Garrett Anstreicher Quick Derivation

We have shifted the shocks that govern whether agents acquire MAs or licenses in the model from Type 1 Extreme Value shocks to normal (or log normal) shocks. This makes computing continuation values marginally more complicated, so this document explains how this is done.

Here I explain how I calculate continuation values in phase A of a given period. Continuation values are based off of expected utility in phase B, wherein the agent chooses whether or not to get a **license**. The method for computing continuation values in phase D (in anticipation of the agent choosing whether to get their **MA** in the upcoming phase A) is quite similar. The equations here represent continuation values for agents who DO NOT yet have a license, as continuation values for agents that DO already have a license are trivial.

## Expression of the Phase-A Continuation Value

Subsume all state variables not related to licensure in the vector  $\Omega$ . Denote  $C_{\ell}(\Omega)$  as the cost for acquiring a license — absent the utility shocks — for an agent in state  $\Omega$ . An agent in phase B chooses to acquire a license if

$$\mathbb{E}[V_d(\Omega, 1)] - \exp(C_\ell(\Omega) + \varepsilon) \ge \mathbb{E}[V_d(\Omega, 0)],$$

where  $\varepsilon \sim N(0,1)$  (Quick question: we obviously get a location normalization for the distribution of  $\varepsilon$  if there's a constant in  $C_{\ell}(\Omega)$ . Do we get a location normalization too?). Thus, the agent gets a license if

$$\varepsilon \leq \log (\mathbb{E}[V_d(\Omega, 1)] - \mathbb{E}[V_d(\Omega, 0)]) - C_\ell(\Omega) \equiv T,$$

where T represents the threshold governing whether the agent gets a license. Recall that phase B value functions are given simply by expected phase-D utility minus licensure costs, should the agent choose to get a license. In phase A, then, the agent's continuation value  $EV_A$  can be given by

$$EV_A = \mathbb{E}[V_d(\Omega, 0)] \cdot Pr(\varepsilon \ge T) + \mathbb{E}[V_d(\Omega, 1) - \exp(C_\ell(\Omega) + \varepsilon) | \varepsilon \le T] \cdot Pr(\varepsilon \le T)$$

$$EV_A = \mathbb{E}[V_d(\Omega, 0)] \cdot (1 - \Phi(T)) + \mathbb{E}[V_d(\Omega, 1)] \cdot \Phi(T) - \exp(C_\ell(\Omega)) \mathbb{E}[e^{\varepsilon} | e^{\varepsilon} \le e^T] Pr(e^{\varepsilon} \le e^T).$$

The CDF expressions simplify due to  $\varepsilon$  being distributed standar normal. For the final term, use the fact that with normality, we have

$$\mathbb{E}[e^{\varepsilon}|e^{\varepsilon} \leq e^{T}] \cdot Pr(e^{\varepsilon} \leq e^{T}) = e^{0.5\sigma_{\varepsilon}^{2}} \cdot \left[\Phi\left(\frac{T - \sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}}\right)\right].$$

(Note: the above expression was ripped from Chao's 751 notes. Would be good to confirm.) Since we assume that  $\varepsilon$  is distributed standard normal, the continuation value then reduces to

$$EV_A = \mathbb{E}[V_d(\Omega, 0)] \cdot (1 - \Phi(T)) + \mathbb{E}[V_d(\Omega, 1)] \cdot \Phi(T) - \exp(C_\ell(\Omega)) \sqrt{e} \cdot \Phi(T - 1).$$