Two Coil Mutual Inductance Measurement Setup

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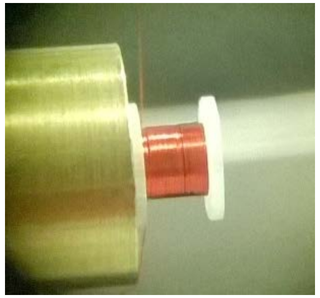
References:

- Journal of Applied Physics 79 , 4221 (1996); [doi: 10.1063/1.362657](https://doi.org/10.1063/1.362657)
- Journal of Applied Physics 83 , 4334 (1998); [doi: 10.1063/1.367193](https://doi.org/10.1063/1.367193)
- Classical electrodynamics, J. D. Jackson

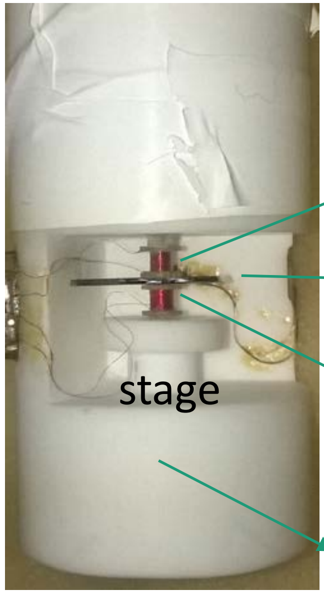
Setup details

1. Machining of bobbins and housing assembly as per technical drawing

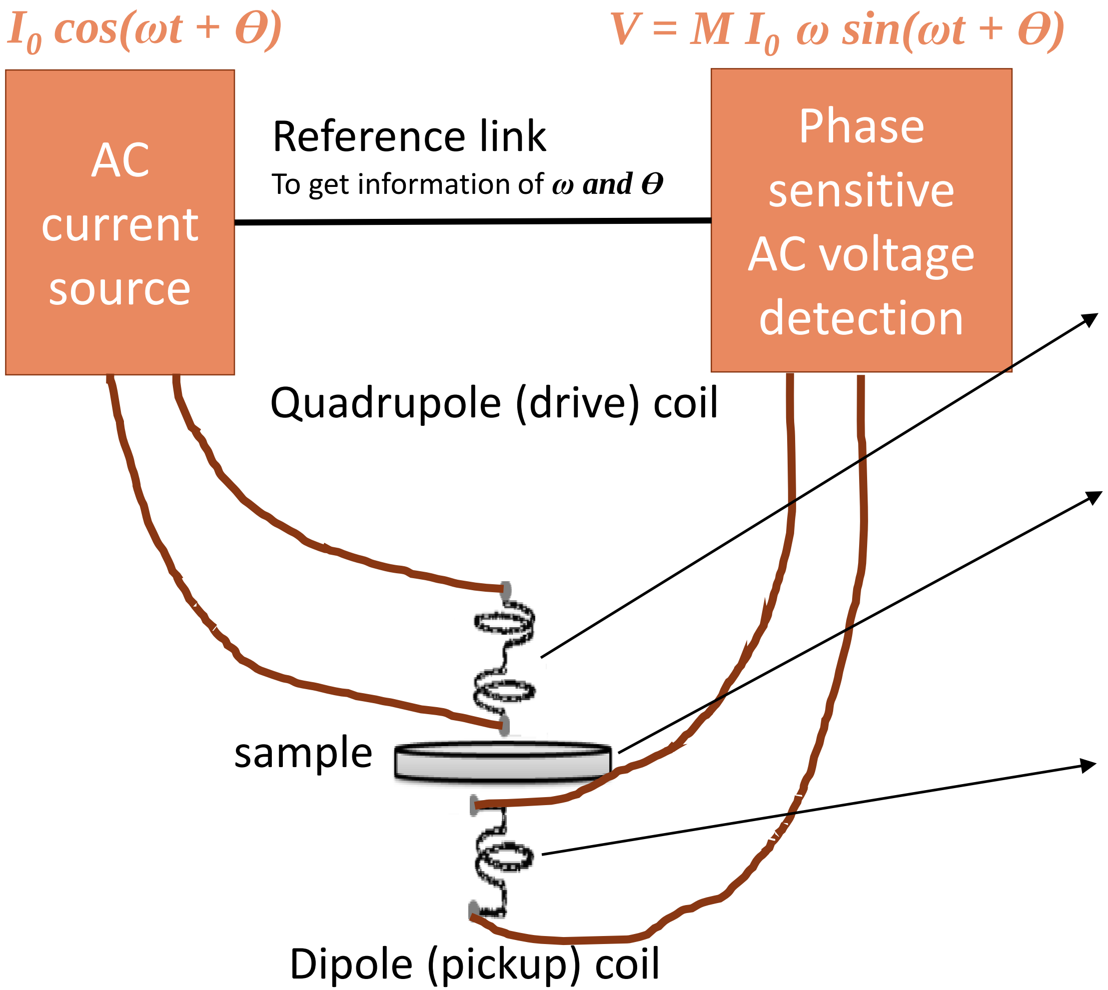
2. Winding miniature coils on a coil winding machine



3. Long probe to reach the center of magnet
4. Co-axial cables (two pairs) and co-ax connectors
5. Temperature sensor
6. AC Current source and phase sensitive voltage detection (lock-in)

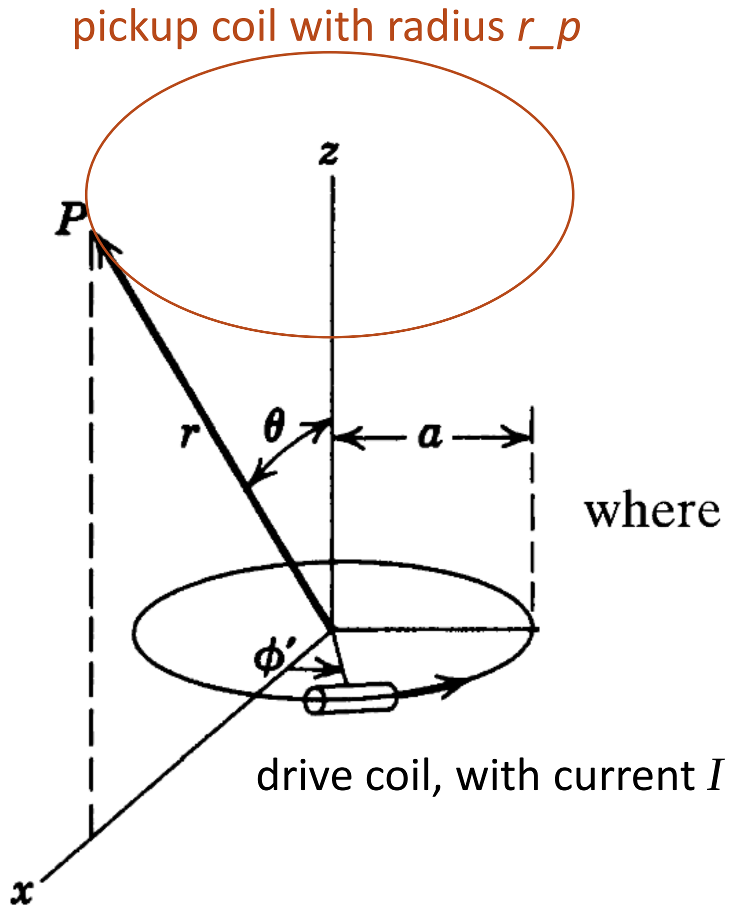


Measurement and analysis schematic for Two-coil setup:



1. Find the vector potential of the magnetic field generated by passing AC current in drive coil
2. Supercurrent and corresponding vector potential generated by superconducting film in response to the magnetic field of drive coil (depends on penetration depth)
3. Total vector potential seen by the pickup coil and the corresponding voltage generated in pickup coil in response to this total vector potential

1 (No sample): Vector potential at pickup coil due to magnetic field generated by passing AC current in drive coil :



The vector potential at each loop of pickup coil due to each ring of drive coil is calculated using following equation:

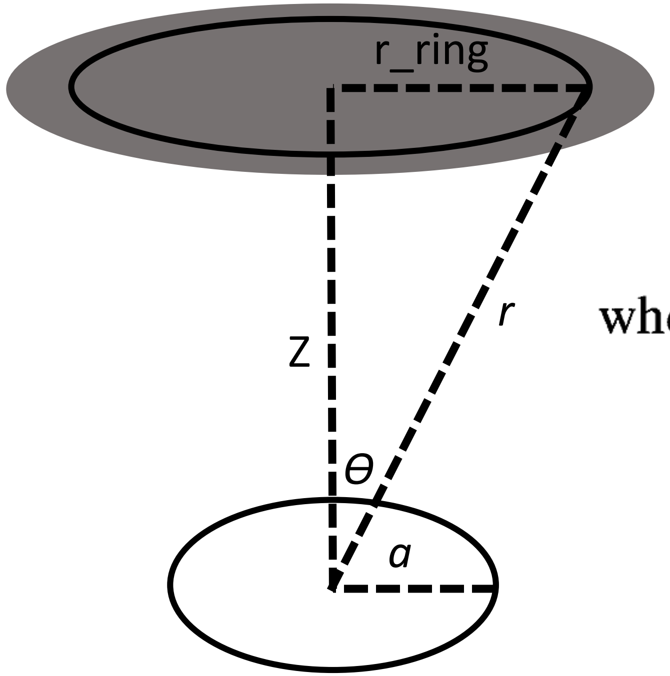
$$A_\phi(r, \theta) = \frac{\mu_0}{4\pi} \frac{4Ia}{\sqrt{a^2 + r^2 + 2ar \sin\theta}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right] \quad (5.37)$$

where the argument k of the elliptic integrals is defined through

$$k^2 = \frac{4ar \sin\theta}{a^2 + r^2 + 2ar \sin\theta}$$

ref: Classical Electrodynamics, J. D. Jackson, page:181

1 (SC film): Vector potential at superconducting film due to magnetic field generated by passing AC current in drive coil :



For numerical simplicity, superconducting film is divided into circular rings and then vector potential at each ring is calculated as follows:

$$A_\phi(r, \theta) = \frac{\mu_0}{4\pi} \frac{4Ia}{\sqrt{a^2 + r^2 + 2ar\sin\theta}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right] \tag{5.37}$$

where the argument k of the elliptic integrals is defined through

$$k^2 = \frac{4ar\sin\theta}{a^2 + r^2 + 2ar\sin\theta}$$

ref: Classical Electrodynamics, J. D. Jackson, page:181

2. Supercurrent generated in superconducting film in response to vector potential created by drive coil :

Much below T_c δ =0, no dissipation

$$\boxed{\lambda_\omega = (i\mu_0\omega\sigma)^{-1/2} = (\lambda^{-2} + i\,\delta^{-2})^{-1/2}}$$

(coupled London and Maxwell equation)

External

Internal

$$\mathbf{J}_{\text{film}}(\mathbf{r}) = -\frac{\mathbf{A}_{\text{film}}(\mathbf{r})}{\mu_0\lambda^2}.$$

$$\mathbf{A}_{\text{film}}(\mathbf{r}) = \mathbf{A}_{\text{drive}}(\mathbf{r}) + \frac{\mu_0}{4\pi}\int d^3\mathbf{r}'\,\frac{\mathbf{J}_{\text{film}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{A}_{\text{drive}}(\mathbf{r}) = -\mu_0\lambda^2\mathbf{J}_{\text{film}}(\mathbf{r}) - \frac{\mu_0}{4\pi}\int d^3\mathbf{r}'\,\frac{\mathbf{J}_{\text{film}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

After doing dΦ integral for azimuthally symmetric system, *K* & *E* are Elliptic integrals:

$$A_{\text{drive}}(\rho) = -\mu_0\lambda^2 J_{\text{film}}(\rho) - \frac{\mu_0}{2\pi}d_{\text{eff}}\int d\rho'\,\frac{J_{\text{film}}(\rho')}{\rho}\ \times(\rho+\rho')\left[\left(1-\frac{k^2}{2}\right)K(k) - E(k)\right]$$

Converting above integral into summation and writing in matrix form: $\mathbf{a}_{ij}\mathbf{c}_j = \mathbf{b}_i$,

depends on rings in film and trial value of λ

calculated in previous step

$$\mathbf{b}_i = \frac{A_{\text{drive},i}}{\mu_0 I_{\text{drive}}}$$

solve for this

$$\mathbf{c}_i = \frac{d_{\text{eff}}R}{I_{\text{drive}}}J_{\text{film},i}\,.$$

$$\mathbf{a}_{ij} = \left[\frac{\lambda^2}{Rd_{\text{eff}}}\delta_{ij} + \frac{1}{2\pi}\frac{s_j}{N}\frac{(\rho_i+\rho_j)}{\rho_i}\times\left(\left(1-\frac{k^2}{2}\right)K(k) - E(k)\right)\right]$$

3. Vector potential at pickup coil

we know the current in each ring now

similar to step 1

$$\mathbf{A}_{\text{pickup}} = \sum_{\text{rings}} \mathbf{A}_{\text{ring}} + \sum_{\text{drive turns}} \mathbf{A}_{\text{turn}}$$

Mutual inductance of each loop of pickup coil

$$\mathbf{M = 2\pi r\, \bar{A} / I}$$

r is the radius of pickup loop, A is the vector potential due to ring of film/loop of drive coil and I is the current in that ring /loop
Total M is obtained by doing this sum for all loops of pickup coil, all rings of film and all loops of drive coil