IMAGE ENHANCEMENT BASED ON EQUAL AREA DUALISTIC SUB-IMAGE HISTOGRAM EQUALIZATION METHOD

Yu Wang, Qian Chen, Baomin Zhang
School of Electronic Engineering & Optoelectronic Technology
Nanjing University of Science & Technology, Nanjing 210094, P.R. China
Email: goldenjade@usa.net

ABSTRACT

Histogram equalization is a simple and effective image enhancing technique. But in some conditions, the luminance of an image may be changed significantly after equalizing process, this is why it never be utilized in video system in the past. A novel histogram equalization technique, equal area dualistic sub-image histogram equalization, is put forward in this paper. First, the image is decomposed into two equal area sub-images based on its original probability density function. Then the two sub-images are equalized respectively. At last, we get the result after the processed sub-images are composed into one image. The simulation result indicates that the algorithm can not only enhance image information effectively but also keep the original image luminance well enough to make it possible to be used in video system directly.

Keywords: image enhancement, image segmentation, histogram equalization, contrast enhancement

1. INTRODUCTION

Histogram is defined as the statistic probabilistic distribution of each gray level in a digital image[1]. It can give us a general overview of an image such as gray scale, gray level distribution and its density, the average luminance of an image, image contrast, and so on[2]. Histogram equalization is a simple and effective method for image enhancement. Based on the image's original gray level distribution, the image's histogram is reshaped into a different one with uniform distribution property in order to increase the contrast[3]. The essentiality of histogram equalization is to decrease the number of gray levels so that the contrast of the image can be enhanced. In the equalizing procedure, the neighboring gray levels with light probabilistic density are combined into one gray level, while the gap between neighbor two gray levels with heavy probabilistic density is enlarged. Thus the processed image can have a uniform gray distribution property[4]. It is obvious that the gray levels with heavy probabilistic density occupied a large scale of the gray dynamic range after the equalization, so the image contrast is enhanced in the whole sense. But it is just because of the gray scale stretching effect that may be cause the average luminance of the image shift significantly, and that it is sure to result in impulse vision sense, some times even degrade the image quality. This is why the histogram equalization technique is seldom utilized in video system in the past[5].

A novel image enhancing method, equal area dualistic sub-image histogram equalization technique, is put forward in this paper[5~8]. It is expected to eliminate the above drawback effectively. First, the original image is decomposed into two equal area sub-images based on its gray level probability density function. Then the two sub-images are equalized respectively. At last, we get the result after the processed sub-images are composed into one image. In fact, the algorithm can not only enhance the image visual information effectively, but also constrain the original image's average luminance from great shift. This makes it possible to be utilized in video system directly.

2. HISTOGRAM EQUALIZATION

2.1 Histogram Equalization Algorithm

Let's suppose that $\mathbf{X}=\{X(i, j)\}$ denotes a digital image, where $X(i, j)$ denotes the gray level of the pixel at (i, j) place. The total number of the image pixels is N, and the image intensity is digitized into L levels that are $\{X_0, X_1, \ldots, X_{L-1}\}$. So it is obvious that $\forall X(i, j) \in \{X_0, X_1, \ldots, X_{L-1}\}$. Suppose n_k denotes the total number of pixels with gray level of X_k in the image, then the probability density of X_k will be

$$p(X_k)=n_k/N, \quad k=0,1,\ldots,L-1 \tag{1}$$

The relationship between $p(X_k)$ and X_k is defined as the probability density function (PDF), and the graphical appearance of PDF is known as the histogram. Based on the image's PDF, its cumulative distribution function (CDF) is defined as

$$c(X_k)=\sum_{i=0}^{k} p(X_k) \tag{2}$$

Where k=0, 1, … , L-1, and it is obvious that $c(X_{L-1})=1$. Thus the transform function of histogram equalization can be defined as

Contributed Paper
Manuscript received November 16, 1998

$$f(X_k)=X_0+(X_{L-1}-X_0)c(X_k),\ k=0,1,\ldots,L-1 \quad (3)$$

Suppose $\mathbf{Y}=\{Y(i, j)\}$ is defined as the equalized image, then

$$\mathbf{Y}=f(\mathbf{X})=\{f(X(i,j))|\forall X(i,j)\in\mathbf{X}\} \quad (4)$$

2.2 The Purpose of Histogram Equalization

In information theory, the entropy of the message source is defined as the expectation of the uncertainty of the message source, that is

$$H(x)=-\sum_i p_i \log p_i \quad (5)$$

Where $\sum_i p_i = 1$, and $p_i \geq 0$. But in digital image processing technology, the gray level distribution is fixed for a certain digital image, that is to say, the message source has happened, and its uncertainty is already dissolved. Then the entropy of the message source should be regarded as the measurement of the image information that can be obtained by human eyes.

Suppose $\mathbf{X}$ denotes a digital image. Its gray intensity is digitized into L levels, and $p_0, p_1, \ldots, p_{L-1}$, denote the probability of each gray level respectively. Let H denotes the entropy of the image information, which is the average information content of the image, then $\sum_{i=0}^{L-1} p_i = 1$, and $H=-\sum_{i=1}^{L-1} p_i \log p_i$. What kind of the distribution of $\{p_i\}$ that can make the H achieve its maximum value is discussed in the following.

For $\forall p>0$, then $\log p \leq p-1$, Suppose $\forall \mu_i, \nu_i > 0$, and

$$\sum_{i=0}^{L-1} \mu_i = \sum_{i=0}^{L-1} \upsilon_i = 1 \quad (6)$$

Let $p_i=\nu_i/\mu_i$, then

$$\sum_{i=0}^{L-1} \mu_i \log\frac{\upsilon_i}{\mu_i} \leq \sum_{i=0}^{L-1} \mu_i \left(\frac{\upsilon_i}{\mu_i}-1\right) = \sum_{i=0}^{L-1} \upsilon_i - \sum_{i=0}^{L-1} \mu_i = 0 \quad (7)$$

Thus

$$\sum_{i=0}^{L-1} \mu_i \log \upsilon_i \leq \sum_{i=0}^{L-1} \mu_i \log \mu_i \quad (8)$$

So

$$-\sum_{i=0}^{L-1} \mu_i \log \mu_i \leq -\sum_{i=0}^{L-1} \mu_i \log \upsilon_i \quad (9)$$

Especially, let

$$\upsilon_i = \frac{1}{L}\sum_{i=0}^{L-1} \mu_i = \frac{1}{L} \quad (10)$$

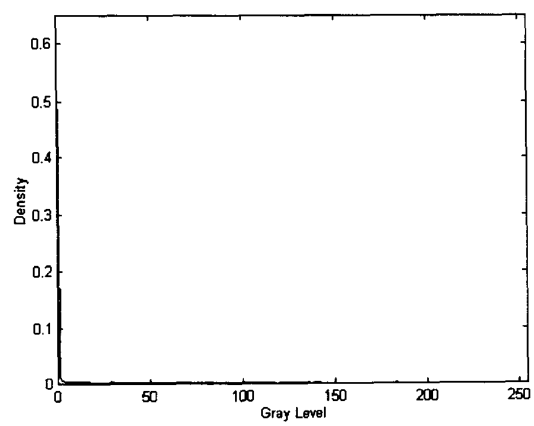
Then

$$-\sum_{i=0}^{L-1} \mu_i \log \mu_i \leq -\sum_{i=0}^{L-1} \mu_i \log \upsilon_i = -\sum_{i=0}^{L-1} \mu_i \log\frac{1}{L} = \log L \quad (11)$$

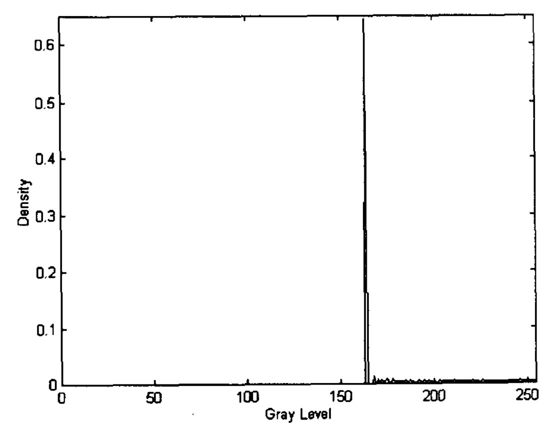
The equation is reasonable only when $\mu_0=\mu_1=\ldots=\mu_{L-1}$.

Based on the above, it can be concluded that the entropy of the message source will get the maximum value when the message has a uniform distribution property. This is why we apply the histogram equalization technique to image enhancement.

An example of histogram equalization is shown in figure 5. By comparing the original image with the equalized one in figure 5a and 5b, we are confirmed that histogram equalization is sure to enhance the image. But it is obvious by comparing the two histograms in figure 1 that the gray level distribution of the equalized image is reshaped into a right handed one, this decreased the dynamic range of the image's gray scale. It can be seen from figure 1 that the luminance of the processed image changed significantly from that of the original one, especially the background gray. This phenomenon is



(a)



(b)

Figure 1: (a) Original Histogram, (b) Equalized Image Histogram

inevitable to cause impulse visual sense if it occurs in video system. That's why the histogram equalization technique is never utilized in video system in the past. In the following section, a novel image enhancing technique, equal area dualistic sub-image histogram equalization method, is brought out. It is expected to eliminate the above drawback.

3. DUALISTIC SUB-IMAGE HISTOGRAM EQUALIZATION

3.1 The Algorithm of Dualistic Sub-Image Histogram Equalization

Suppose image **X** is segmented by a section with gray level of $X=X_e$, and the two sub-images are $\mathbf{X}_L$ and $\mathbf{X}_U$, so we have $\mathbf{X}=\mathbf{X}_L \cup \mathbf{X}_U$. Here

$$\mathbf{X}_L=\{X(i,j)|X(i,j)<X_e, \forall X(i,j)\in \mathbf{X}\} \tag{12}$$

$$\mathbf{X}_U=\{X(i,j)|X(i,j)\geq X_e, \forall X(i,j)\in \mathbf{X}\} \tag{13}$$

It is obvious that sub-image $\mathbf{X}_L$ is composed by gray level of $\{X_0, X_1, \ldots, X_{e-1}\}$, while sub-image $\mathbf{X}_U$ is composed by gray level of $\{X_e, X_{e+1}, \ldots, X_{L-1}\}$. The aggregation of the original image's gray level distribution probability is decomposed into $\{p_0, p_1, \ldots, p_{e-1}\}$ and $\{p_e, p_{e+1}, \ldots, p_{L-1}\}$ correspondingly.

Suppose

$$p=\sum_{i=0}^{e-1} p_i \tag{14}$$

Then the normalized gray level distribution probability for sub-image $\mathbf{X}_L$ and $\mathbf{X}_U$ will be the following

$$\left\{\frac{p_i}{p}, \quad i=0,1,\ldots,e-1\right\} \tag{15}$$

And

$$\left\{\frac{p_i}{1-p}, \quad i=e,e+1,\ldots,L-1\right\} \tag{16}$$

So the corresponding cumulative distribution function will be

$$c_L(X_k)=\frac{1}{p}\sum_{i=0}^{k} p_i, \qquad k=0,1,\ldots,e-1 \tag{17}$$

And

$$c_U(X_k)=\frac{1}{1-p}\sum_{i=e}^{L-1} p_i, \quad k=e,e+1,\ldots,L-1 \tag{18}$$

Based on the cumulative distribution function, the transform functions for the two sub-images' histogram equalization are listed below

$$f_L(X_k)=X_0+(X_{e-1}-X_0)c(X_k), \quad k=0,1,\ldots,e-1 \tag{19}$$

And

$$f_U(X_k)=X_e+(X_{L-1}-X_e)c(X_k), \quad k=e,e+1,\ldots,L-1 \tag{20}$$

At last, the result of the dualistic sub-image histogram equalization is obtained after the two equalized sub-images are composed into one image. Suppose **Y** denotes the processed image, then

$$\mathbf{Y}=\{Y(i,j)\}=f_L(\mathbf{X}_L)\cup f_U(\mathbf{X}_U) \tag{21}$$

Where

$$f_L(\mathbf{X}_L)=\{f_L(X(i,j))|\forall X(i,j)\in \mathbf{X}_L\} \tag{22}$$

$$f_U(\mathbf{X}_U)=\{f_U(X(i,j))|\forall X(i,j)\in \mathbf{X}_U\} \tag{23}$$

Namely

$$Y(i,j)=\begin{cases} X_0+(X_{e-1}-X_0)c_L(X), & \text{if } X<X_e \\ X_e+(X_{L-1}-X_e)c_U(X), & \text{else} \end{cases} \tag{24}$$

3.2 The Segmentation Method to Achieve Dualistic Sub-Image

Suppose the gray level distribution probability of the processed image is $\{p/e, p/e, \ldots, p/e\}\cup\{(1-p)/(L-e), (1-p)/(L-e), \ldots, (1-p)/(L-e)\}$, then the average information content of the processed image is

$$\begin{aligned} H &= -\sum_{i=0}^{e-1}\frac{p}{e}\log\frac{p}{e}-\sum_{i=e}^{L-1}\frac{1-p}{L-e}\log\frac{1-p}{L-e} \\ &= -p\log\frac{p}{e}-(1-p)\log\frac{1-p}{L-e} \end{aligned} \tag{25}$$

Let $p'=e/L$, then

$$\begin{aligned} H &= -p\log\frac{p}{Lp'}-(1-p)\log\frac{1-p}{L(1-p')} \\ &= -p\log p-(1-p)\log(1-p) \\ &\quad +p\log p'+(1-p)\log(1-p') \\ &\quad +\log L \\ &\leq \log L \end{aligned} \tag{26}$$

The equation is reasonable only when $p=p'$.

So it is concluded from the above that the average information content of the processed image will achieve its maximum value only when the ratio of the gray level of the segmentation plane versus the gray scale equals the cumulative probability of the gray level. But this condition can seldom be satisfied for common images, furthermore, the average luminance of the original image should be considered firstly in order to keep it from varying greatly.

The dualistic segmentation entropy of the gray level distribution probability aggregation [9] may be taken into account in the problem of choosing segmentation gray plane, since the histogram can indicate some image figures such as gray level distribution, the average luminance, and so on. Suppose the gray level distribution probability aggregation of the original image is segmented into two sub-aggregations, $\{p_0, p_1, \ldots, p_{e-1}\}$ and $\{p_e, p_{e+1}, \ldots, p_{L-1}\}$. Let $p=\sum_{i=0}^{e-1} p_i$, then the Shannon

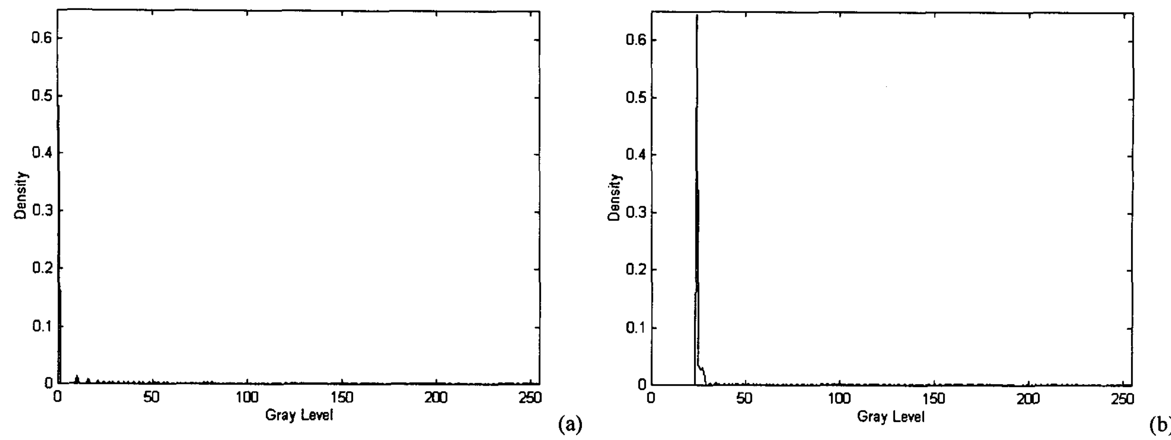


Figure 2: (a) DSIHE Histogram, (b) BBHE Histogram

entropy function of the aggregation's dualistic segmentation is

$$\begin{aligned} h &= -p\log p - (1-p)\log(1-p) \\ &\leq \log 2 \end{aligned} \tag{27}$$

The equation is reasonable only when p=1/2, i.e., the segmentation entropy will achieve the maximum value when the two sub-images have equal area. There is no doubt that the two sub-images represent the dark and bright area of the original image respectively. So the gray level can be remained in their original scales respectively after sub-image histogram equalizing. Furthermore, the contrast of the original image is also enhanced effectively after processing. This is the basis for us to bring forward the equal area dualistic sub-image histogram equalization (DSIHE) technique. The processed image using this method is shown in figure 5d. It is easy to see that the visual quality of the processed image in figure 5d is far better than that of the processed image using traditional equalization method as shown in figure 5b. And from the histogram shown in figure 2a we can know that the average luminance of the original image is kept from significant shift since the histogram doesn't change greatly. This is the superior property of the equal area dualistic sub-image histogram equalization technique.

4. ANALYSIS ON THE CHANGE OF LUMINANCE

Suppose **X** denotes the original image, and the gray level distribution probability after traditional histogram equalization is $p(X)=1/(X_{L-1}-X_0)$, $\forall X \in \{X_0, X_1, \ldots, X_{L-1}\}$, then the average luminance of the equalized image will be

$$\begin{aligned} E(\mathbf{Y}) &= \int_{X_0}^{X_{L-1}} Xp(X)dX \\ &= \int_{X_0}^{X_{L-1}} \frac{X}{X_{L-1}-X_0} dX \\ &= \frac{X_{L-1}+X_0}{2} \end{aligned} \tag{28}$$

So it is easy to see that the average luminance of the image is always the middle gray level of the dynamic range after traditional equalization processing, and that it has nothing to do with the original gray level distribution probability. So the average luminance of the original image maybe changes significantly after histogram equalization.

Suppose $\{p_i \mid i=0, 1, \ldots, e-1\} \cup \{p_j \mid j=e, e+1, \ldots, L-1\}$ is the gray level distribution probability of the processed image using the dualistic sub-image histogram equalizing technique, then $p_i=1/2(X_{e-1}-X_0)$, $p_j=1/2(X_{L-1}-X_e)$, and the average luminance of the processed image is

$$E(\mathbf{Y}) = E(\mathbf{Y} \mid X < X_e) + E(\mathbf{Y} \mid X \geq X_e) \tag{29}$$

And we can also deduce the following equations from equation (28)

$$E(\mathbf{Y} \mid X < X_e) = (X_0 + X_{e-1})/4 \tag{30}$$

$$E(\mathbf{Y} \mid X \geq X_e) = (X_e + X_{L-1})/4 \tag{31}$$

Then

$$\begin{aligned} E(\mathbf{Y}) &= (X_0 + X_{e-1})/4 + (X_e + X_{L-1})/4 \\ &= (X_e + (X_{L-1} + X_0)/2)/2 \end{aligned} \tag{32}$$

i.e., the average luminance of the processed image using equal area dualistic sub-image histogram equalization technique is the average of the segmentation gray level and the middle gray level of the image's gray scale. Since the original image is decomposed into one dark image and one bright image with equal area property, the gray levels can be remained in the two segmented gray-scales respectively. So it is sure that the average

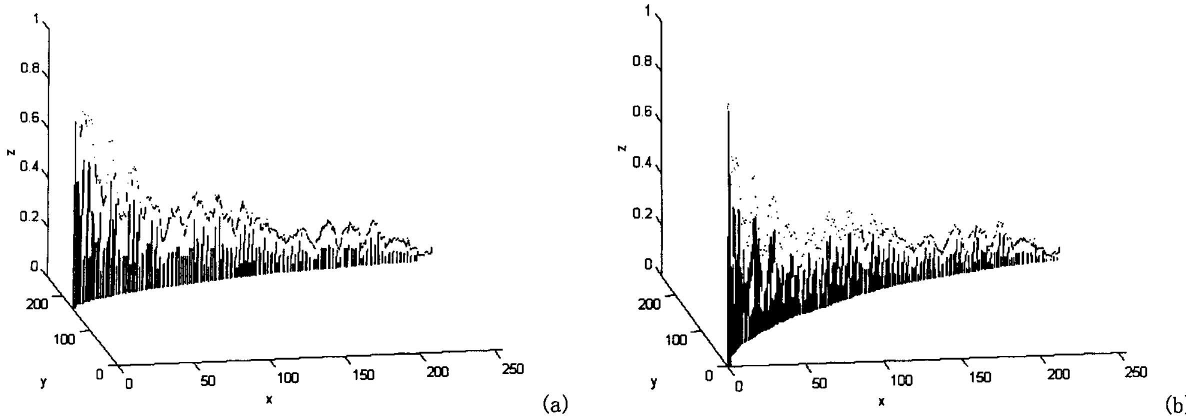


Figure 3: Two-Dimension Histogram

(a) Traditional Equalizing, (b) Dualistic Sub-Image Equalizing

luminance of the original image could be kept from significant shift especially for the large area of the image with the same gray level.

Suppose that axis x denotes the gray level of the original image, axis y denotes the gray level of the processed image, and the coordinate (x, y) on the x-y plane denotes a pair of corresponding pixels in the two images. Then axis z is defined as the probability of the corresponding pixels in the original image and the processed image whose gray level equal to the x and y coordinate value respectively. This is what we called two-dimension histogram [1]. It is easy to know from the definition that the two dimensional histogram can not only reflect the one dimensional gray level distribution property of the two images, but also can reflect the difference between the original image and the processed one. The two images are exactly the same if all of the pixel pairs locate on the diagonal of the x-y plane. The further the pixel pair is away from the diagonal and the bigger the probability is, the greater the average luminance changes after processing. The two-dimension histograms of the two equalization results are shown in figure 3. Since there are a large amount of pixels with the same gray value in the image's background, the amplitude of the two-dimension histogram is enlarged in order to make the small probability be visible. And the probability of gray zero that is very far bigger than one is cut off, and it is shown as one in the histogram. Looking into figure 3, it is easy to know that the dualistic sub-image histogram equalization can preserve the average luminance of the original image more effectively than the traditional histogram equalization.

The brightness preserving bi-histogram equalization (BBHE) method is described in reference 5. And the brightness preserving bi-histogram equalization processing result we simulated is shown in figure 5c. Comparing figure 5c with figure 5d, it is easy to see that the quality of the image processed by equal area dualistic sub-image histogram equalization is better than that of the brightness preserving histogram equalization. In order to get an objective comparison and evaluation of the different processing effect, some relevant data is shown in table 1 such as mean, average information content (AIC), background gray level (BGL), and so on. Here HE, BBHE, DSIHE are the abbreviations of traditional histogram equalization, brightness preserving bi-histogram equalization, equal area dualistic sub-image histogram equalization respectively. According to the data in table 1, the equal area dualistic sub-image histogram equalization is the best to keep the original image's luminance from great shift especially for the background that occupies most part of the image with the same gray zero. And the data of the average information content indicates that the equal area dualistic sub-image histogram equalization is the best processing technique to preserve the original image details. So it is the best processing technique of the three in all of the three items. It is easy to see why the image luminance changes from the gray level transform curve shown in figure 4. The two-dimension histogram of the BBHE processing is also shown in figure 4d.

Table 1: Data of the Processed Images for Evaluation

Item / Image	Mean	AIC(bits)	BGL
Original	28	3.6408	0

HE	180	3.1956	164
BBHE	53	3.2982	24
DSIHE	45	3.5488	0

5. CONCLUSION

Histogram equalization is a simple and effective image enhancing technique. But in some conditions, the luminance of an image may be changed significantly after equalizing process. This is sure to cause impulse visual sense if it occurs in video system, so it is never utilized in video system in the past. In this paper, we bring forward a novel histogram equalization technique, which is called equal area dualistic sub-image histogram equalization technique. It can resolve the above drawback effectively. The simulation result indicates that the algorithm can not only enhance image information effectively but also keep the original image luminance well enough to make it possible to be used in video system directly. Furthermore, the comparison of different equalizing results is carried out in the previous section. And it is no doubt that the DSIHE is the best equalizing technique.

6. REFERENCES

1. K. R. Castleman, *Digital Image Processing*, Prentice Hall, Englewood Cliffs, New Jersey, 1996
2. A. Rosenfeld, A. C. Kak. *Digital Picture Processing*, Academic Press, New York, 1976
3. J. H. Xu, *Image Processing and Analysis*, Academic Press(China), Beijing, 1992
4. R. J. Jing et al., *Computer Image Processing*, Zhejiang University Press, Hangzhou, 1992

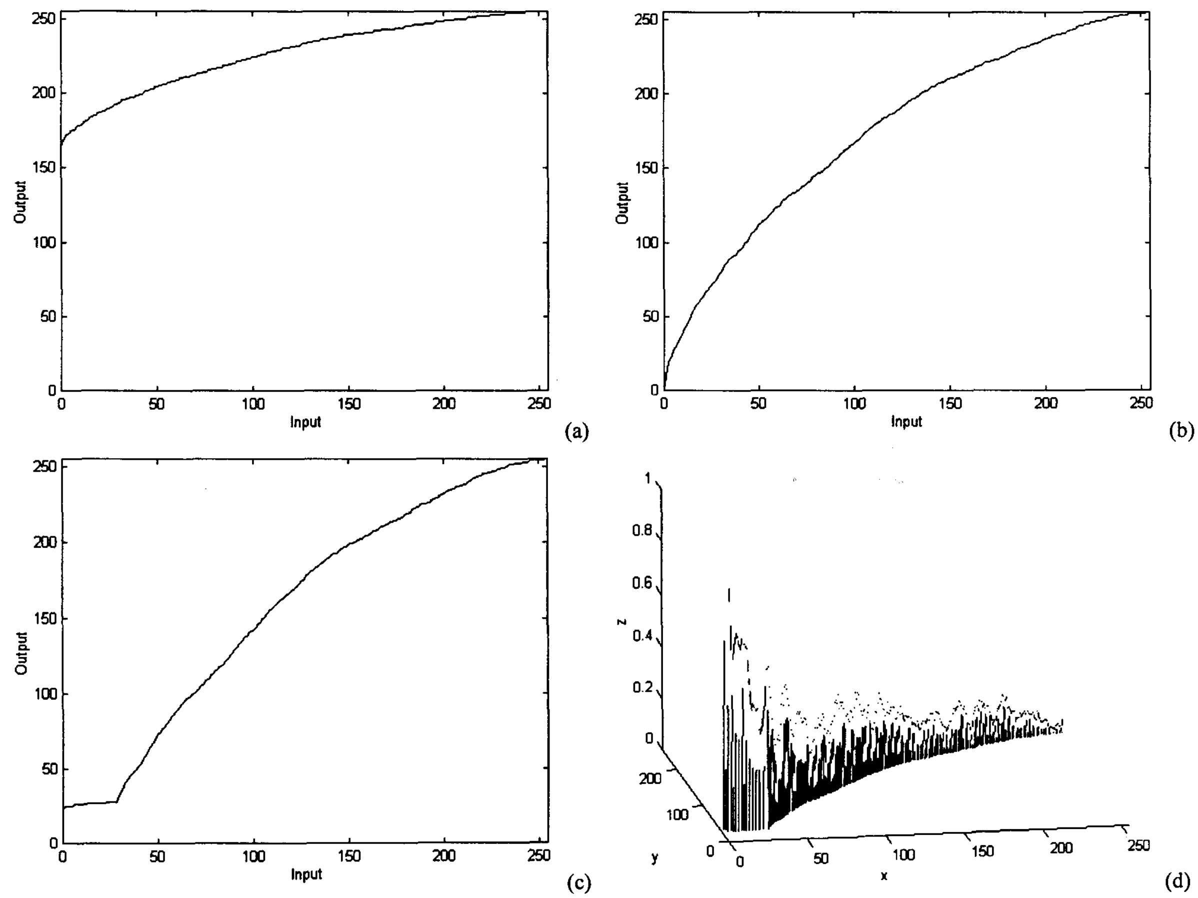
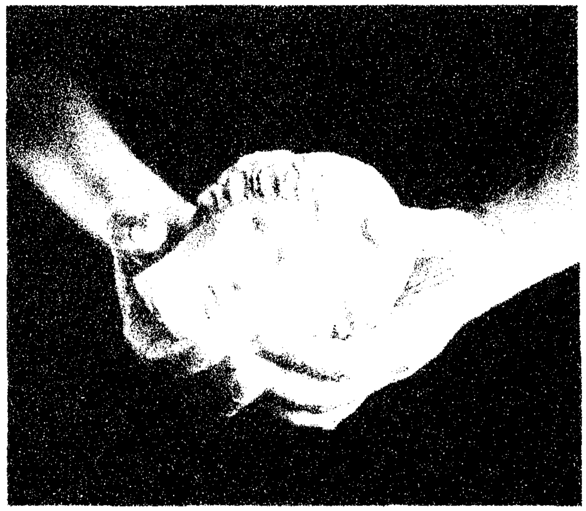


Figure 4: Gray Level Transform Curve

(a) Traditional HE, (b) DSIHE, (c) BBHE, (d) 2D Histogram of BBHE

(a) Original Image



(b) Traditional HE Processing Image

(c) BBHE Processing Image

(d) DSIHE Processing Image

Figure 5: Different Processing Result

5. Y. T. Kim, "Contrast enhancement using brightness preserving bi-histogram equalization," IEEE trans. CE, 43(1):1~8, 1997

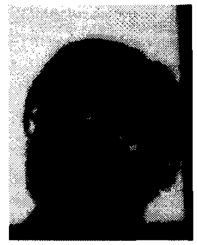
6. T. K. Kim et al., "Contrast enhancement system using spatially adaptive histogram equalization with temporal filtering," IEEE trans. CE, 44(1):82-87, 1998

7. S. M. Pizer et al., "Adaptive histogram equalization and its variations," Computer Vision, Graphics and Image Processing, 39:355-368, 1987

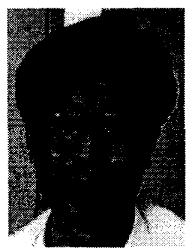
8. Y. Q. Li, "Application of adaptive histogram equalization to X-ray chest image," Proc. of the SPIE, 2321: 513-514, 1994

9. M. J. Wu, *Introduction to Fractal Information*, Shanghai Science & Technology Bibliography Press, Shanghai, 1994

Yu Wang Received B. Sc. degree and M. Sc. degree from the School of Elec. Eng. & Opto. Tech., Nanjing Univ. of Sci. and Tech. in 1994 and in 1997, respectively. And he is currently pursuing his Ph.D. degree in the School of Elec. Eng. & Opto. Tech., NUST, Nanjing, P.R. China. He is interested in digital image processing, digital video signal processing, image data fusion, optoelectronic data acquisition and processing.



Qian Chen Associate professor, received Ph.D. degree from the School of Elec. Eng. & Opto. Tech., NUST, in 1996. And now he is the vice dean of the Department of Optoelectronic Technology, NUST, Nanjing, P. R. China. He is interested in real-time digital image processing, optoelectronic imaging technology, electro-optical displaying technology, optoelectronic signal processing and transmission.



Bao-Min Zhang Professor, graduated from the Department of Precise Instrument, National Institute of Military Engineering, Harbin, P.R. China, in 1960. He is the committeeman of the Chinese Electronic Society and of the Chinese Optical Society. He is awarded the 20th century out-standing scientist gold medal by the Cambridge International Biographical Center for his remarkable contribution to the advance of photo-electronics. His research area includes optoelectronic detecting theory and technology, image noise theory, image quality evaluation.