Gamma
3CO24 (102102663)

Assignment - Parameter estimation

sol1) Given a random sample $(X_1, - X_n)$ from normal distribution

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\left(\frac{(X_i - \mu)^2}{2\sigma^2}\right)\right)$$

Taking natural log of likelihood funn

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(X_i - \mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\right)$$

To find MLE's we differentiate log likelihood funn w.r.t θ_1 and θ_2

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^2}\right) = 0$$

This implies

$$\sum_{i=1} x_i - n\mu = 0$$

$$\theta_1 / \mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$

for θ_2, $\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(X_i - \theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2}\right.$

This implies

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$$\sum_{i=1}^{n} \frac{(x_i-\theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0$$

$$\frac{1}{\theta_2^2} \sum_{i=1}^{n} (x_i-\theta_1)^2 = \frac{n}{\theta_2}$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2$$

sample variance

sol 2) To find MLE of parameter θ for a binomial distribution $B(m, \theta)$ where m is known as positive integer

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m-x_i)\ln(1-\theta)\right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

solving for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} (x_i) = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

MLE of θ is sample mean of obs.