

Homework Assignment #1 – Due Monday Feb 11, 2019 by 10am.

Part 1 (2 points)

Choose a type of biological network that interests you.

For this network, provide:

- A description of the meaning of a node in this network.
- A description of the meaning of an edge in this network.
- A reference to a paper that discusses this type of network. Summarize in a few lines whether and why the network representation is useful in this paper.

Part 2 (4 points)

A bacterial cell has the shape of a sphere with radius R . This cell produces ATP (using membrane-bound ATP synthetases) at a rate A per unit surface. The cellular internal machinery consumes ATP at a rate B per unit volume. Assume that A and B , rather than having fixed values, depend themselves on R as follows: $A = A_0/R$; $B = B_0/R$, with A_0 and B_0 constant.

- Express the ATP production (p) and consumption (c) rates as a function of R . What is the maximal radius (R_{max}) energetically sustainable by this cell?
- At what cell radius (R_s) will the cell produce maximal surplus of ATP?
- If the cell is growing from a radius R_s to a radius R_{max} , what is the total surplus ATP produced throughout its growth process? (hint: this question requires computing an integral)

Part 3 (4 points)

Consider the following coupled logistic equations for two replicators, with concentrations x and y :

$$\begin{aligned}\frac{dx}{dt} &= r_1 x \left(1 - \frac{x + y}{K_1} \right) \\ \frac{dy}{dt} &= r_2 y \left(1 - \frac{x + y}{K_2} \right)\end{aligned}$$

- List the possible steady states of these equations (assuming $K_1 > K_2$), and determine which of these steady states is stable/unstable.
- Solve the equations numerically, using Matlab, Python or an equivalent programming language (See sample scripts provided with lectures). Try 6 different sets of values of the intrinsic growth parameters (r_1 , r_2), and carrying capacity (K_1 , K_2), and plot the time dependent abundances of the two species under each set of parameters.
- Find a set of parameters that will lead to coexistence of the two species.
- Show through examples or reasoning what parameter(s) determine the winner of the competition in case the two species don't coexist.