Diffraction
Summary Report
PHYS375

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Dates Performed: April 6 & 13
Instructor: Dr. Hill

Abstract

This experiment was performed to determine how light diffracts when incident with various obstacles such as: a single slit, double slits, multiple slits, a sharp edge, a circular aperture, and a thread. Huygens-Fresnel principle states that every point on a wave front acts as a source of spherical forward propagating beam. A consequence of this principle is that light will create a diffraction pattern (similar visually to interference) for all obstacles. The resulting diffraction patterns from the various obstacles are quantitatively explored. Diffraction is an important property of light that, among other things, sets conditions on optical elements for resolving images such as distant galaxies.

1 Objectives

First Objective

Use a photodiode to capture diffraction patterns of a laser incident with various obstacles

Second Objective

Compare the diffraction patterns to theory, and analyze them to calculate geometrical quantities of the obstacles.

2 Apparatus

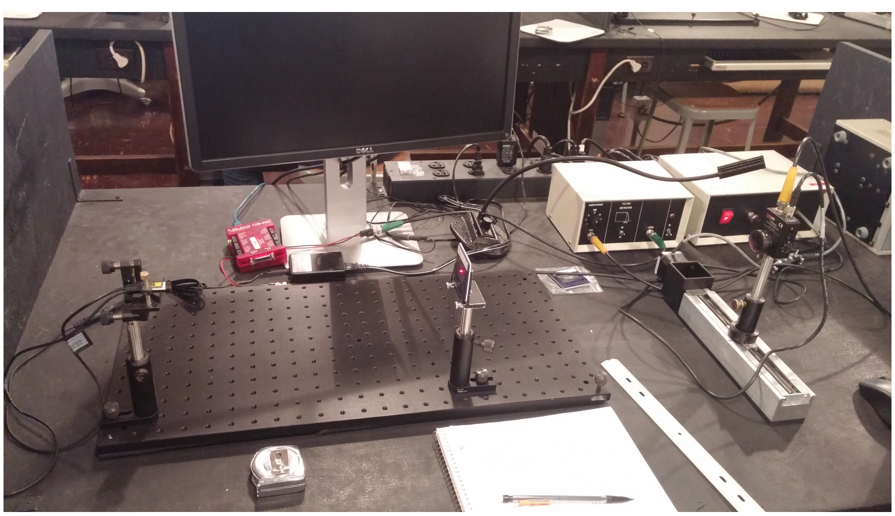

Figure 1: Experimental Setup: The laser, obstacle, and photodiode are all collinear and separated such that the diffraction is Fraunhofer diffraction

The laser, amplified photodiode, and obstacles (will vary) should be aligned collinear with the obstacle in the middle. Aside from aligning these components, it is vital to ensure that the distance between all the components is such that Fraunhofer diffraction will be observed (i.e. far field). Fraunhofer diffraction is assumed when the following condition is met:

$$\frac{1}{2}\left(\frac{1}{d}+\frac{1}{d'}\right)\delta^2 \ll \lambda \tag{1}$$

Where d is the distance between the laser and the obstacle, d' is the distance between the obstacle and the amplified photodiode, delta is width of single slit, and lambda is the wavelength of the laser.

3 Procedure

Set up the apparatus with the obstacle of interest (single slit, double slit, multi-slit, thread, circular aperture, and razor) and use a screen to visualize the resulting diffraction pattern. Ensure that the iris of the amplified photodiode is small enough so it will measure all of the structure (i.e. the iris should be smaller than the width of the bright and dark fringes). Measure the distance between the laser and the obstacle and the distance between the obstacle and the amplified photodiode housing.

Next, from Matlab, run a script that will scan 4 to 8 cm so that a couple finges can be measured. For an example of such a script see *Scan_Laser.m* under labarchives, data & lab notes, lab 5, day 1. Scans can be taken in both directions, but (for comparison) when plotting and analyzing the location vector should be flipped. When the razor is the obstacle, scans should be taken in one direction with and without the razor so that the difference in signal voltage can be obtained. For the razor, a lense will need to put into the optical system before the razor to collimate the beam.

For the single slit, double slit, multi-slit and thread the obstacles should be measured (with uncertainty) using a travelling microscope. The width of each slit and the gap between each slits should be measured; these are the geometrical quantities the analysis should reveal.

4 Experimental Data

The lab notes, raw voltage data (.mat files) and Matlab scripts used in the experiment can be found on labarchives, under "Data & Lab Notes", Lab 5. The voltage files correspond the obstacles in the following manner

Measurement	**Base file name**
Single Slit	volt_1_1
Double Slit	volt_double_1
Triple Slit	volt_triple_1
Circular aperture	volt_circ
Thread	hair_1
Razor	volt_razor1

On the next page are plots of voltage signal from the photodiode compared with a fitted version of the theory for each of the obstacles.

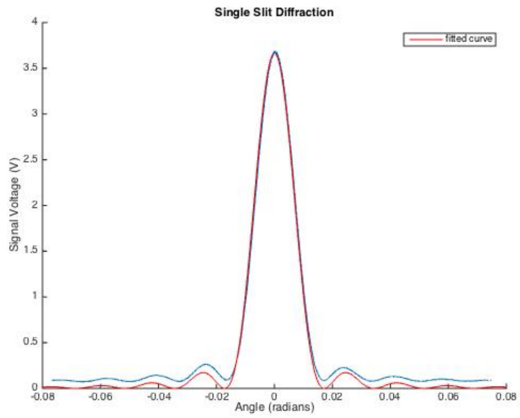

Figure 2: Single slit diffraction with theory curve

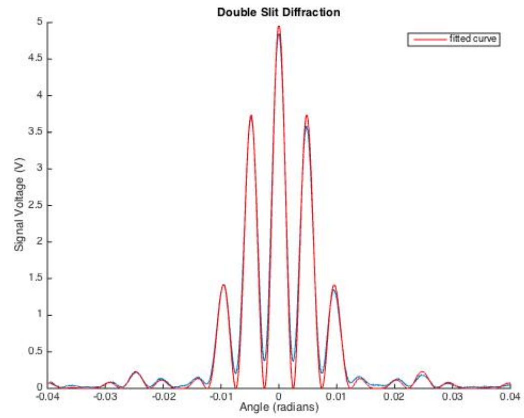

Figure 3: Double slit diffraction with theory curve

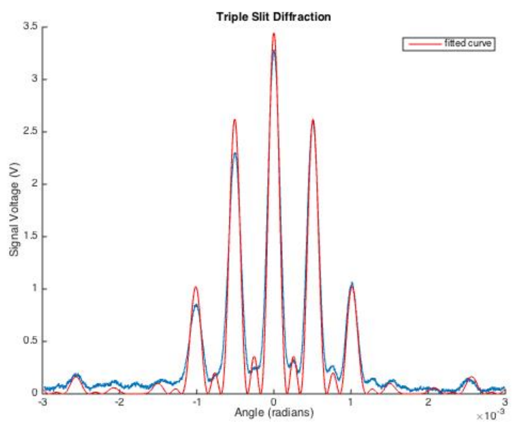

Figure 4: Triple slit diffraction with theory curve

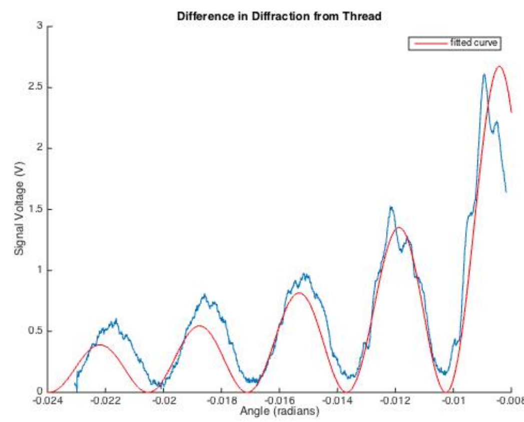

Figure 5: Thread diffraction, difference in signal with and without thread

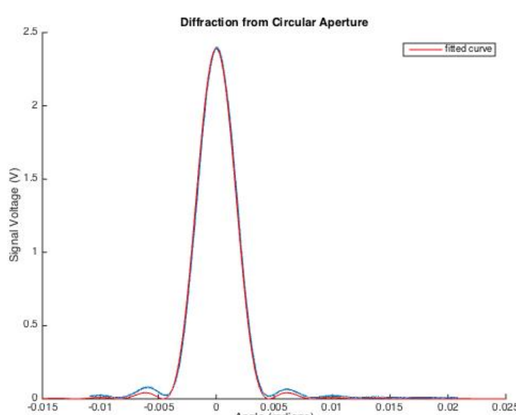

Figure 6: Triple slit diffraction with theory curve

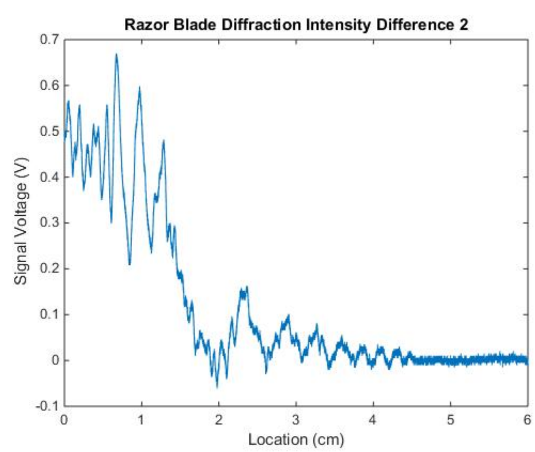

Figure 7: Razor diffraction, the difference between the background and signal

5 Numerical Analysis

The general equation for fraunhofer diffraction due to a multi-slit obstacle is given by

$$I = I_0 \left(\frac{sin\beta}{\beta} \right)^2 \left(\frac{sinN\gamma}{Nsin\gamma} \right)^2 \tag{2}$$

Where $\beta = 1/2kbsin\theta$, $\gamma = 1/2khsin\theta$, and N is number of slits. Furthermore, k is the wavenumber, b is the width of each slit (we assume all slits are the same width and have the same gap), h is the distance between slits (center to center), and lastly, θ is the angle measured from normal. These are summarized in the following plot

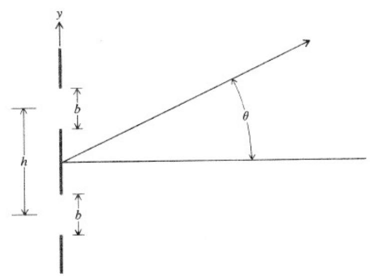

Figure 8: Parameter description

The insensity of diffraction from a circular aperture is given by

$$I = I_0 \left[\frac{2J_1(\rho)}{\rho} \right]^2 \tag{3}$$

Where J_1 is the Bessel function of the first kind and $\rho = \frac{2\pi}{\lambda} R$ (R is the radius of the circular aperture).

In Matlab, these equations were then used to fit the raw data (blue in previous plots). For all plots besides the single slit diffraction and the razor blade diffraction (which wasn't fitted), the curve-fitting tool needed reasonable bounds for the parameters. With the bounds, the fit Matlab returned was unrecognizable; however, with the bounds the fits were very good. With the fits, Matlab would output a 95% confidence interval, which will be used later to calculate the error.

The table below summarizes the geometrical qualities of the obstacles given by the Matlab fits

Table 1: Computed Values

Obstacle	**Parameters**
Single Slit	Slit Width $= .037 \pm .0003$mm
Double Slit	Slit Width $= .0377 \pm .0003$mm Slit Gap $= .128 \pm .001$mm
Triple Slit	Slit Width $= .0356 \pm .0003$mm Slit Gap $= .124 \pm .001$mm
Circular aperture	Diameter $= .170 \pm .0003$mm
Thread	Thread Width $= .186 \pm .0003$mm

These calculated values are compared with the measurements taken with the traveling microscope which yielded the following values

Table 2: Measured Values

Obstacle	**Parameters**
Single Slit	Slit Width $= .033 \pm .001$mm
Double Slit	Slit Width $= .0385 \pm .001$mm Slit Gap $= .130 \pm .001$mm
Triple Slit	Slit Width $= .0385 \pm .001$mm Slit Gap $= .123 \pm .001$mm
Circular aperture	Diameter not measured
Thread	Thread Width $= .182 \pm .005$mm

The nominal values for all of the slit widths was .04 mm and slit gaps was .125 mm

6 Error Analysis

The error propagation for the measurements of the traveling microscope was done in accordance with the following method of error propagation where the function was an average. Let f be a function of some random variables x,y,z. Then its uncertainty is derived from:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} * \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} * \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} * \sigma_z\right)^2} \tag{4}$$

However, the error analysis for the geometrical parameters was tricky since it was fitted by a Matlab and the dependence on the θ, which was calculated by measuring the distance between the optical elements, is buried inside two sine functions. Rather than try to compute partials with respect to θ and λ, the Matlab fits were re-run with the extreme cases of error added to both θ and λ. So, first the error in θ was computed using equation 4 applied to the equation

$$\theta = atan((\text{Lateral Position}/\text{Distance to Photodiode}) \tag{5}$$

It was found that the average uncertainty in θ was .0002 radians. This was then added to the angles (as to replicate being maximally off—yet within the found uncertainty), and the wavelength was changed from 635 to 640. Under these conditions it was found that widths had an uncertainty of .0003 mm while gaps had an uncertainty of .001mm

7 Discussion

There was quantitative analysis done for the razor blade, but the importance of this scan can be seen in Figure 7. Notice that after a location of 2cm (where the razor blade came into play) the difference between the background voltage and the razor signal voltage oscillates, decaying with an increase in location. Visually this could be seen on a screen. The diffraction from the sharp edge of the razor caused vertical fringes that alternated bright and dark, similar to diffraction patterns from other obstacles.

Comparing Table 1, Table 2, and the nominally given values for geometric quantities it is not clear they are true to theory. This is mainly due to the extremely small uncertainty in all of the values. When the raw data is compared, visually, to theory as in Figures 2 through 7 it is clear that the theory indeed holds. The issue with the error propagation was the complexity of fitting and the obscurity that is buried in how the fits are determined. If this experiment were to be repeat it would have been beneficial to have a computer determine the error in the geometrical values for each data point (all 4 to 8 thousand per scan) and then use these to do compute a weighted average for the geometrical quantities.

Errors that could have affected the data in a significant way include: asymmetric slit widths and gaps, orientation of the obstacles, and the alignment of the photodiode. If the slit widths or gaps were asymmetric then the waves would not destructively and constructively interfere as the theory suggests. Rather there would be subtleties that when unaccounted for. If the obstacles where not properly oriented (i.e. the slits were not perfectly vertical to the table) then the photodiode would me measuring on different portions of the diffraction pattern that would skew the signal voltages. Lastly, if the photodiode was not square to the resulting beam then as it scanned across it would be measuring the beam at different distances from the obstacles, which would not result in the expected voltage signal.