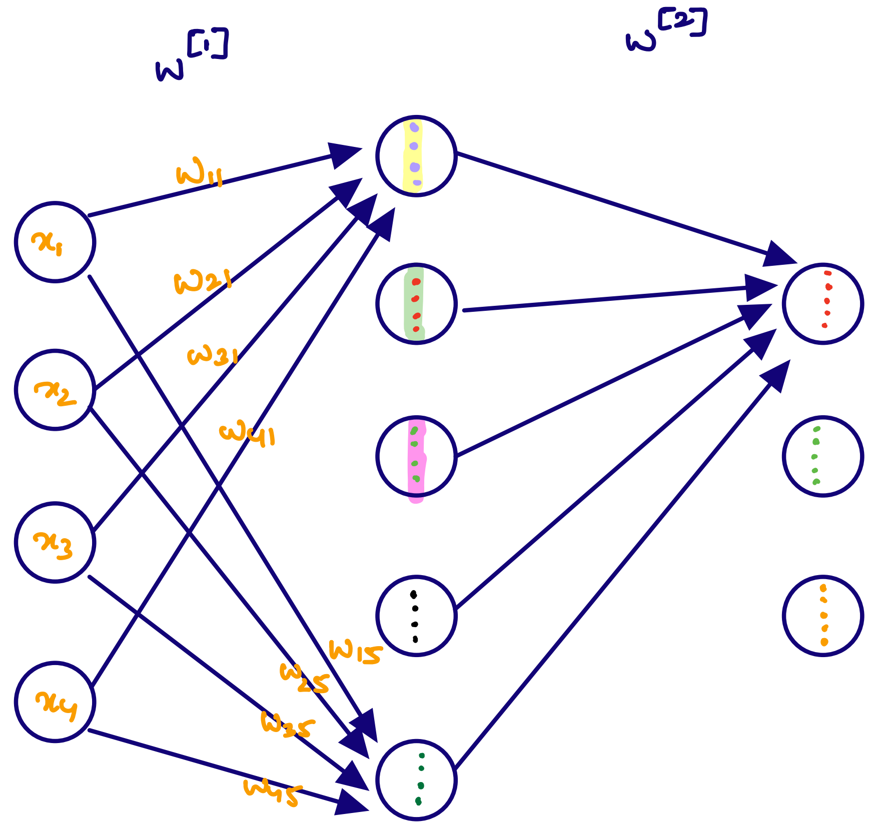


$$W^{[1]} = \begin{bmatrix} w_{11} & w_{12} & \cdot & \cdot & w_{15} \\ w_{21} & w_{22} & \cdot & \cdot & w_{25} \\ w_{31} & w_{32} & \cdot & \cdot & w_{35} \\ w_{41} & w_{42} & \cdot & \cdot & w_{45} \end{bmatrix}$$

4×5

no. of neuron in i/p layer

no. of neuron in hidden layer

$$W^{[1]} = \begin{bmatrix} | & | & | & | & | \\ w_1^{[1]} & w_2^{[1]} & w_3^{[1]} & w_4^{[1]} & w_5^{[1]} \\ | & | & | & | & | \end{bmatrix}$$

$$W^{[2]} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

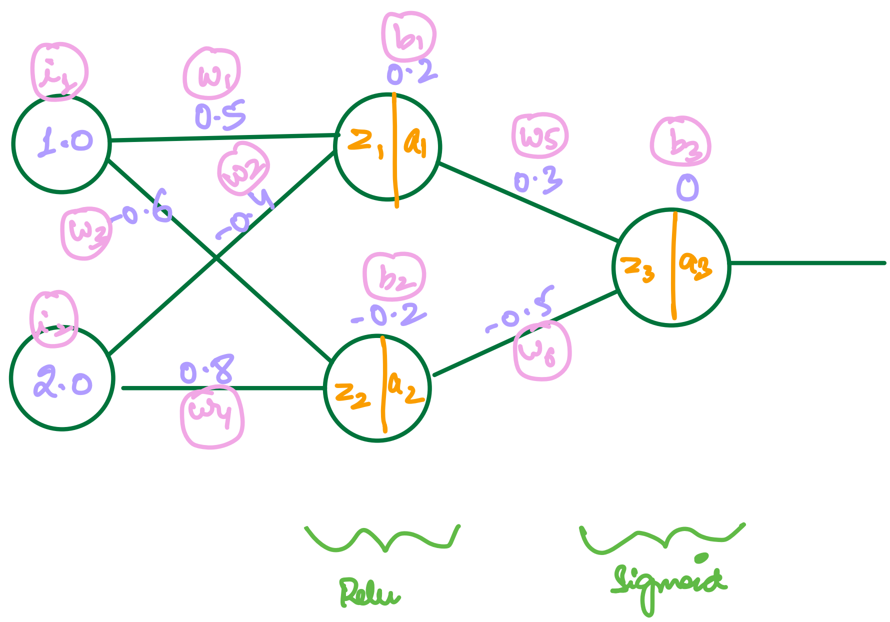
4. Answer *any TWO* of the followings

[a] Consider a two-layer neural network used for binary classification. The network has an input layer with 2 neurons, a hidden layer with 2 neurons, and an output layer with 1 neuron. The activation function for the hidden layer is ReLU (Rectified Linear Unit), and for the output layer, it's a sigmoid function. The network is trained using the binary cross-entropy loss function and stochastic gradient descent (SGD) with a learning rate of 0.01. The initial weights and biases are as follows: Weights from input to hidden layer: W_1 = [[0.5, -0.6], [-0.4, 0.8]], Bias for hidden layer: b_1 = [0.2, -0.2], Weights from hidden to output layer: W_2 = [0.3, -0.5], Bias for output layer: b_2 = 0. Consider the network is trained with a single training sample (X = [1.0, 2.0], Y = 0). Perform the forward pass to calculate activations at hidden layer and output layer, and then compute the loss. **[4] [CO2]**

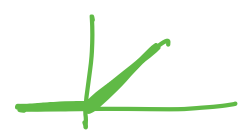
[b] Consider the neural network in 4[a] again and perform the backpropagation to update the weights and biases. Calculate the updated weights W_1, W_2, and biases b_1, b_2 after one iteration. Show your calculations for the forward pass, loss calculation, and backpropagation steps. **[4] [CO2]**

sel 1st neuron sel 2nd neuron

$$\begin{bmatrix} 0.5 & -0.6 \\ -0.4 & 0.8 \end{bmatrix}$$



$z_1 = i_1 * w_1 + i_2 * w_2 + b_1 \quad = 1 * 0.5 + 2 * (-0.4) + 0.2 = -0.1$



$a_1 = \text{Relu}(z_1) = 0$

$\text{Relu}(z_1) : \begin{cases} 0 & \text{if } z \le 0 \\ z_1 & z > 0 \end{cases}$

$z_2 = i_1 * w_3 + i_2 * w_4 + b_2 = 1 * (-0.6) + 2 * (0.8) - 0.2$
$= 0.8$

$a_2 = \text{Relu}(z_2) : 0.8$

$z_3 = a_1 * w_5 + a_2 * w_6 + b_3 = 0 * 0.3 + 0.8 * (-0.5) + 0 = -0.4$

$a_3 = \text{Sigmoid}(z_3) = \frac{1}{1+e^{-z_3}} = \frac{1}{1+e^{0.4}} = \underline{0.401}$ → Dog

≥0.5 → Cat
<0.5 → Dog

→ forward pass

LR

weights random

- times loop
- how good ur weights are? → loss fxⁿ → value
- weights update → GD

→ loss function

$$BCE = -\sum_{i=1}^{m}\left(y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right)$$

$m=1$

$$J = -\left(y\log\hat{y} + (1-y)\log(1-\hat{y})\right)$$

$$= -\left(0\log 0.401 + (1-0)\log(1-0.401)\right)$$

$$= -\log 0.599 = 0.225$$

$\hat{y} = a_3 = 0.401$

TASK:

Weights update? → GD

$x = x - \eta \dfrac{dy}{dx}$

$y=(x-5)^2$

Value of x for which y is minⁿ

Value of weight for which loss is minimum.

$$w = w - \eta\frac{\partial J}{\partial w}$$

Backpropagation

$$* \quad \frac{\partial J}{\partial w_5} = \underset{①}{\frac{\partial J}{\partial a_3}} \cdot \underset{②}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{③}{\frac{\partial z_3}{\partial w_5}}$$

1. $\frac{\partial J}{\partial a_3}$

$$J = -\left(y \log \hat{y} + (1-y)\log(1-\hat{y})\right)$$

$$\hat{y} = a_3$$

$$J = -\left(y \log a_3 + (1-y)\log(1-a_3)\right)$$

$$\frac{\partial J}{\partial a_3} = -\left(\frac{y}{a_3} - \frac{(1-y)}{(1-a_3)}\right) = -\left[\frac{y(1-a_3) - a_3(1-y)}{a_3(1-a_3)}\right]$$

$$= -\left[\frac{y - \cancel{ya_3} - a_3 + \cancel{a_3 y}}{a_3(1-a_3)}\right]$$

$$= \frac{a_3 - y}{a_3(1-a_3)}$$

2. $\frac{\partial a_3}{\partial z_3}$

$$a_3 = \frac{1}{1+e^{-z_3}}$$

$$\frac{\partial a_3}{\partial z_3} = (a_3)(1-a_3)$$

sigmoid

$$y = \frac{1}{1+e^{-x}} = (1+e^{-x})^{-1}$$

$$\frac{\partial y}{\partial x} = \frac{-1}{(1+e^{-x})^2}(-e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{(1+e^{-x})} \cdot \frac{e^{-x}}{(1+e^{-x})}$$

$$= \left(\frac{1}{1+e^{-x}}\right)\left(1 - \frac{1}{1+e^{-x}}\right)$$

$$= y(1-y) \rightarrow \text{Remember this}$$

3. $\frac{\partial z_3}{\partial w_5}$

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial w_5} = a_1$$

$$\frac{\partial J}{\partial w_5} = (1)\,(2)\,(2)$$

$$= \frac{a_3 - y}{a_3(\cancel{1-a_3})} \cdot a_3(\cancel{1-a_3}) \cdot a_1 = (a_3 - y)\, a_1$$

$$w_5 = w_5 - \eta \frac{\partial J}{\partial w_5}$$

$$= w_5 - \eta\, (a_3 - y) a_1$$

$$= 0.3 - 0.01\,(0.401 - 0) \cdot 0 = 0.3 \checkmark$$

$$* \quad \frac{\partial J}{\partial w_6} = \underset{①}{\frac{\partial J}{\partial a_3}} \cdot \underset{②}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{④}{\frac{\partial z_3}{\partial w_6}}$$

same

④ $\frac{\partial z_3}{\partial w_6}$?

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial w_6} = a_2$$

$$\frac{\partial J}{\partial w_6} = ① ② ④$$

$$= \frac{a_3 - y}{\cancel{a_3(1-a_3)}} \cdot \cancel{a_3(1-a_3)} \cdot a_2 = (a_3 - y) a_2$$

$$w_6 = w_6 - \eta \frac{\partial J}{\partial w_6}$$

$$= w_6 - \eta\, (a_3 - y) a_2$$

$$= -0.5 - (0.01)(0.401 - 0)(0.8) = -0.5032 \checkmark$$

$$* \quad \frac{\partial J}{\partial w_1} = \frac{\partial J}{\partial a_3} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$\qquad\qquad (1) \qquad\quad (2) \qquad\quad (5) \qquad\quad (6) \qquad\quad (7)$$

(5) $\dfrac{\partial z_3}{\partial a_1}$

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial a_1} = w_5$$

(6) $\dfrac{\partial a_1}{\partial z_1}$

$$a_1 = \mathrm{Relu}(z_1) = \begin{cases} 0 & \text{if } z_1 \leq 0 \\ z_1 & \text{if } z_1 > 0 \end{cases}$$

$$\frac{\partial a_1}{\partial z_1} = \begin{cases} 0 & \text{if } z_1 \leq 0 \\ 1 & \text{if } z_1 > 0 \end{cases}$$

(7) $\dfrac{\partial z_1}{\partial w_1}$

$$z_1 = i_1 w_1 + i_2 w_2 + b_1$$

$$\frac{\partial z_1}{\partial w_1} = i_1$$

$$\frac{\partial J}{\partial w_1} = (1)\ (2)\ (5)\ (6)\ (7)$$

$$= \frac{a_3 - y}{a_3(1-a_3)} \cdot a_3(1-a_3) \cdot w_5 \cdot \left\{ \begin{matrix} 0 & \text{if } z_1 \leq 0 \\ 1 & \text{if } z_1 > 0 \end{matrix} \right\} \cdot i_1$$

$$= (a_3 - y)\, w_5 \cdot \begin{cases} 0 & \text{if } z_1 \le 0 \\ 1 & \text{if } z_1 > 0 \end{cases} \cdot i_1$$

$$= (0.401 - 0)(0.3)(0)\cdot(1.0) = 0$$

$$w_1 = w_1 - \eta \frac{\partial J}{\partial w_1}$$

$$= 0.5 - (0.01)(0) = 0.5 \checkmark$$

$$* \quad \frac{\partial J}{\partial w_2} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{5}{\frac{\partial z_3}{\partial a_1}} \cdot \underset{6}{\frac{\partial a_1}{\partial z_1}} \cdot \underset{8}{\frac{\partial z_1}{\partial w_2}}$$

8 $\quad \dfrac{\partial z_1}{\partial w_2}$?

$$z_1 = i_1 w_1 + i_2 w_2 + b_1$$

$$\frac{\partial z_1}{\partial w_2} = i_2$$

$$\frac{\partial J}{\partial w_2} = 1\ 2\ 5\ 6\ 8$$

$$= \underbrace{\frac{a_3 - y}{a_3(1-a_3)}} \underbrace{(a_3)(1-a_3)} \underbrace{w_5} \underbrace{\begin{matrix} 0 & \text{if } z_1 \le 0 \\ 1 & \text{if } z_1 > 0 \end{matrix}} \underbrace{i_2}$$

$$= (a_3 - y)(w_5)\begin{pmatrix} 0 & \text{if } z_1 \le 0 \\ 1 & \text{if } z_1 > 0 \end{pmatrix}(i_2)$$

$$= (0.401 - 0)(0.3)(0)(2.0) = 0$$

$$w_2 = w_2 - \eta \frac{\partial J}{\partial w_2}$$

$$= -0.4 - (0.01)(0) = -0.4 \checkmark$$

$$* \quad \frac{\partial J}{\partial w_3} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{9}{\frac{\partial z_3}{\partial a_2}} \cdot \underset{10}{\frac{\partial a_2}{\partial z_2}} \cdot \underset{11}{\frac{\partial z_2}{\partial w_3}}$$

9 $\quad \frac{\partial z_3}{\partial a_2}$?

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial a_2} = w_6$$

10 $\quad \frac{\partial a_2}{\partial z_2}$?

$$a_2 = \text{Relu}(z_2) = \begin{cases} 0 & \text{if } z_2 \le 0 \\ z_2 & \text{if } z_2 > 0 \end{cases}$$

$$\frac{\partial a_2}{\partial z_2} = \begin{cases} 0 & \text{if } z_2 \le 0 \\ 1 & \text{if } z_2 > 0 \end{cases}$$

11 $\quad \frac{\partial z_2}{\partial w_3}$?

$$z_2 = i_1 w_3 + i_2 w_4 + b_2$$

$$\frac{\partial z_2}{\partial w_3} = i_1$$

$$\frac{\partial J}{\partial \omega_3} = 1 \quad 2 \quad 9 \quad 10 \quad 11$$

$$= \underbrace{\frac{a_3 - y}{a_3(1-a_3)}} \quad \underbrace{(a_3)(1-a_3)} \quad \underbrace{\omega_6} \quad \underbrace{\begin{matrix} 0 & \text{if } z_2 \le 0 \\ 1 & \text{if } z_2 > 0 \end{matrix}} \quad \underbrace{i_1}$$

$$= (a_3 - y)\,(\omega_6)\begin{pmatrix} 0 & \text{if } z_2 \le 0 \\ 1 & \text{if } z_2 > 0 \end{pmatrix}(i_1)$$

$$= (0.401 - 0)\,(-0.5)\,(1)\,(1) = -0.2005$$

$$\omega_3 = \omega_3 - \eta \frac{\partial J}{\partial \omega_3}$$

$$= -0.6 - (0.01)(-0.2005) = -0.6 + 0.002005 = -0.597$$

$$* \quad \frac{\partial J}{\partial \omega_4} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{9}{\frac{\partial z_3}{\partial a_2}} \cdot \underset{10}{\frac{\partial a_2}{\partial z_2}} \cdot \underset{12}{\frac{\partial z_2}{\partial \omega_4}}$$

12 $\quad \dfrac{\partial z_2}{\partial \omega_4}$?

$$z_2 = i_1 w_3 + i_2 w_4 + b_2$$

$$\frac{\partial z_2}{\partial \omega_4} = i_2$$

$$\frac{\partial J}{\partial \omega_4} = 1 \quad 2 \quad 9 \quad 10 \quad 12$$

$$= \underbrace{\frac{a_3 - y}{a_3(1-a_3)}} \quad \underbrace{(a_3)(1-a_3)} \quad \underbrace{\omega_6} \quad \underbrace{\begin{matrix} 0 & \text{if } z_2 \le 0 \\ 1 & \text{if } z_2 > 0 \end{matrix}} \quad \underbrace{i_2}$$

$$= (a_3 - y)\,(w_6) \begin{pmatrix} 0 & \text{if } z_2 \le 0 \\ 1 & \text{if } z_2 > 0 \end{pmatrix} (i_2)$$

$$= (0.401 - 0)\,(-0.5)\,(1)\,(2) \;=\; -0.401$$

$$w_4 = w_4 - \eta \frac{\partial J}{\partial w_4}$$

$$= 0.8 - (0.01)(-0.401) \;=\; 0.8 + 0.00401 \;=\; 0.80401 \checkmark$$

$$* \quad \frac{\partial J}{\partial b_3} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{13}{\frac{\partial z_3}{\partial b_3}}$$

13 $\quad \frac{\partial z_3}{\partial b_3}$?

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial b_3} = 1$$

$$\frac{\partial J}{\partial b_3} = 1 \; 2 \; 13$$

$$= \frac{a_3 - y}{a_3\cancel{(1-a_3)}} \cdot a_3\cancel{(1-a_3)} \cdot 1 \;=\; (a_3 - y)$$

$$b_3 = b_3 - \eta \frac{\partial J}{\partial b_3}$$

$$= b_3 - \eta\,(a_3 - y)$$

$$= 0 - 0.01\,(0.401 - 0) \;=\; -0.00401 \checkmark$$

$$* \quad \frac{\partial J}{\partial b_1} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{5}{\frac{\partial z_3}{\partial a_1}} \cdot \underset{6}{\frac{\partial a_1}{\partial z_1}} \cdot \underset{14}{\frac{\partial z_1}{\partial b_1}}$$

14 $\dfrac{\partial z_1}{\partial b_1}$?

$$z_1 = i_1 w_1 + i_2 w_2 + b_1$$

$$\frac{\partial z_1}{\partial b_1} = 1$$

$$\frac{\partial J}{\partial b_1} = 1 \; 2 \; 5 \; 6 \; 14$$

$$= \frac{(a_3 - y)}{a_3(1-a_3)} \cdot a_3(1-a_3) \cdot w_5 \cdot \begin{cases} 0 & \text{if } z_1 \leq 0 \\ 1 & \text{if } z_1 > 0 \end{cases} \cdot 1$$

$$= (a_3 - y) \cdot w_5 \begin{cases} 0 & \text{if } z_1 \leq 0 \\ 1 & \text{if } z_1 > 0 \end{cases} \cdot 1$$

$$= (0.401 - 0)(0.3)(0) \cdot 1 = 0$$

$$b_1 = b_1 - \eta \frac{\partial J}{\partial b_1}$$

$$= 0.2 - (0.01)(0) = 0.2 \checkmark$$

$$* \; \frac{\partial J}{\partial b_2} = \underset{1}{\frac{\partial J}{\partial a_3}} \cdot \underset{2}{\frac{\partial a_3}{\partial z_3}} \cdot \underset{9}{\frac{\partial z_3}{\partial a_2}} \cdot \underset{10}{\frac{\partial a_2}{\partial z_2}} \cdot \underset{15}{\frac{\partial z_2}{\partial b_2}}$$

15 $\dfrac{\partial z_2}{\partial b_2}$?

$$z_2 = i_1 w_3 + i_2 w_4 + b_2$$

$$\frac{\partial z_2}{\partial b_2} = 1$$

$$\frac{\partial J}{\partial b_2} = 1 \; 2 \; 9 \; 10 \; 12$$

$$= \frac{a_3 - y}{a_3(1-a_3)} \quad a_3(1-a_3) \quad w_6 \begin{cases} 0 & \text{if } z_2 \leq 0 \\ 1 & \text{if } z_2 > 0 \end{cases} \cdot 1$$

$$= (a_3 - y)\, w_6 \begin{cases} 0 & \text{if } z_2 \leq 0 \\ 1 & \text{if } z_2 > 0 \end{cases}$$

$$= (0.401 - 0)(-0.5)(1) = -0.2005$$

$$b_2 = b_2 - \eta \frac{\partial J}{\partial b_2}$$

$$= -0.2 - (0.01)(-0.2005) = -0.197995 \quad ✓$$