



# MCMC: An Introduction

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① Probability Practice

② History 1

③ Markov Chains

④ History 2

⑤ MCMC

⑥ Applications

# 1 Probability Practice

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## 3 Markov Chains

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## 5 MCMC

## 6 Applications

# Coin Flipping

- Let's try flipping some coins!

## WLLN

## Theorem (Weak Law of Large Numbers)

*If  $X_1, \dots, X_n$  are independent and identically distributed random samples,  $\bar{X}_n \xrightarrow{P} \mu$  as  $n \rightarrow \infty$ . That is, the sample average converges to the mean of the distribution each  $X_i$  follows*

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Pavel Nekrasov believed independence was necessary for this to hold and also claimed this had more metaphysical implications.



Figure 1: Pavel Nekrasov

Markov proved this is not the case using Markov Chains!



Figure 2: Andrey Markov

# Markov's Example

*He was too young to have been blighted  
by the cold world's corrupt finesse;  
his soul still blossomed out, and lighted  
at a friend's word, a girl's caress.  
In heart's affairs, a sweet beginner,  
he fed on hope's deceptive dinner;  
the world's éclat, its thunder-roll,  
still captivated his young soul.  
He sweetened up with fancy's icing  
the uncertainties within his heart;  
for him, the objective on life's chart  
was still mysterious and enticing—  
something to rack his brains about,  
suspecting wonders would come out.*

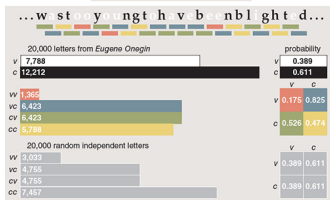


Figure 3: The distribution of consonants and vowels Markov found

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# Definition

A Markov Chain is a sequence of events whose probability of transitioning to another event is only dependent on the current event.

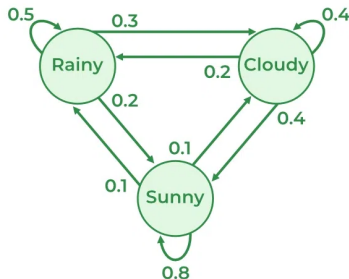


Figure 4: Basic Markov Chain

# Matrix View

The matricial view aids in describing Markov Chains using stochastic matrices. Consider the weighted case of our coin flip game:

$$\begin{bmatrix} .75 & .25 \\ .25 & .75 \end{bmatrix}$$

If we wish to assess the states of our heads or tails game at time step  $k$ , we merely have to raise this matrix to the power  $k$ .

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# Inventor of Monte Carlo

Over a game of solitaire during a bed-ridden illness, Ulam invented what we call Monte Carlo.

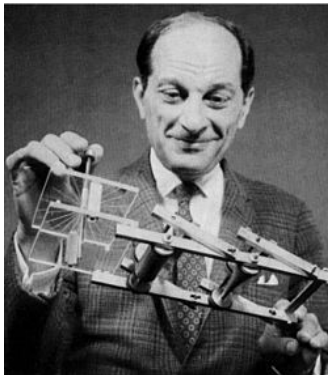


Figure 5: Stanislaw Ulam, creator of Monte Carlo methods

# Monte Carlo

The Monte Carlo method can be summarized as follows:

- Define a domain
- Generate samples from a distribution over domain
- Apply a function to determine outputs
- Accept sample or reject sample according to your desire

# Example

We can use this to approximate  $\pi$ ! (Buffon's Needle)

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# Combine the Two!

The MCMC method combines the two ideas:

- Start with an initial state  $X_0$
- Propose a next step  $X'$
- Compute acceptance probability  $\alpha = \min \left( 1, \frac{\mathbb{P}[X_0 \rightarrow X']}{\mathbb{P}[X' \rightarrow X_0]} \right)$
- Sample  $U \sim \text{Uniform}(0, 1)$
- If  $U \leq \alpha$ , transition to  $X'$ . Else, stay put and try again!

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# Gerrymandering

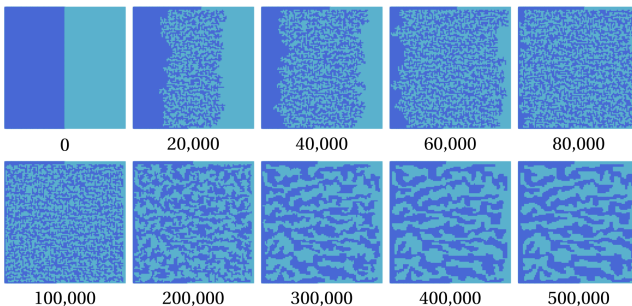
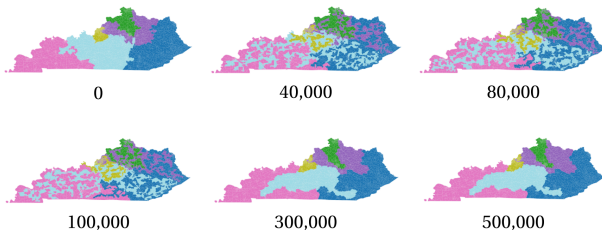


Figure 6: Generating sample of a state (guess which one?)

# Gerrymandering



**Figure 7:** Generating sample of a state (guess which one?)

# Gerrymandering

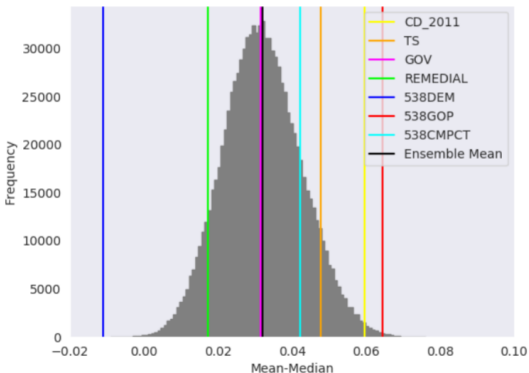
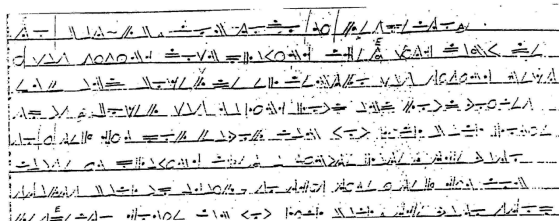


Figure 8: Ensemble approach in Gerrymandering

# Cryptography



**Figure 9:** An encrypted message from inside a prison in the Bay Area, California

# Cryptography

ENTER HAMLET HAM TO BE OR NOT TO BE THAT IS THE QUESTION WHETHER TIS  
NOBLER IN THE MIND TO SUFFER THE SLINGS AND ARROWS OF OUTRAGEOUS  
FORTUNE OR TO TAKE ARMS AGAINST A SEA OF TROUBLES AND BY OPPOSING END

Figure 10: Hamlet as we know and love it

# Cryptography

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100 ER ENOHDLA E OHDLO UOZEOUNORU O UOZEO HD OITO HEOQSET IUROFHE HENO ITORUZAEN
200 ES ELOHRNDE OHRNO UOVEOULOSU O UOVEO HR OITO HEOQAET IUSOPHE HELO ITOSUVDEL
300 ES ELOHANDE OHANO UOVEOULOSU O UOVEO HA OITO HEOQRET IUSOFHE HELO ITOSUVDEL
400 ES ELOHINME OHINO UOVEOULOSU O UOVEO HI OATO HEOQRET AUSOWHE HELO ATOSUVMEL
500 ES ELOHINME OHINO UODEOULOSU O UODEO HI OATO HEOQRET AUSOWHE HELO ATOSUDMEL
600 ES ELOHINME OHINO UODEOULOSU O UODEO HI OATO HEOQRET AUSOWHE HELO ATOSUDMEL
900 ES ELOHANME OHANO UODEOULOSU O UODEO HA OITO HEOQRET IUSOWHE HELO ITOSUDMEL
1000 IS ILOHANMI OHANO RODIORLOS R O RODIO HA OETO HIOQUIT ERSOWHI HILO ETOSRDMIL
1100 ISTILOHANMITOHANOT ODIO LOS TOT ODIOTHATOEROTHIOQUIRTE SOWHITHILOTEROS DMIL
1200 ISTILOHANMITOHANOT ODIO LOS TOT ODIOTHATOEROTHIOQUIRTE SOWHITHILOTEROS DMIL
1300 ISTILOHARMITOHAROT ODIO LOS TOT ODIOTHATOENOTHIOQUINTE SOWHITHILOTENOS DMIL
1400 ISTILOHAMRITOHAMOT OFIO LOS TOT OFIOTHATOENOTHIOQUINTE SOWHITHILOTENOS FRIL
1600 ESTEL HAMRET HAM TO CE OL SOT TO CE THAT IN THE QUENTIOS WHETHEL TIN SOCREL
1700 ESTEL HAMRET HAM TO BE OL SOT TO BE THAT IN THE QUENTIOS WHETHEL TIN SOBLER
1800 ESTER HAMLET HAM TO BE OR SOT TO BE THAT IN THE QUENTIOS WHETHER TIN SOBLER
1900 ENTER HAMLET HAM TO BE OR NOT TO BE THAT IS THE QUESTION WHETHER TIS NOBLER
2000 ENTER HAMLET HAM TO BE OR NOT TO BE THAT IS THE QUESTION WHETHER TIS NOBLER

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Figure 11: Recovering Hamlet using MCMC!