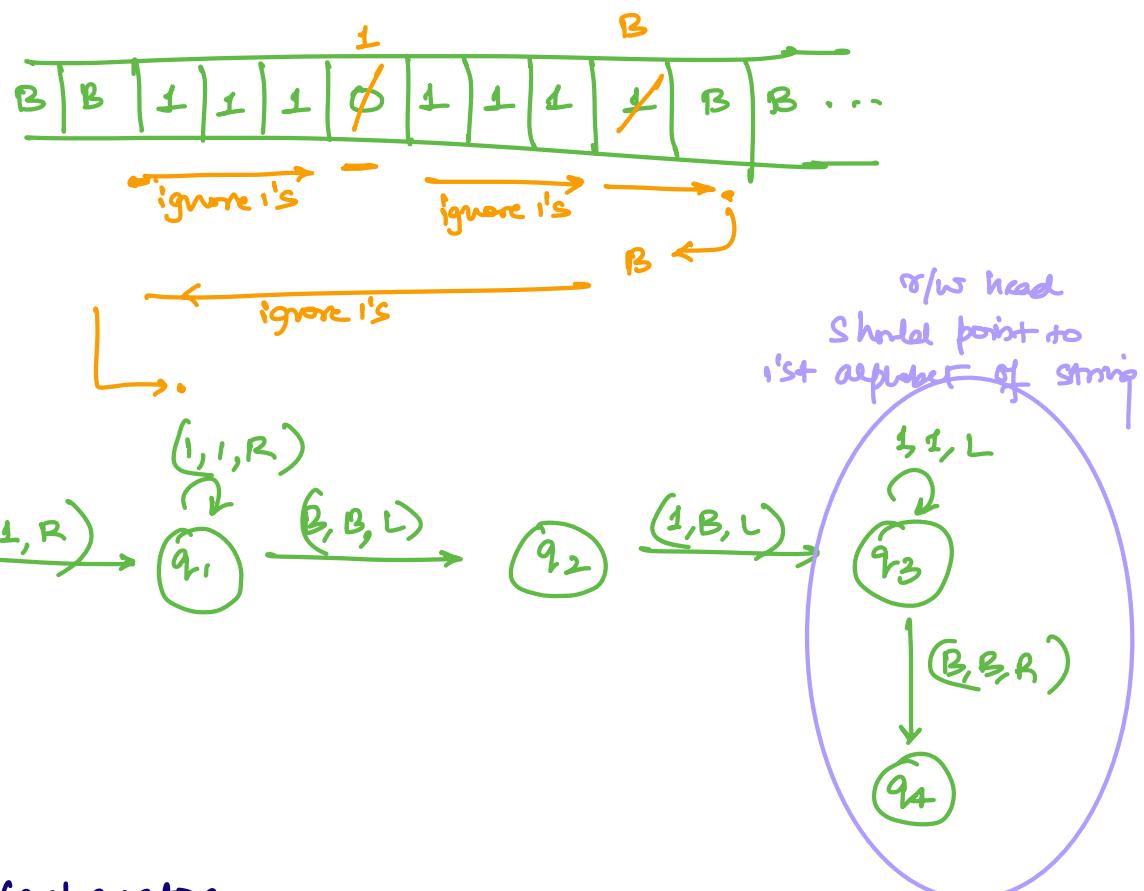
Eg: TM as adder

$$3 \rightarrow 111$$

$$|x|= \text{no.}$$

$$4 \rightarrow 1111$$

$$2 \rightarrow 11$$



Eg: TM as comparator

$$a \ b$$

$$a = b$$

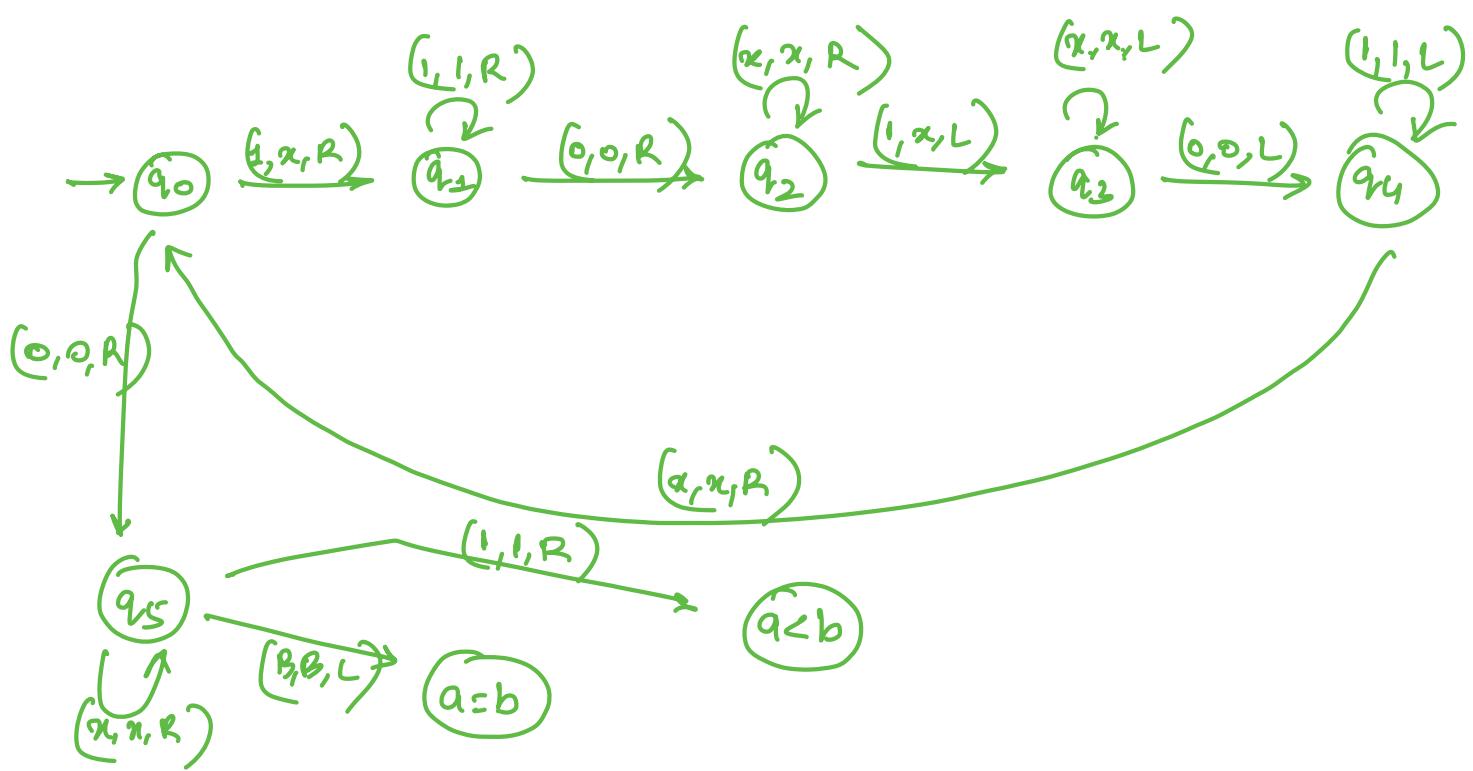
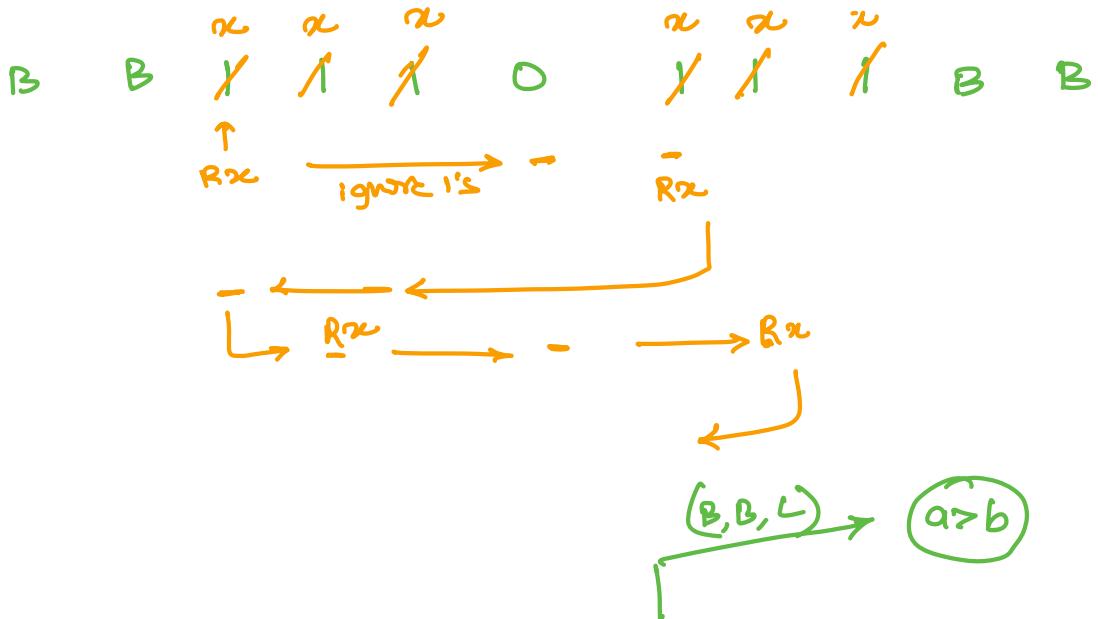
$$a > b$$

$$a < b$$

$$\frac{111}{3} = \frac{0111}{3} \quad a = b$$

$$\frac{1111}{4} = \frac{0111}{3} \quad a > b$$

$$\frac{111}{3} = \frac{01111}{5} \quad a < b$$



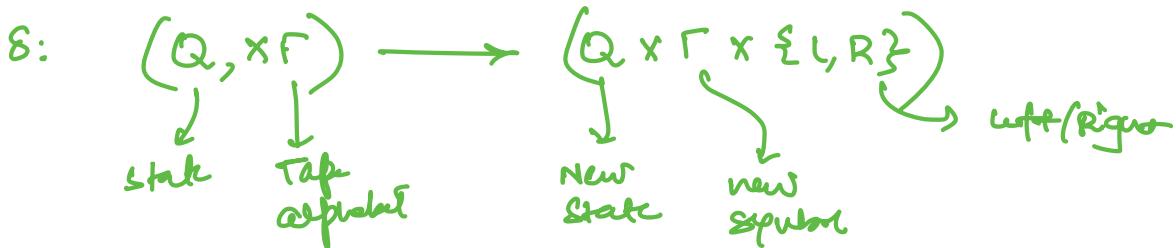
$B \ \overbrace{X \ 1 \ 1}^n \ \alpha \ 0 \ \overbrace{X \ 1 \ 1}^n \ \alpha \ 1 \ B$

$B \ \overbrace{1 \ 1 \ 1 \ 1}^n \ \alpha \ \overbrace{X \ X \ X}^n \ B$

$\underbrace{\quad}_{4} \quad \underbrace{\quad}_{3}$

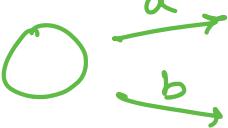
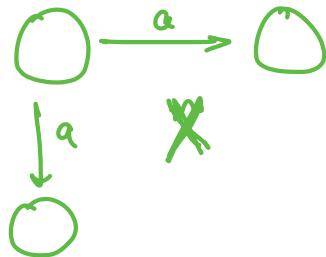
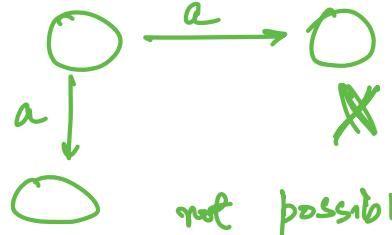
Definition of Standard TM

$M = (\mathbb{Q}, \Sigma, \Gamma, \delta, q_0, B, f)$ → Initial state Final state $f \subseteq Q$



Properties:

1. Tape is unbounded, you can take any no. of left & right steps.
2. TM is deterministic

fully Deterministic?	Partially Deterministic?
<ol style="list-style-type: none"> 1.  <p>from every state for every input alphabet there will be a transition</p>	<ol style="list-style-type: none"> 1. for every state for every alphabet you don't have to show transitions <p>Eq: TM, PDA</p>
<ol style="list-style-type: none"> 2.  <p>$(Q, \Sigma) \rightarrow$ exactly one transition</p> <p>Eq: DFA</p>	<ol style="list-style-type: none"> 2.  <p>not possible</p>

Non Deterministic[?] Eg: NFA, ENFA

FA + 1 Stack = PDA

PDA + 1 Stack = TM

FA + 2Stacks = TM

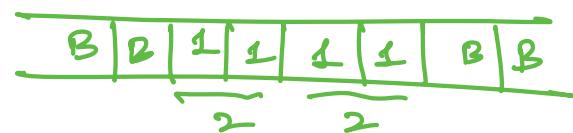
FA + Queue = TM

Eg: TM as a Copier $\rightarrow a \rightarrow 2a$

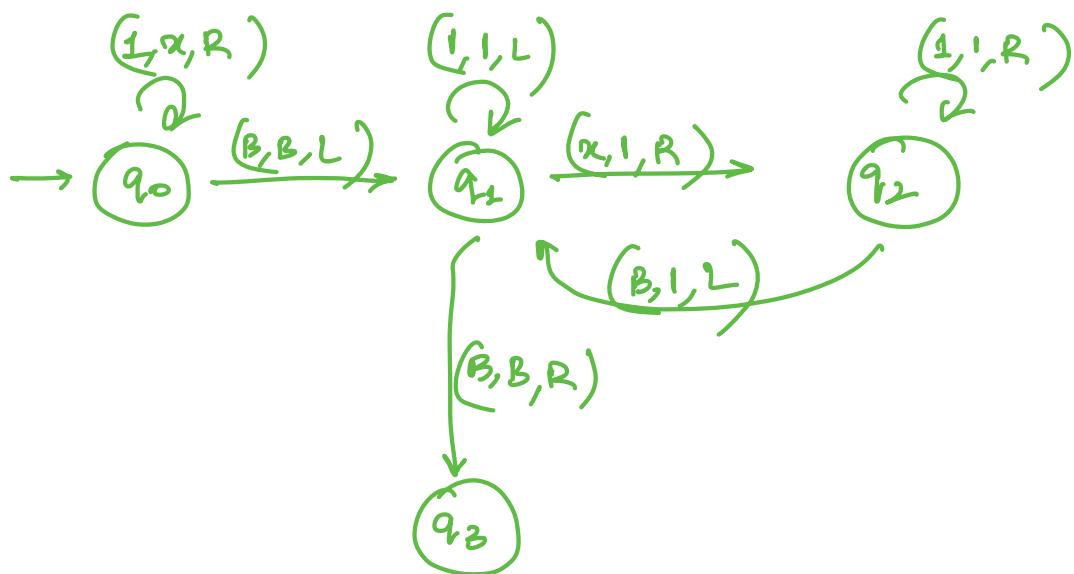
Input:



Output



$\cdots B \ B \ \cancel{1} \ \cancel{1} \ \cancel{1} \ \cancel{1} \ B \ B \cdots$



$$3+3 = 2^{\#3}$$

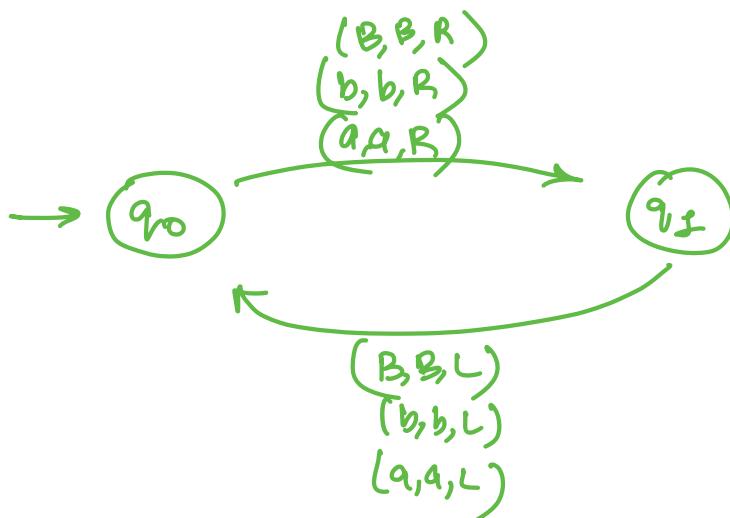
$$2^{\#3}$$

$a^{\#4}$ | TM which can do multiplication of a
TM is mathematically complete

Non Halting TM



FA, PDA,
I/P string
halt surely



halting problem
bcz of I/P string
you can move
in both dir's

.. B B a b B B ..
 ↗ ↗ ↑

$$q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_0$$

$a^n b^n$ TM m/c

I/P String: aabb

Input string is in the language

TM will halt
bcz it will halt at final state

I/P String: aab

Input string is not in the language

TM halt at
a non final state

TM never halts. ??

↓ Problem

Problem with TM halting
TM is you don't know
how long to wait
↓
non halting TM problem

Case 1:

TM is doing
some
computation

Ufeel: ∞ loop
stuck
Stop the TM

Case 2:

TM is stuck in
∞ loop

Ufeel:
Computation

Turing Thesis

Turing is a scientist, hypothesis 1930

The
Imitation
Game

Any computation that can be
carried out by mechanical means
can be performed by TM.

→ TM is as powerful as a computer

Prove?

$$P_1 \rightarrow T M_1$$

$$P_2 \rightarrow T M_2$$

P_3

|

$$P_\infty \rightarrow T M_\infty$$

Issue: ∞ problems

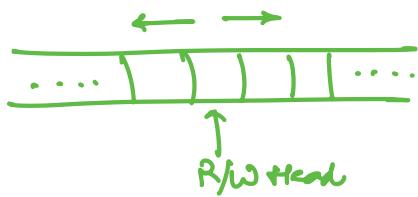
Alternative, you can come up with a problem that is
not solvable by your TM.

→ nobody was able to come up with a problem
which is not solvable by TM.

- TM & Computers are equally powerful.

Modifications / Variants of Standard TM:

Standard TM:



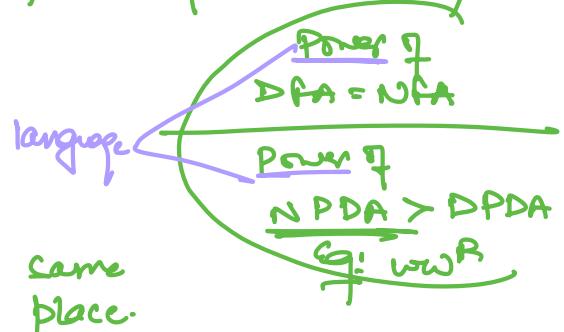
Power of TM: language accepted by TM.

→ not the time complexity or space complexity

① TM with stay option

Standard TM: left, right

Modified TM: will remain at the same place.



$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,S\}$$

Power of this TM = Power of Standard TM

② TM with semi infinite tape

Standard TM:

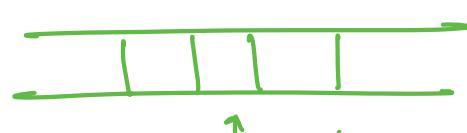
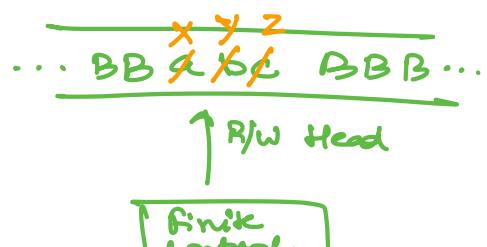


Semi-infinite TM:

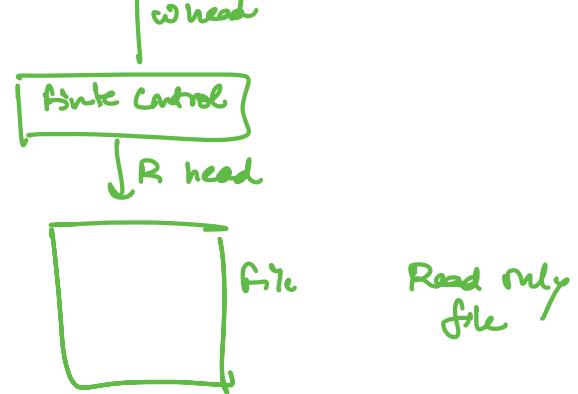


③ Offline TM

Standard TM:

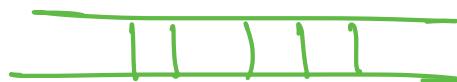


writing Tape

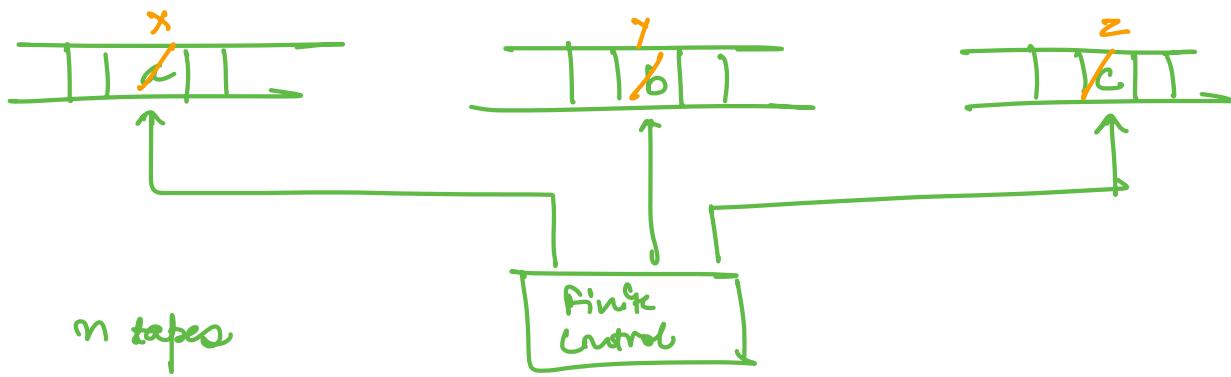


④ Multitape TM

Standard TM:



Multitape TM:



$$S: Q \times \Gamma^n \longrightarrow Q \times \Gamma^n \times \{L, R\}^m$$

$\overset{abc}{\downarrow}$ $\overset{xyz}{\downarrow}$
 m symbols new symbols
 from n m
 tapes tapes

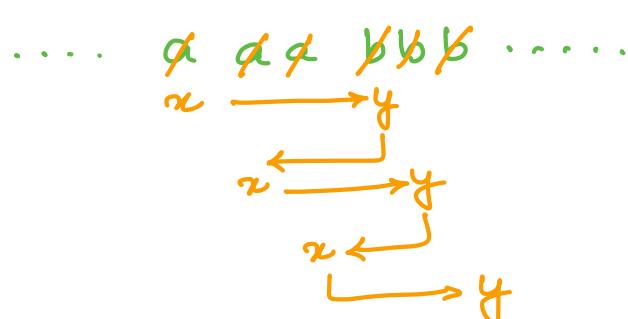
left, right
 for n tapes.

Power of Standard TM = Power of multitape TM

Benefit of Multitape TM is time is reduced.

Eg: $a^n b^n$

$a^3 b^3$:



Standard TM

You want to match n pairs
for every pair you have to move n steps.
for n pairs you have to move n^2 steps.

$$TC = O(n^2)$$

Multitape TM:

$a^2 b^3$

- ① Copy the entire input to other tape

...aaa bbb...

...aaa bbb...

$O(n)$

- ② Set the Read/write head

...aaa bbb...



...aaa bbb...



$O(n)$

- ③ Scan symbols one by one in both the tapes

...aaa bbb...



...aaa bbb...

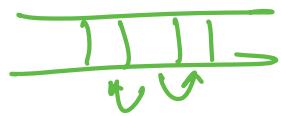


$O(n)$

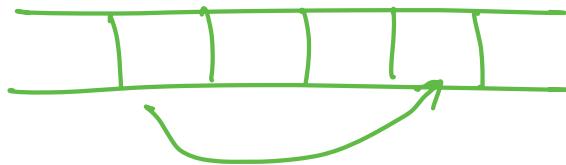
Total time: $3n = O(n)$

⑤ jumping TM

Standard TM:



jumping TM:

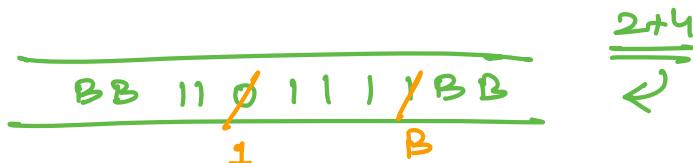


$$\delta: Q \times \Gamma = Q \times \Gamma \times \{L, R\} \times \{\tau, n\}$$

\downarrow
steps in jump

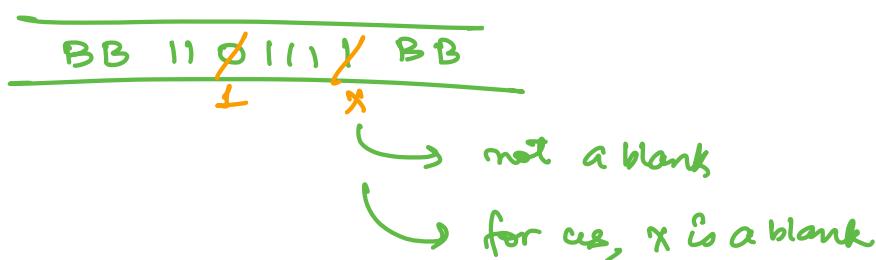
⑥ Non Erasing TM

Standard TM:



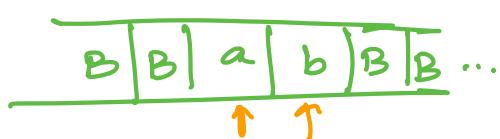
Input Symbol \leftrightarrow blank

Non Erasing TM: Remove the option of changing input to blank.



⑦ Always writing TM

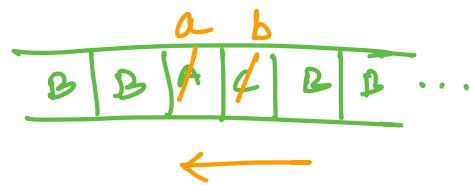
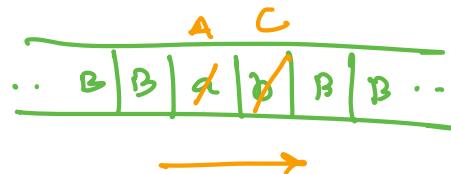
Standard TM:



You may not change
the tape alphabet

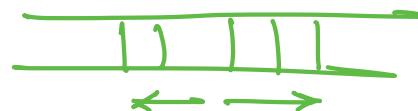
Always writing TM:

Definitely change the input alphabet

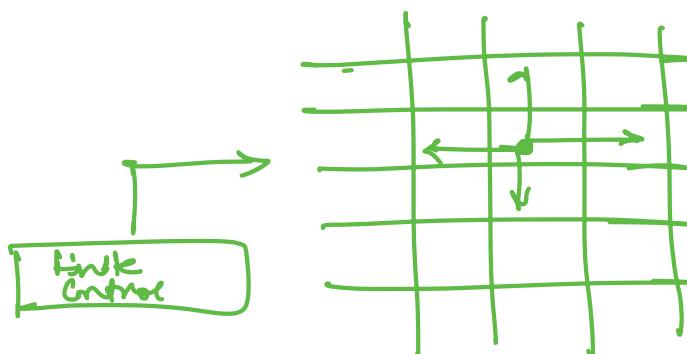


⑧ Multi dimensional TM

Standard TM:



Multi dimension:

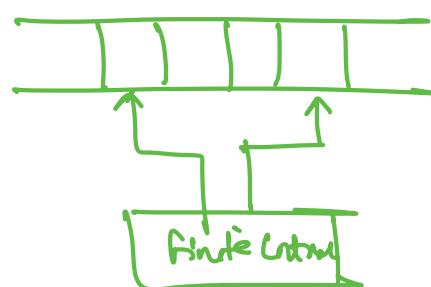


Power is same

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, U, D\}$$

⑨ Multi head TM

Single tape, read the content from multiple places at same time



10

Automata with queue

TM = Automata + Queue

11

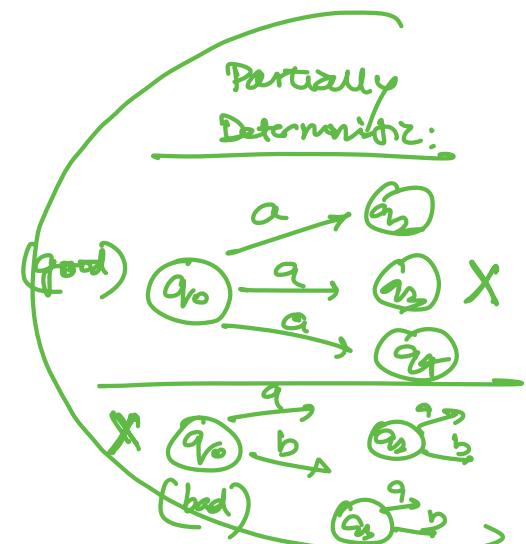
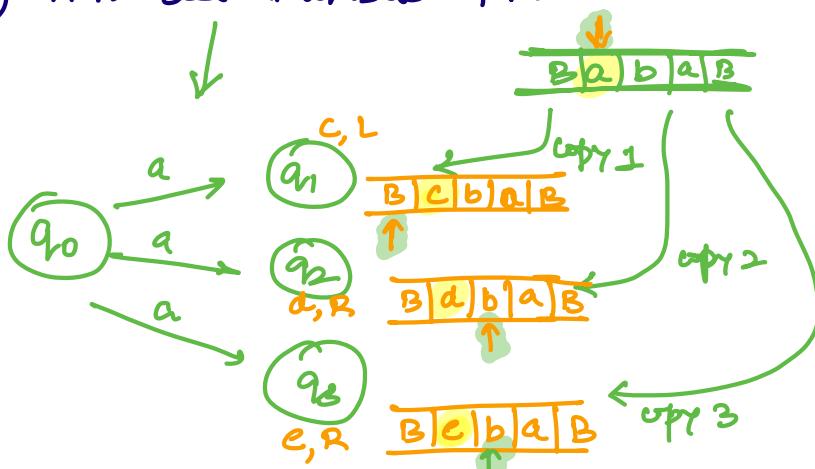
Any TM can be minimized to a TM with only 3 states

12

Any standard TM can be converted to a multitape TM with stay option and atmost 2 states.

13

Non deterministic TM



On looking at one state and one symbol we can make multiple copies & can simultaneously go in many states and can change the tape symbol

$$\delta: Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}$$

Non Deterministic TM & Deterministic TM have equal power.

$$DTM \cong NTM$$

