A Look at Limits in Random Graphs

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Outline

1. thanks to fmjh, msri, univ of paris saclay, lecturers, tas and deford
2. looks at limits – regular definition, sequence definition, sequence of function convergence (limits of uniformly cont funcs are uniformly cont)
3. what do “similar graphs” have in common, motivate notion of closeness under dloc topology
4. local limit examples
5. describe uniformly pointed version of a graph concept
6. examples of convergence here
7. unimodularity
8. limit of unimodular graphs is unimodular
9. interlude regarding bus paradox tomas told me and how it allows us to transform the theorems from uniformly pointed graphs to uniformly rooted ones.
10. uniformly rooted graphs

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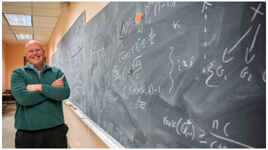

1. Wandering in the Forest
2. What Do We Want?
3. Local Limits

The Prairie

If $f(x)$ is a function defined on an interval containing a (except possibly at a), then

$$\lim_{x \to a} f(x) = L$$

if for every $\epsilon > 0$ there exists some $\delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{when} \quad |x - a| < \delta$$

i.e. under certain conditions your function will get arbitrarily *close* to some limiting object

The Treeline

If $x_n = \{x_1, x_2, x_3, x_4, \ldots\}$ is a sequence, then

$$\lim_{n \to \infty} x_n = x$$

if for every $\epsilon > 0$ there exists some $N > 0$ such that

$$|x_n - x| < \epsilon \quad \text{when} \quad n \geq N \quad \text{for every natural number } n$$

i.e. under certain conditions your sequence will get arbitrarily *close* to some limiting object

Okay I Am Standing Next to a Tree Now

Theorem

If $f_n = \{f_1, f_2, f_3, f_4, \dots\}$ is a sequence of continuous functions such that f_n converges uniformly to f, then f is continuous.

i.e. under certain conditions your sequence will get arbitrarily *close* to some limiting object *and* you know something about the object

The Forest?

Theorem (?)

If $f_n(G_n) = \{f_1($$), f_2($$), f_3($$), f_4($$), \ldots\}$ *is a sequence of* ________ *functions such that* $f_n(G_n)$ *converges* ________ *to* $f($$)$, *then* $f($$)$ *is* ________.

i.e. we need new concepts!

What does it mean for graphs to be 'close'?

Interlude