Michelson Interferometer
Summary Report
PHYS375

Garrett SUTHERLAND

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Instructor:	Dr. Hill

Abstract

This experiment was performed to determine how light interferes when traveling two arms of a Michelson interferometer, to calculate the wavelength of the diode laser, and to measure the splitting of the sodium D doublet. First the laser light was sent through the Michelson interferometer and the resulting fringes were scanned as a function of the adjustable mirror. After making similar measurements on the light from a sodium lamp the lever arm factor was determined to be $4.96 \pm .18$. With this and the scans of the laser light, analysis determined that the wavelength of the laser light was 634 ± 6 nm. The analysis of sodium lamp light scans also revealed that the difference in the two wavelengths of the sodium D doublet, $|\Delta\lambda|$, was $.526 \pm .003$ nm.

1 Objectives

First Objective

Determine the wavelength of the laser diode by using the resulting fringes from interference.

Second Objective

Measure the splitting of the sodium D line doublet and calculate the difference between the two wavelengths.

2 Apparatus

See figure 1 on the next page

Materials			
Michelson interferometer	Photodiode detector	15mm plano-convex lense	Sodium lamp
LabJack Interface	Motor	Diode laser	Signal filter

Align the diode laser beam with the Michelson interferometer. This entails setting the laser to the correct height so that the beam enters the beam splitter. Place the screen at the output of the interferometer. There should be three spots that appear in a horizontal line with one being the brightest. Looking at these spots, use a piece of paper to block arm 1 of the interferometer. While still observing the spots, next block arm 2. The interferometer is aligned when the brightest spot does not move when blocking different arms of the interferometer. If the spot does move, then the fixed mirror should be adjusted. Be careful to only slightly adjust the fixed mirror up/down or left/right to make the bright spot stationary, which means the two beams fall on top of one another.

Next, place the 15mm plano-convex lense roughly 8.5cm after the diode laser along its path of propagation such that the beam hits the curved side first. The beam must hit the plano-convex lense so that it's normal to the surface so make sure the beam is centered on the lense. The output of the plano-convex lense will be a significantly larger beam and should still be aligned to enter the Michelson interferometer.

Looking at the screen there should be some sort of pattern of bright and dark spots. Depending on the alignment,

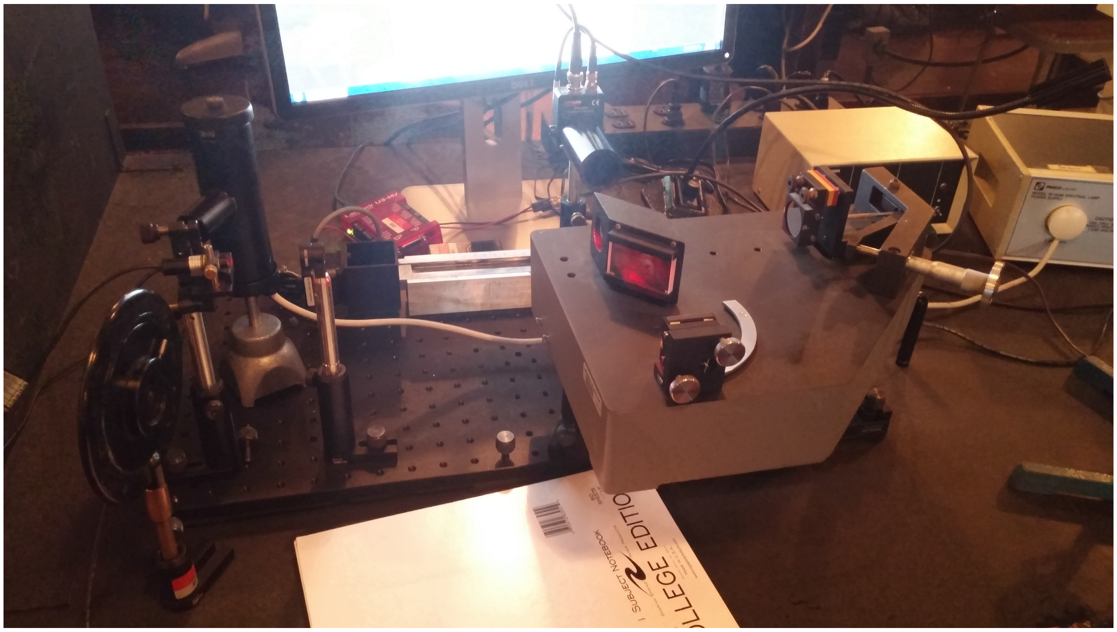

Figure 1: Experimental Setup

there could be slightly curved lines that are vertical (Figure 2 a) or horizontal (Figure 2 b), or there could be a bullseye (Figure 2 c and d). If there are curved lines and no bullseye it means that the fixed mirror must be slightly adjusted. A bullseye pattern is the one that indicated the fixed mirror is properly aligned. If there are curved vertical lines the fixed mirror needs to be adjusted left to right, while if there are curved horizontal lines the fixed mirror needs to be adjusted up and down. Slightly adjust the mirrored mirror until there is a bullseye pattern on the screen.

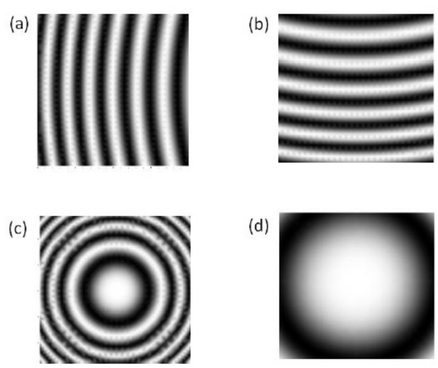

Figure 2: Interference Patterns

Next, the adjustable mirror should be moved until the circle in the middle of the bullseye is maximum. Use the micrometer to move the adjustable mirror in and out until this maximum is found. When the inner circle is largest it means that the path length in arm 1 is equal to the path length in arm 2, which is where the experiment should take place.

Lastly, the amplified photodiode should be placed so that the center of the bullseye falls on the detector at the output of the Michelson interferometer and be set to a gain of 50. The output of the amplified photodiode should be run to a filter so that high frequency noise (from instrumentation) can be removed from the signal. This is the complete setup for the measurement of the laser light interference, and this setup is shown in Figure 1

For the measurements of the sodium lamp light the sodium lamp with need to be placed after the plano-convex lense so that its light is directed into the Michelson interferometer. An extra iris may need to be placed after the sodium lamp and before the Michelson interferometer to increase the coherence length so make sure there is enough room in the case this is necessary. It is likely that the fixed mirror will need minor adjustments, but it can be adjusted using the screen and techniques previously explained.

3 Procedure

Calibrate Motor

Set the micrometer to an integer reading (for convenience), and record this value. Open Matlab so the commands *tic* and *toc* can be used as a timer. Next, with the motor off, couple the motor to the micrometer. Note, the motor may need to be turned on to align the prongs with the micrometer before it can be turned off and coupled. Type the *tic* command into Matlab and simultaneously enter the command and turn the motor on. Enter *toc* to stop the timer in Matlab when the micrometer has travel .5mm. Record the time it took for the motor to move the micrometer .5mm and repeat this measurement three times so the results can be averaged and the speed of the motor can be determined.

Interference of Laser Light

With the motor coupled to the micrometer, start the motor and then run a Matlab script that takes a voltage reading while simultaneous recording the time of the measurement. The script should thus be an array of timestamps and voltage measurements. Below is the Matlab script that was use:

```
h=load_labjack;
volt_time=[];
i=1;
tic
while toc<120
volt_time(i,:)=[toc lj_get(h)];
pause(0.02);
i=i+1;
end
volt_time;
plot(volt_time(:,1),volt_time(:,2))
```

Repeat this scan a couple times, saving the data each time with a unique name so that the data is not overwritten after multiple scans.

Interference of Sodium Lamp Light

With the sodium lamp (and iris if necessary) in place, the same type of scans as for the laser light should be taken; however, this time the Matlab script should be adjusted so that the scans run for approximately 16 minutes instead of only 2 minutes. This will allow the envelope function to be observed. To change the script above, just replace the 120 at the introduction of the while loop to 960 (16*60).

4 Experimental Data

The lab notes, raw voltage and time data (.mat files) from the photodiode for both the laser light interference and the sodium lamp light interference, and Matlab scripts used in the experiment can be found on labarchives, under "Data & Lab Notes", Lab 4.

The first set of data taken was the used for the calibration of the motor:

Distance (mm)	Time (s)
$.5 \pm .005$	$119.54 \pm .5$
$.5 \pm .005$	$120.27 \pm .5$
$.5 \pm .005$	$119.73 \pm .5$

The raw voltage scans of the laser and sodium lamp fringes are on the following page. Figures 3 and 4 are for laser light, while Figures 5 and 6 are for sodium lamp light. Note that Figure 3 is a two-minute scan, Figure 4 is a four minute scan and Figures 5 and 6 are sixteen minute scans.

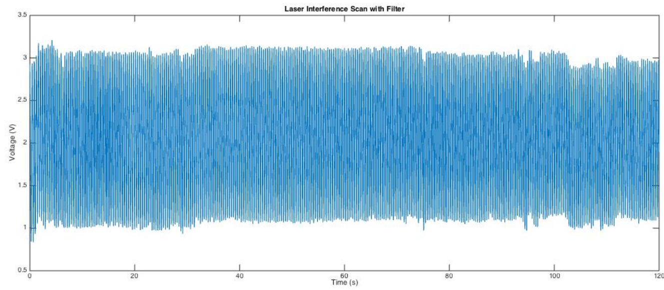

Figure 3: Laser Interference

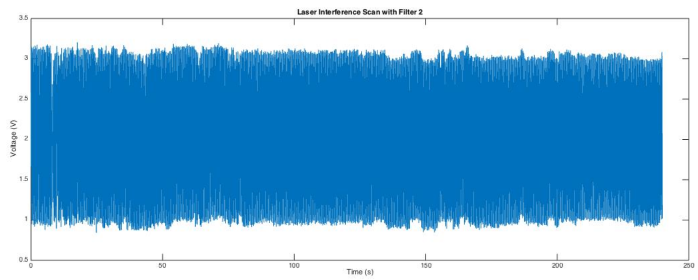

Figure 4: Laser Intererence 2

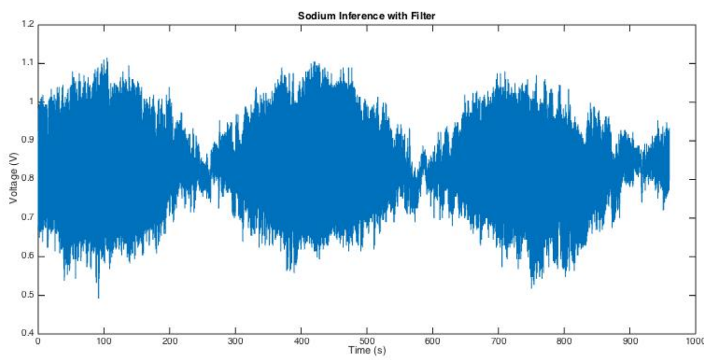

Figure 5: Sodium Lamp Interference

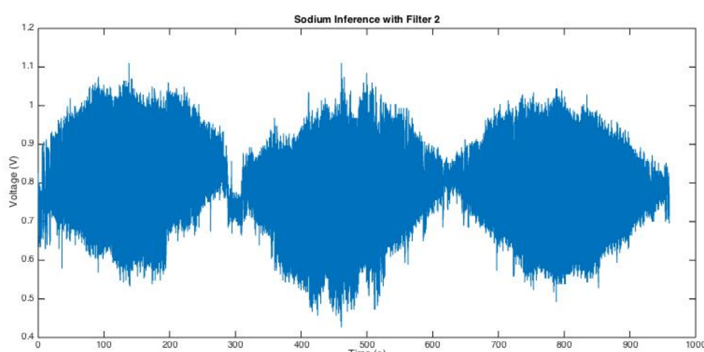

Figure 6: Sodium Lamp 2

5 Numerical Analysis

The first step in analyzing the data was to determine the speed the motor moved the adjustable mirror of the Michelson interferometer. Using the table of data from the previous section and averaging the results, the speed (distance/time) with error, as described in the next section, was . . .

$$speed = \frac{.5}{119.8467} = .0042 \pm .00003 \frac{\text{mm}}{\text{s}} \tag{1}$$

The preliminary determination of the wavelength of the laser was computed using the following equation

$$\lambda \approx \frac{1}{5}\left(\frac{2d}{m}\right) \tag{2}$$

Where d is the distance the micrometer moves and m is the number of rings that appear. One fringe is counted when the interference pattern goes from bright to dark to bright, which is a phase difference of 2π. The number of fringes corresponds to the number of peaks in Figures 3 and 4, so this counting should not be done by hand. Instead, Matlab has a *findpeaks* function that will do this tedious work.

Using both trials of the laser interference the wavelength of the diode laser was computed in Matlab using the following script. Where volt_time_filter1 and volt_time_filter1 are the arrays with the time and voltage measurement for the two trials.

```
%Trial 1
time=volt_time_filter1(:,1);
signal=volt_time_filter1(:,2);
[pks, indices]=findpeaks(signal);
number_of_peaks=length(indices);
speed=0.0042;
for i=2:indices
del_t=time(indices(i))-time(indices(i-1));
end
del_t=mean(del_t);
dist=del_t*speed;
wave(1)=2/5*dist*1000000;
%Trial 2
time=volt_time_filter2(:,1);
signal=volt_time_filter2(:,2);
[pks, indices]=findpeaks(signal);
number_of_peaks=length(indices);
speed=0.0042;
for i=2:indices
del_t=time(indices(i))-time(indices(i-1));
end
del_t=mean(del_t);
dist=del_t*speed;
wave(2)=2/5*dist*1000000;

wave=mean(wave)
```

The output of this script, *wave*, is the averaged diode waver wavelength which for the data plotted in Figure 3 and 4 was . . .

$$\lambda = 628.9865 \tag{3}$$

However, analyzing the sodium lamp light interference will reveal the true value for the lever arm factor and thus will allow for a more accurate determination of the laser's wavelength. Turning to the sodium lamp, the period of the envelope function needed to be determined to analyze the splitting of the D line doublet. Using Matlab to perform a Fourier transform of the data plotted in Figure 5 and 6, the following spectrums were found: While these spectrums are messy and blow up at a frequency of zero—indicating no periodic functionality—there is some reassurance when the frequency corresponding to the largest peak is used to create a sine function. Using the frequency given by marker one in Figure 7 and 8 a sine function was then adjusted by hand (manipulating the amplitude, phase and constant) and superimposed on plots of the signal. Notice the periods match up extremely well.

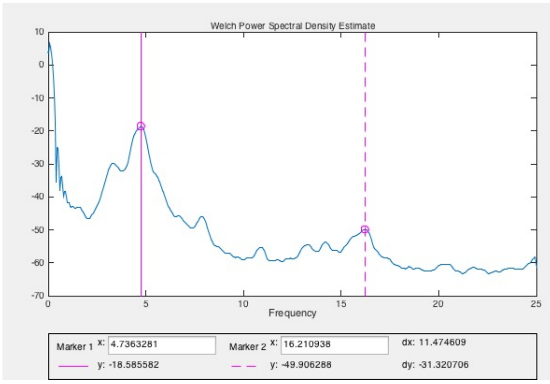

Figure 7: Frequency Spectrum Trial 1

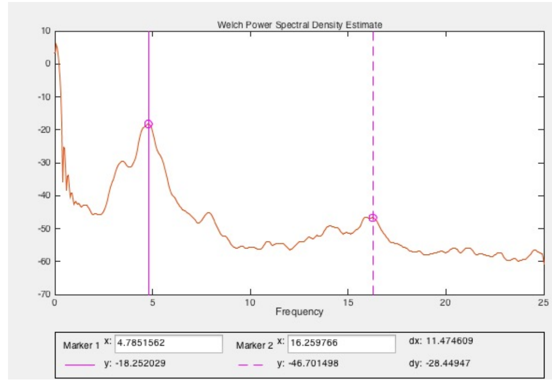

Figure 8: Frequency Spectrum Trial 2

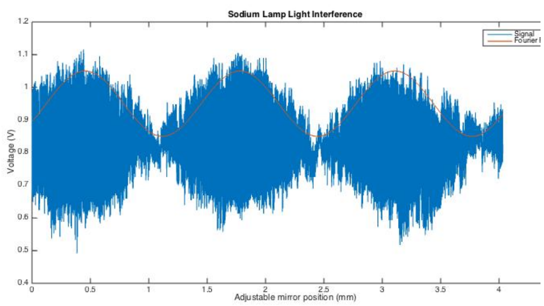

Figure 9: Fourier Frequency Fit 1

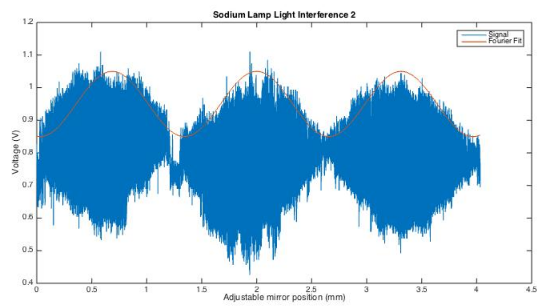

Figure 10: Fourier Frequency Fit 2

Averaging the two frequencies, since the error in the spectral decomposition cannot be determined to use for weights, we obtain the following

$$frequency = 4.7607 \quad \text{with standard deviation} \quad \sigma = .024414 \tag{4}$$

Comparing to the equation for the sum of two self interfering wavelengths

$$I = I_{const} + I_{mod} cos(k_{avg}\Delta s) cos(\Delta k \Delta s/2) + I_{osc} cos(k_2 \Delta s) \tag{5}$$

Since $cos(\Delta k \Delta s/2)$ is the term that creates envelope, we can determine Δk since the frequency is know

$$\Delta k = 2 * 4.7607 = 9.5215 \frac{1}{\text{mm}} \tag{6}$$

In turn this can be used to determine the difference between the two sodium D doublet wavelengths using the following formula

$$\Delta \lambda = 2\pi \left(\frac{1}{k_1} - \frac{1}{k_2} \right) \approx \frac{2\pi \Delta k}{k_{avg}^2} \tag{7}$$

And using that the mean of the two wavelengths is 589.2937nm, k_{avg} can be computed and

$$k_{avg} = \frac{2\pi}{589.2937/1000000} = 10662 \frac{1}{\text{mm}} \tag{8}$$

So

$$|\Delta \lambda| \approx \frac{2\pi * 9.5215}{10662^2} = 5.2627 * 10^{(} - 7)\text{mm} = .526 \pm .003\text{nm} \tag{9}$$

If we assume that the mean wavelength of the sodium D doublet is exactly 589.2937, and let 5 be a variable in equation 2 and then solving for this variable, then the interferometer level arm reduction factor can be determined. That is ...

$$factor = \frac{2d}{\lambda m} \tag{10}$$

However, if the script that determines wavelength is run on the sodium lamp data the results are roughly 100 and 620 nm. Clearly from this huge discrepancy it can be concluded that the *findpeaks* function does not work well on this data. So an interval must be counted by hand. Counting the fringes in a twenty second interval should suffice. For the section of data counted by hand, there were 57.5 fringes (bright dark to bright is one, hence the half). So leverage factor was found to be the following

$$factor = \frac{2 * 20 * .0042}{589.2937/1000000 * 57.5} = 4.96 \pm .18 \tag{11}$$

With the lever arm factor revised, the wavelength of the laser light can be more accurately computed by changing the factor of 5 in the previously shown code to the factor stated in equation 12.

Running the code again with the revised lever arm factor yields

$$\lambda_{trial1} = 634.0868\text{nm} \quad \text{and} \quad \lambda_{trial1} = 634.0312\text{nm} \tag{12}$$

Finally, using a weighted average based on the uncertainty in the wavelength from each trial the wavelength of the laser light is determined to be

$$\lambda = 634 \pm 6\text{nm} \tag{13}$$

6 Error Analysis

The error propagation done throughout the numerical analysis section was done in accordance with the following method of error propagation: let f be a function of some random variables x,y,z. Then its uncertainty is derived from:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} * \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} * \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} * \sigma_z\right)^2} \tag{14}$$

For the calibration of the motor the error in the time and distance needed to be propagated because of the averaging, and then needed to be propagated for the computation of the speed. For the averaging of the time and distance the error was computed in the following manner, where X is quantity being average and the denominator in the terms under the square root is three because there were three measurements made:

$$\sigma_{X_{avg}} = \sqrt{\left(\frac{\sigma_{X_1}}{3}\right)^2 + \left(\frac{\sigma_{X_2}}{3}\right)^2 + \left(\frac{\sigma_{X_3}}{3}\right)^2} \tag{15}$$

Since for both the time and distance measurements all three measurements carried the same error (ie $\sigma_{X_1} = \sigma_{X_2} = \sigma_{X_3}$) the formula simplified to ...

$$\sigma_{X_{avg}} = \sqrt{3 * \left(\frac{\sigma_X}{3}\right)^2} \tag{16}$$

Using this formula for both the time and distance measurements the following error in the average of the time and distance were found

$$\sigma_{time_{avg}} = .2887\text{s} \qquad \sigma_{distance_{avg}} = .0034\text{mm} \tag{17}$$

With the average time, distance and the error in both the error in the speed was computed as follows

$$\sigma_{speed} = \sqrt{\left(\frac{\sigma_{distance_{avg}}}{time_{avg}}\right)^2 + \left(\frac{distance_{avg}\sigma_{time_{avg}}}{t_{avg}^2}\right)^2} = 2.6202 * 10^{-5} \approx .00003\frac{\text{mm}}{\text{s}} \tag{18}$$

With the error in the speed of the motor the error in the wavelength of the laser can also be computed. Since equation 2 was used to compute the wavelength, the error propagation, given by equation 9, must also be applied to equation 2 (with the revised lever arm factor). Matlab is the most efficient way to implement this ...

```
%% Error of Wavelength
speed=.0042;

syms d m
lam=2/4.96*1000000*d/m;
```

```
dldd=diff(lam,d);
dldm=diff(lam,m);

%Trial 1
time=volt_time_filter1(:,1);
signal=volt_time_filter1(:,2);
[pks, indices]=findpeaks(signal);
number_of_peaks=length(indices);
distance=(time(indices(end))-time(indices(1)))*speed
elapsed=(time(indices(end))-time(indices(1)))

sig_speed=2.6202*10^-5;
sig_t=.0001;
sig_d=sqrt((sig_speed*elapsed).^2+(sig_t*speed).^2)
sig_m=5;

partiald=eval(subs(dldd,{d,m},{distance,number_of_peaks}));
partialm=eval(subs(dldm,{d,m},{distance,number_of_peaks}));
sig_wave(1)=sqrt((partiald*sig_d).^2+(partialm*sig_m).^2);

%Trial 2
time=volt_time_filter2(:,1);
signal=volt_time_filter2(:,2);
[pks, indices]=findpeaks(signal);
number_of_peaks=length(indices);
distance=(time(indices(end))-time(indices(1)))*speed
elapsed=(time(indices(end))-time(indices(1)))

sig_speed=2.6202*10^-5;
sig_t=.0001;
sig_d=sqrt((sig_speed*elapsed).^2+(sig_t*speed).^2)
sig_m=10;

partiald=eval(subs(dldd,{d,m},{distance,number_of_peaks}));
partialm=eval(subs(dldm,{d,m},{distance,number_of_peaks}));
sig_wave(2)=sqrt((partiald*sig_d).^2+(partialm*sig_m).^2);
sig_wave
```

The uncertainty in the time was given by the precision of the Matlab's *tic* and *toc* functions. So the error was taken to be .0001 seconds as this was the last digit that Matlab provides. Further more, the uncertainty in the number of fringes was estimated by looking at the Matlab function *findpeaks* over laid with the data and counting the number of erroneous peaks. Notice that this value for the four minute scan is double the value for the two minute scan.

The error in $\Delta\lambda$ was computed by applying equation 9 to equation 8. However, since the error in Δk was not known, the standard deviation was used instead. Also it was taken as given that the average wavelength of the sodium D doublet was *exactly* 589.2937 so the error in k_{avg} was identically error.

$$\sigma_{\Delta\lambda} = \sqrt{\left(\frac{2\pi\sigma_{\Delta k}}{k_{avg}^2}\right)^2} = .003\text{nm} \tag{19}$$

To calculate the error in the lever arm factor equation 9 was applied to equation 11, assuming an uncertainty of two fringes over the twenty-second interval, the uncertainty in the speed given by equation 1 and no uncertainty in the wavelength. This was computed in Matlab as follows

```
syms speed m
factor=2*20*speed/(589.2937/1000000*m);
dfds=diff(factor,speed);
dfdm=diff(factor,m);

sig_speed=.00003;
sig_m=2;

partials=eval(subs(dfds,{speed,m},{.0042,57.5}));
partialm=eval(subs(dfdm,{speed,m},{.0042,57.5}));
sig_factor=sqrt((partials*sig_speed)^2+(partialm*sig_m)^2)
```

The result was

$$\sigma_{factor} = .1761 \approx .18 \tag{20}$$

7 Discussion

The wavelength that was computed using the revised lever arm factor,634nm, is consistent with observation in that the laser light was red. The wavelength of red light is between 620-750 nm (as stated by wikipedia). It is also reassuring since in previous work the laser was mentioned to have a nominal wavelength of 635nm. However, the computation for the lever arm factor is hard to justify since the best way would have been to use all the data instead of a twenty second interval. If the sodium lamp light scans were taken at a higher sampling rate then perhaps the *findpeaks* command would have been more useful and led to a more accurate lever arm factor.

As for the splitting of the sodium D line doublet, it would be a stretch to say the result of $|\Delta\lambda| = .526 \pm .003$nm matches theory given the narrow margin of error. The accepted value for $|\Delta\lambda|$ is 0.5974nm $(589.5924 - 588.9950)$ which does not fall in the window of error of the computed value. However the computed value derived from experimental measurements was not egregious far from what was expected. If the sodium scan could be run for a significantly longer period of time, so that more periods of the envelope function could be analyzed, the results would likely be closer to theory.