PHYSICS 375

LAB 3 FULL REPORT

# Polarization of Light


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| Partners: | NA |


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**Abstract**

This experiment was performed to explore the polarization of light, and various theories that govern the mathematical construct of polarization. The intensity of a linearly polarized light as a function of a linear polarizer's axis of polarization angle should follow Malus' law for the intensity. The experimental data matches this theory exceptionally well. Malus' law is further verified by using it to describe a system of three polarizers. Lastly, the theory of reflected light summarized by the Fresnel equations are used to calculate the index of refraction of a glass tile. The experimental data lead to an index of refraction of $n = 1.5135 \pm 0.0028$ for the glass tile. While the type of glass used in the experiment is not known, it does match the index of refraction for most types of glasses that Wikipedia provides.

# 1 Introduction

## 1.1 Definitions

**Polarization**

Polarization refers to the direction in which the electric field oscillates in an electromagnetic field. This oscillation occurs in the plane perpendicular to the propagation of the electromagnetic wave, and can take on various patterns of repetition such as elliptical, circular, and linear or exhibit no repetition in which case the light is unpolarized.

**Scattering Plane**

The scattering plane is the plane that contains the $\overrightarrow{k}$ vectors of the incident, reflected and transmitted electromagnetic (in this scattering case) waves. These $\overrightarrow{k}$ vectors are the direction of propagation of the waves.

**Tranverse Electric Polarization**

The transverse electric or TE polarization is when the electric field is parallel to a boundary plane. This is equivalent to saying the electric field does not lie in the scattering plane.

**Tranverse Magnetic Polarization**

The transverse magnetic or TM polarization is when the magnetic field is parallel to a boundary plane. This is equivalent to saying the electric field does lie in the scattering plane, since the electric field is always perpendicular to the magnetic field.

**Brewster's Angle**

When an incident electromagnetic wave strikes an interface at the Brewster's angle—defined as the angle between the incident and the normal to the surface—the component of the electric field that lies in the scattering plane is perfectly transmitted.

## 1.2 Theory

**Malu's Law**

The polarization of the light is explored by measuring the intensity of a gaussian beam from a diode laser after propagating through various polarizers. Malu's Law provides the neccesary equations that link the intensity of transmitted light to the angle between the direction of the electromagnetic waves polarization and the polarizer's direction of polarization. This is summarized by the following formulation:

$$I = I_0 cos^2(\Delta\theta) \tag{1}$$

Given that $\theta_1$ is the angle of the beam's polarization, and that $\theta_2$ is the angle of the polarizer's polarization then $\Delta\theta$ is define as ...

$$\Delta\theta = |\theta_1 - \theta_2| \tag{2}$$

**Jones Calculus**

Jones calculus is linear algebra that compactly represents the polarization of light. It is comprised of Jones matrices and Jones vectors. The Jones vector corresponds to the polarization state of the light while Jones matrices are used to describe how various optics transform the polarization of light. The way we represent the electric field, and hence the polarization, of light propagating in the z direction is as follows:

$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix} = \begin{pmatrix} E_{0x}e^{i(kz-wt+\phi_x)} \\ E_{0y}e^{i(kz-wt+\phi_y)} \end{pmatrix} = \begin{pmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{pmatrix} e^{i(kz-wt)} \tag{3}$$

The last vector on the right is the Jones vector. Given what the Jones vector represents it is easy to see that multiplying the vector by a factor of i corresponds to a phase shift of $\frac{\pi}{2}$ and likewise multiplying by a factor of -i corresponds to a phase shift of $\frac{-\pi}{2}$. The following table summarizes what polarizations various Jones vectors correspond to.

| Polarization | Jones Vector |
|---|---|
| Linearly polarized in x direction (Horizontal) | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ |
| Linearly polarized in y direction (Vertical) | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ |
| Linearly polarized at 45 degrees | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ |
| Linearly polarized at -45 degrees | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ |
| Right hand circular | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ |
| Left hand circular | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ |

The Jones matrices summarize how these polarization states transformed when they interact with various optics such as linear and circular polarizers. Below are some common optical elements and the Jones matrix that describes them.

| Optical Element | Jones Matrix |
|---|---|
| Horizontal transmission linear polarizer | $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ |
| Vertical transmission linear polarizer | $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ |
| $\pm$ 45 degree transmission linear polarizer | $\frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$ |
| Right circular polarizer | $\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ |
| Left circular polarizer | $\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ |

**Reflection Amplitude**

When an incident light wave incurs a planar boundary there the wave separates into a reflected and transmitted wave. Exploring how much of the wave is reflected and transmitted requires consideration of the polarization of the light, the angle of incidence, and indices of refraction of both materials. The Fresnel's equations beautifully summarize the relation between all of these variables. Let E be the amplitude of the incident wave and E' be the amplitude of the reflected wave.

$$r_s = \left[\frac{E'}{E}\right]_{TE} = \frac{cos(\theta) - \sqrt{n^2 - sin^2(\theta)}}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{4}$$

$$r_p = \left[\frac{E'}{E}\right]_{TM} = \frac{-n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}}{n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{5}$$

Where $r_s$ and $r_p$ are the coefficients of reflection for the TE and TM polarization respectively. Even more useful to the following experiment is the *reflectance*, or the fraction of the incident light that is reflected. The reflectance is given by the following equation ...

$$R_s = |r_s|^2 = \left|\frac{E'}{E}\right|_{TE}^2 \qquad R_p = |r_p|^2 = \left|\frac{E'}{E}\right|_{TM}^2 \tag{6}$$

The reflectance is precisely what will be quantitatively explored in the following experiment. The expected plots of the reflectances (Fowles, 1968, p. 45):

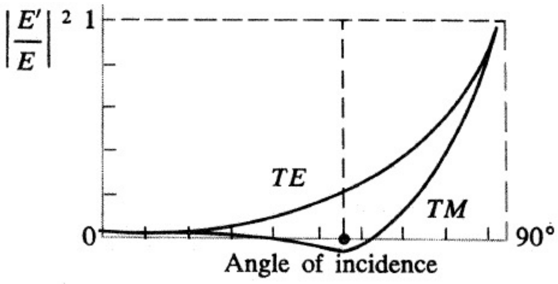


Figure 1: External reflection $\frac{n_1}{n_2} < 1$

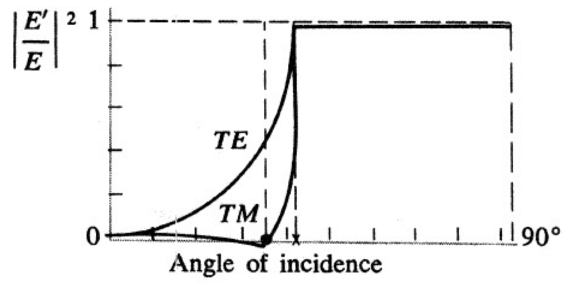


Figure 2: Internal reflection $\frac{n_1}{n_2} > 1$

## 2 Objectives

**First Objective**

Find the angle of polarization for both the fixed and rotating polarizers and determine the polarization of light coming from various sources, such as: an incandescent bulb, a computer monitor, and the laser.

**Second Objective**

Characterize the polarization of the laser light by measuring the intensity of transmitted light as a function of polarizer angle. Repeat using a sequence of polarizers, to get a feel for how the relative angle between polarizers effect the intensity of the transmitted laser light.

**Third Objective**

Quantitatively explore how the intensity of light reflected from a glass tile is influenced by the polarization of the light. Use this information to determine the index of refraction of the glass tile.

## 3 Apparatus

The three objectives utilized different setups. The first objective did not use an apparatus; rather it made use of the three polarizers to look at light reflected from the floor, the light emitted by a computer monitor, an incandescent light, and lastly the laser. The second objective made use of a setup comprised of the laser, a polarizer attached to a stepper motor and the amplified photodiode. This setup would have another polarizer put in, and later would have a liquid sugar inserted into the optical system. Lastly, the third objective made use of a setup with the laser, a glass tile on a rotary stage and the amplified photodiode.

| Materials | | |
|---|---|---|
| 1 Large Polarizer with Manual Rotation Adjustment | Optical Breadboard | Diode Laser |
| 1 Polarizer attached to Rotation Stage | LabJack Interface | Flash Light |
| 1 Small Polarizer with Manual Rotation Adjustment | Glass Plate | Corn Syrup |
| Amplified Photodiode | Computer with Matlab | Ruler |
| Stepper Motor (for rotation stage) | Calipers | Motor Controller |

On the following page find images and schematics of the setups for the second and third objectives

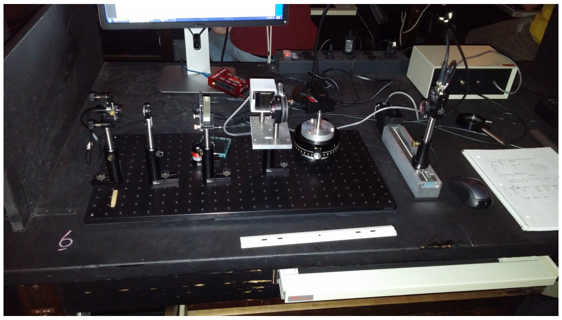

Figure 3: Setup for objective 2

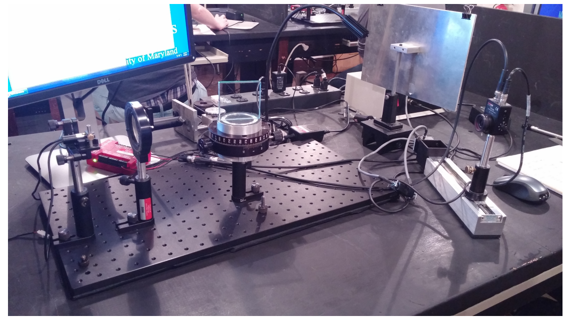

Figure 4: Setup for objective 3

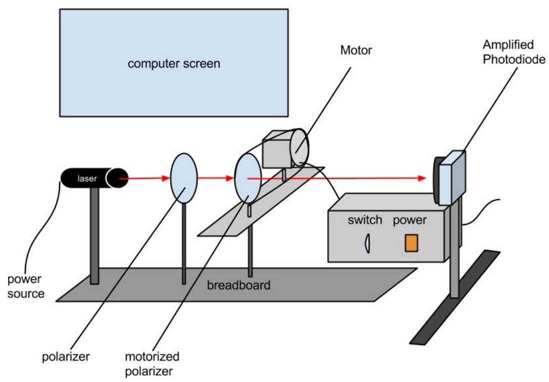


Figure 5: Schematic for objective 2

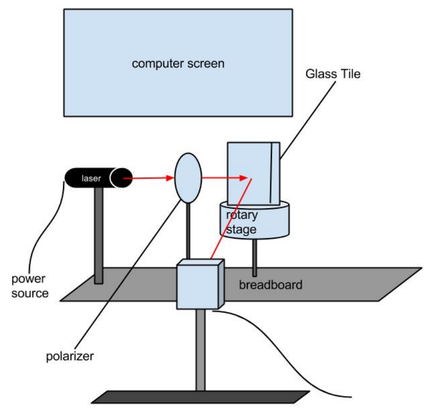


Figure 6: Setup for objective 3

## 3.1 Experimental Setup

**Polarization of Various Light**

Take all of the polarizers, the fixed polarizer, the large and the small rotating polarizers and make sure they are mounted on optical posts. This will make noting the angle of polarization when looking at various light sources easier. The only setup need for this section is mounting the large rotating polarizer in the path of the laser beam so the intensity after can be examined using a screen to visualize the beam.

**Polarization of Laser Light**

Align the diode laser beam with a line of holes in the breadboard so that one direction of the laser is well defined. This can be done by placing a ruler on half of each hole in the breadboard and ensuring it blocks half the beam along the whole length of the breadboard. Adjust the laser beam laterally using the fine adjustment knob on the side of the laser mount. Next aligned the beam so it is flat (i.e. it does not propagate upwards towards the ceiling or down towards the floor). Using a ruler to check the height of the beam when it leaves the diode laser and when it gets to the end of the breadboard adjust the laser up or down as necessary using the fine adjustment knob on the top of the laser mount.

Next, place the motor controlled polarizer in the path of the laser such that the normal to the polarizer corresponsed to the propagation direction of the laser. With the laser hitting the center of the polarizer set up the amplified photodiode at the end of the breadboard so the intensity after the polarizer can be measured at various angles. At this point the first measurement of the intensity verse the angle of polarization will be made.

Still pursuing the second objective, add an adjustable polarizer after the laser but before the motor controlled polarizer. Adjust the polarizer so the intensity of the laser beam on a screen behind the polarizer is maximized.

This will be done by eye but try to note the uncertainty in this angle. At this point the same measurements from before will be caried out.

**Sequence of Polarizers**

Next, under the second objective, remove the motor controlled polarizer and place a second adjustable polarizer such that its polarization axis is $90°$ separated from the axis of polarization of the first polarizer. The transmitted light should be at a minimum; take a voltage reading of this setup.

Next, keeping the two adjustable polarizers in place, add the motor controlled polarizer in-between them. Measurements of the intensity as a function of the polarization angle of the motor controlled polarizer will ensue here.

Lastly remove the second adjustable polarizer, and place a corn syrup cell in between the adjustable polarizer and the motorized polarizer (Figure 3). Take a measurement of the laser beam's intensity as a function of the motor controlled polarizer angle (this is the exact setup pictured in Figure 3). Repeat this process with two corn syrup cells.

**Polarization of Reflected Light**

With the laser aligned as before, remove the corn syrup cells and the motor controlled polarizer. Mount the rotary stage and place the glass tile in the rotary stage. The rotary stage scale must be set so the zeros coincide, while simultaneously the glass slide (the normal to its face) is made square to beam. This can be done by aligning the reflection off the first face of the glass slide to the spot of emission of the diode laser. Next, place the amplified photodiode so that the reflection (from the front face of the glass tile) enters the iris—this can only be done at angles $15° < \theta < 75°$ as other optical elements get in the way. Note the reflection from the front face of the glass tile will always be the spot closer to the laser. Next, adjust the initial polarizer so that its axis of polarization is horizontal (after measurements it will be switched to vertical for more measurements). This setup is exactly the one show in Figure 4. An averaging measurement will ensue here.

# 4 Procedure

**Polarization of Various Light**

Take all polarizers—except the motorized one—into a hallway. Looking through a polarizer look at an overhead light and note the polarization of the light. Incandescent lights are unpolarized so as the axis of polarization is rotated the transmitted light should remain (roughly) constant. Next look at the light reflected from the floor. Since the boundary plane is horizontal the reflected light will also be horizontal. Knowing this we are in a position to determine the axis of polarization of the polarizers. Rotate the adjustable polarizers through a large angle while observing the light reflected from the floor. If rotated through $360°$ there will be two angles at which the transmitted light is minimal. This means the axis of polarization if perpendicular to the polarization of the incoming light, which was already determined to be horizontal. Record the angle on the adjustable polarizer at these minimums of the transmitted light; this is a vertical axis of polarization. Repeat this for all three polarizers. Note that the fixed polarizer you cannot write an angle down, but an idea of the axis of polarization can be determined.

Knowing the axis of polarization specifically for the adjustable polarizers and roughly for the fixed polarizer, the polarization of other light can be determined. Take an adjustable polarizer and look at a computer screen while rotating through another large angle. When the light transmitted is at a minimum the light's polarization is perpendicular, or $90°$ separated from the axis of polarization.

Next, mount one of the adjustable polarizers so that it is in the path of the laser. Using a screen to visualize when the transmitted light is at a minimum, record the angle of the adjustable polarizer at the minimum transmission—this is $90°$ from the lasers polarization. Repeat this measurement a couple times looking for the two dimmest angles to minimize uncertainty.

**Polarization of Laser Light**

For the first part of the second objective the laser, motorized polarizer, and amplified photodiode will need to be setup as in Figure 3 and Figure 5 (note the *polarizer* is not yet included). First, using the screen manually rotate (the motor will need to be off) the motorized polarizer until the light transmitted is maximum (brightest). Now the rotation must be calibrated. To calibrate the rotation, the number of steps the motor takes must be adjusted from 400 to ensure the number of steps rotates the polarizer $360°$ exactly. So, using a Matlab program such as:

```
h=load_labjack;
steps=1
stop=400
while steps<=stop
    lj_step(h);
    voltage_reading(1,steps)=lj_get(h);
    pause(.02);

    steps=steps+1;
end
```

Note the initial angle of the motorized polarizer, then run the program and adjust *stop* until the polarizer rotates exactly 360°. For the equipment used, 400 steps was 7° short, 405 steps was 1° to far, and 404 steps was just right.

Next, a Matlab program needs to control the rotation of the polarizer in such a way that the belt will not slip. The best way to accomplish this is to have the speed ramp up to full speed and ramp down to a stop. Below is the Matlab program that was used to control the motorized polarizer

```
%% Scan
h=load_labjack;
steps=1;
tic
total_steps=400;
voltage_reading=zeros(1,total_steps);
rampsteps = 30;
steadysteps = 404;
while steps<=rampsteps
    t=toc;
    nu=5*(t+.01);

    lj_step(h);
    if steps>0
        pause(1/(400*nu));
    end
    steps=steps+1;
end
while rampsteps<steps && steps<steadysteps + rampsteps
    lj_step(h);
    voltage_reading(1,steps)=lj_get(h);
    pause(1/(400*nu));
    steps=steps+1;
end
for steps=rampsteps+steadysteps:steadysteps + 1.65*rampsteps
    lj_step(h);
    pause((1-((steps-(steadysteps+rampsteps))/(rampsteps+1)))^(-2)/400*nu)
    steps=steps+1;
end
```

Now, with the amplified photodiode turned on and the gain set to 50, run the above Matlab program and save the data. It is important to have the photodiode turned on, otherwise, like happened; the first trials will not be of use (note the base file for the motorized polarizer starts at volt_third). This measurement should be taken multiple times but it is important the motorized polarizer follows the exact same path. So the previous program, or one stripped of the voltage reading commands should be run after the direction is switched on the motor controller.

The next measurement is the same as previous with an extra polarizer added. An extra adjustable polarizer should be placed between the laser and the motorized polarizer, and be adjusted so that the maximum laser light is transmitted (use the screen method). This setup is exactly Figure 5. Run the same Matlab program as before, and be sure to save the data from each trial! There were two trials that were overwritten and needed to be repeated because the data wasn't saved. These measurements with the extra polarizer will help verify (presumably) Malus' law.

Lastly, turn the laser and motor off and record the background light that the amplified photodiode picks up so this can be subtracted off the previously recorded data.

**Sequence of Polarizers**

With two polarizers crossed as the setup describes take a voltage measurement of this optical configuration. Note, it would be best to write and use a Matlab program that averages a couple measurements, but a single value was recorded when the experiment was performed.

Next, with the motorized polarizer between the two polarizers, use the Matlab scan used previously to record the voltage against the motorized polarizer's angle. Remembering to use the same program to "rewind" the polarizer, repeat this measurement, and save the files, multiple times.

With one corn syrup cell in place, measure the voltage as a function of the motorized polarizer's angle like the previous measurement. Repeat this measurement a couple times and save the data. Repeat this whole process with two corn syrup cells in the path of the laser.

**Polarization of Reflected Light**

With the initial polarizer's polarization axis set to horizontal (with a recorded uncertainty!), set the rotary stage to an angle of 15° and roughly move the amplified photodiode into place. Matlab must be running the voltage recording command continuously so that the amplified photodiode can be move into the proper place—where it attains a maximum reading. When this stop is found be sure not to bump the table and use a Matlab script to record ten measurements of voltage, this can then be averaged. The script used was:

```
steps=1;
voltage_reading=[];
while steps<=10
    voltage_reading(1,steps)=lj_get(h);
    pause(.02)
end
```

After getting the voltage at 15°, turn the laser off and take another measurement. This way the background light can be substracted off. It is important to take this background measurement before the amplified photodiode is moved to ensure the best results. These measurements should be taken with the rotary stage at angles of 15°, 25°,35°,45°,55°, 65°, 75°. Figure 4 demonstrates the importance of taking the background voltage at each position since the bright LCD computer screen will effect each measurement differently.

Repeat the previous measurements at the same rotary stage angles but with the initial polarizer's axis vertical this time.

# 5 Experimental Data

## 5.1 Raw Data

**Voltage Files**

The raw voltage data (.mat files) from the photodiode for the various measurements can be found on labarchives, under "Data & Lab Notes", Lab 3, under Day 1 and Day 2. Below is a table that matches the base .mat files with the measurement being taken.

| **Measurement** | **Base file name** |
|---|---|
| Motorized polarizer | volt_third |
| Adjustable polarizer before motorized polarizer | volt_2_first |
| 2 Crossed polarizers | volt_back |
| All 3 polarizers | volt_3_first |
| 1 Corn syrup cell | volt_sug1 |
| 2 Corn syrup cells | volt 1 sug |
| Horizontal polarizer measuring reflection | raw: raw15hor averaged: volt horizontal 15 deg |
| Vertical polarizer measuring reflection | raw: raw15vert averaged: volt vert 15 |

It should be noted that voltage files created from measurements with the motorized polarizer contain thirty elements of zero in the beginning of the arrays—these were removed for the purpose of analysis. This occurred because the array that stored the voltage files was indexed with "steps" when voltage reads were only taken for "steadysteps".

**Polarizer of Various Light**

The visual characterization of polarizers was done by noting at what angle the polarizer let through the least amount of light reflected from the floor. The measurements of dimmest are summarized below:

| **Large Adjustable** | **Small Adjustable** | **Fixed** |
|---|---|---|
| $268 \pm .5°$ | $98.5 \pm .5°$ | Horizontal |
| $269 \pm .5°$ | $99.5 \pm .5°$ | Horizontal |
| $90 \pm .5°$ | $282.5 \pm .5°$ | Horizontal |
| $91.5 \pm .5°$ | $282 \pm .5°$ | Horizontal |

Notice that the fixed polarizer only has a qualitative measurement, as there were no markings of angle since it was not adjustable. Since light that is reflected from the floor is polarized horizontally, the polarizers were oriented vertical when they were the dimmest.

Knowing the polarization of the polarizers the polarization of the laser, and the motorized polarizer could be determined. Though the fixed polarizer was used to cross the motorized polarizer so only qualitative measurements were made. The measurements of those polarizations are listed in the table below:

| **Laser as Mounted** | **Motorized Polarizer at** $0°$ |
|---|---|
| $327 \pm .5°$ | Vertical |
| $148 \pm .5°$ | Vertical |

Below are the superimposed plots that result from the polarization of laser light section. Figure 7 shows the measurement of voltage when the optical system was just the laser; the motor controlled polarizer and amplified photodiode. The green plot is the fitted sinusoidal function which has the equation $3.423 * sin(1.986x - 4.454) + 3.46$. Next, an adjustable polarizer was placed in front of the motor controlled polarizer. The measurements from this optical system are plotted in Figure 8 over top of the measurements in Figure 7. Notice that the max voltage with an initial polarize is lower than without an initial polarizer since the extra polarizer rejects some of the light from the laser (a significant portion despite being aligned to transmit the max amount of light). Notice how the sine function repeats with period $\pi$ just as $cos^2(x)$ does which is what Malus' law predicts.

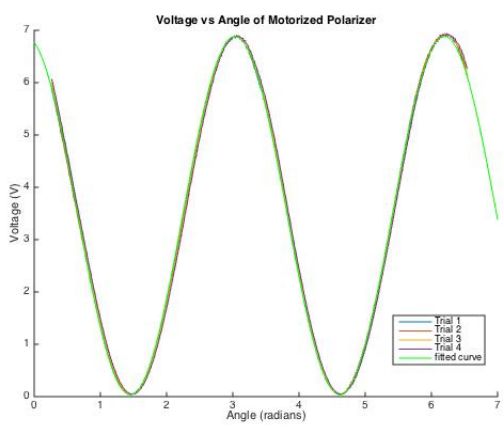


Figure 7: Just the motorized polarizer

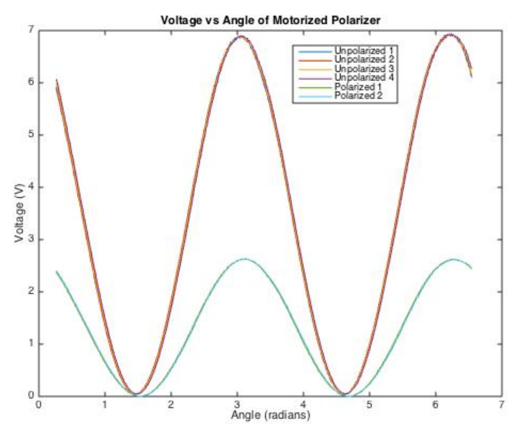


Figure 8: Intial polarizer superimposed over Figure 7

The background voltage, which was subtracted off, was plotted ("background light") and can be found on labarchives with the rest of the data; however, it is not particularly enlightening so it was omitted. Lastly, note the small phase shift between the two sets of curves in Figure 8. This phase shift indicates that the initial polarizer, which was supposed to be set up to transmit the maximum amount of light (i.e. its polarization axis is aligned with the lasers), did not have its axis of polarization exactly matched up with the laser's axis of polarization. Further more, it can be deduced that the lasers intrinsic polarization axis was slightly more than

$143.5 \pm .5°$—the angle the initial polarizer was set at—since without the initial polarizer the minimum light transmitted was achieved with less rotation. This roughly validates the laser polarization we found early of $148 \pm .5°$. There was, however, a noticeable difference in the starting angle after 5 scans which would indicate the belt was slipping and would also indicate the analysis of the phase shift maybe simplistic.

**Sequence of Polarizers**

With the two adjustable polarizers crossed, such that the small adjustable polarizer was set at $234 \pm .5°$ and the large adjustable polarizer was set at $143 \pm .5°$, the voltage reading was .0053V. While at first glance $234° - 143° \neq 90°$, this is where the minimum transmitted light occured. This can be understood as $143° - 91° = 52°$ and $282° - 234° = 48°$. Putting these together, since they were aligned anti-parallel and not parallel, $48° + 52° = 100° \approx 90°$. While this is a huge margin of error, the measurements of the polarization axes were rough. Despite the uncertainty of the angle the adjustable polarizer being .5° the uncertainty in which location was the dimmest could not be well judged by eye.

The expected functional form of two adjustable polarizers crossed and the motorized polarizer between can be worked out using Malus' law.

Take the laser after the first polarizer, call this $I_0$ of the intensity, and call the amplitude of intensity after the motorized polarizer $I_1$. Then passing through the motorized polarizer follows Malus' law ...

$$I_1 = I_0 cos^2(\theta) \tag{7}$$

Where $\theta$ is the angle of the motorized polarizer with respect to the initial polarizer. The take this amplitude $I_1$ and use Malus' law again to describe the beam going through the next polarizer. Let the final amplitude be $I_2$ ...

$$I_2 = I_1 cos^2(\phi) \tag{8}$$

Where $\phi$ is the angle between the motorized polarizer and the final polarizer. Combining these equations we can find the functional form of the intensity for the whole optical system (all three polarizers).

$$I_2 = I_0 cos^2(\theta) cos^2(\phi) \quad \text{Since } \phi - \theta = \frac{\pi}{2} \quad I_2 = I_0 cos^2(\theta) sin^2(\theta) \tag{9}$$

Below is the data from all three polarizers superimposed with the expected functional form (equation 9) overlaid:

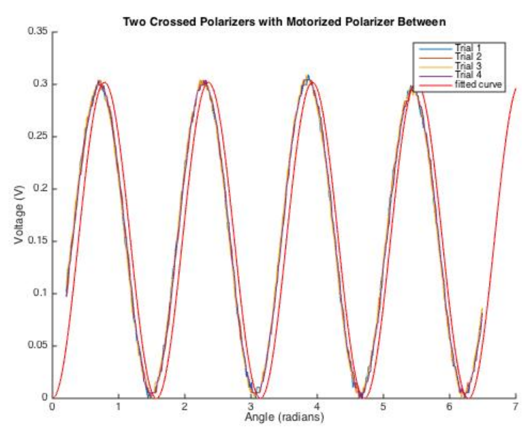


Figure 9: All three polarizers

There is an unexpected phase difference between the theory and the data, which would indicate that the angle of the motorized polarizer did not coincide with the initial polarizer. The period matches what the theory suggests. The amplitude, according to the theory, should be approximately $1 * cos^2(45) cos^2(45) = .25$ since the maximum beam going into the motorized polarizer is given by the lower blue plot in Figure 8. This beam this then put through the motorized polarizer at $45°$ and then through the last polarizer at $45°$. However, the amplitude seen from the data is closer to .15 indicating there may be additional losses at each polarizer. Setting aside these minor effects, the theory matches the data quite well.

The measurements with the corn syrup cells resulted in the following superimposed plots. Due to restriction of lab time there were only two trials taken when there were two corn syrup cells in the optical system. Ideally the number of trials for one and two corn syrup cells would have matched for consistency.

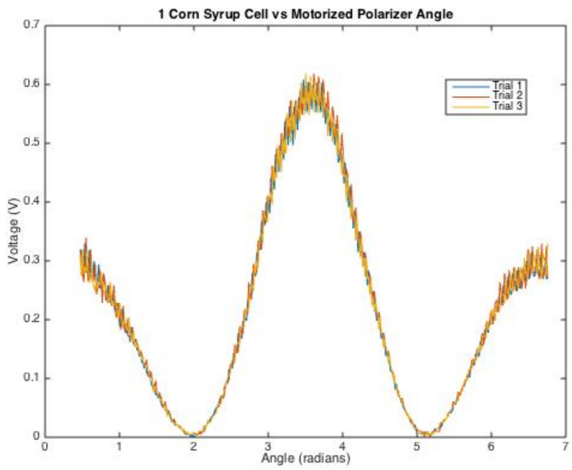


Figure 10: 1 Corn Syrup Cell

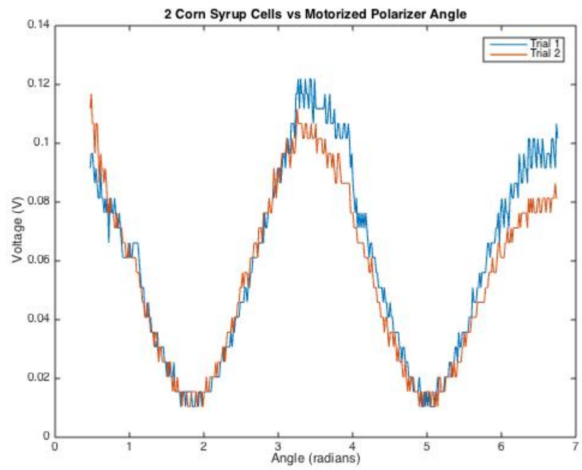


Figure 11: 2 Corn Syrup Cells

The most obvious difference between the plots in Figures 10 and 11 compared to Figures 7 and 8 is the phase shift. Indeed the chiral structure of the sugar in the corn syrup caused the initially linearly polarized light to become circularly polarized. If we examine what happens when circularly polarized light is transmitted through a linear polarizer using Jones Calcus there is a resulting factor of i that appears and that is equal to a phase shift.

**Polarization of Reflected Light**

This portion of the experiment was completed the second day in the lab and was intended to be the most quantitative. With an initial polarizers' polarization axis set both horizontal and (then) vertical, measurements of the reflected light were taken when the rotary stage was at angles of $15°$, $25°$,$35°$,$45°$,$55°$, $65°$, $75°$. Recall the placement of the amplified photodiode was very particular and sensitive to minor adjustments, which made finding the perfect placement difficult. Below are plots of the voltage verse the angle of the rotary stage (and the hence the glass tile) less the background voltage:

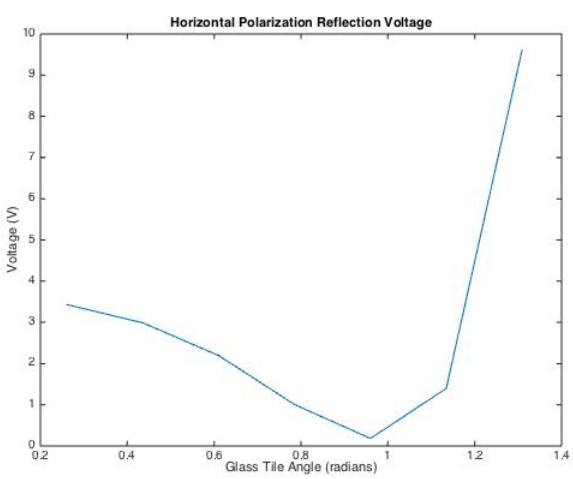


Figure 12: Reflected Horizontally Polarized Light

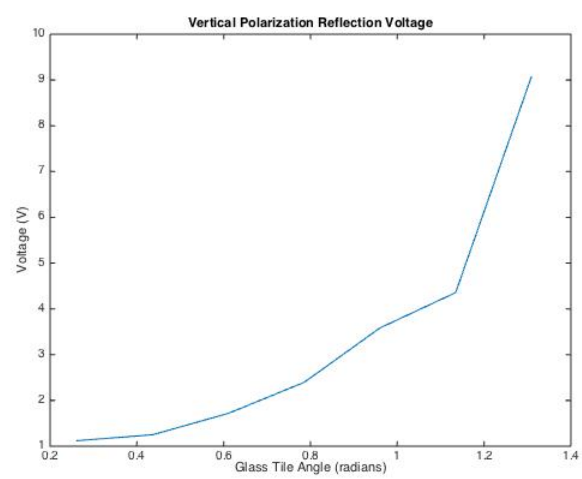


Figure 13: Reflected Vertically Polarized Light

Before looking back to the theory of reflected light, the TM and TE mode of polarization should be determined so that the comparison can take place in the yet to come analysis. When the initial polarization axis was set to horizontal (Figure 12), the electric field of the laser light had components that lay in the scattering plane making this the TM polarization. When the initial polarization axis was set to vertical the electric field of the laser did not fall in the scatter plane—it would be coming out of and going into the scattering plane rather—thus Figure 13 represents the TM mode.

Now, referring back to the theory of scattered light the similarity between Figures 12 and 13 and Figure 1 is clear. It also makes sense in relation to Brewster's Angle. Recall that for TM polarized light there is an angle (called Brewster's angle) where this light is perfectly transmitted. This angle can be estimated from Figure 12; it appears that 1 radian is roughly Brewster's angle in this setup.

# 6 Numerical Analysis

The majority of the analysis is done on the reflected polarized light section. The first step in analyzing the reflected light, which is plotted in Figure 12 and Figure 13 is to find the standard deviation of the voltages at each angle of measurement. Recall the plotted data in Figures 12 and 13 are the averages, which were computed via Matlab to get one number. However to get the uncertainty in these voltages a standard deviation command of "std2" must be run on each set of data (10 points).

With the uncertainty in the voltage measurements we are set to compare the experimental data to the theory of reflected light. In this case, $n_2 > n_1$ thus we will be using the external reflection given by Figure 1. Again, the TE mode corresponds to vertically polarized laser light and the TM mode corresponds to the horizontally polarized laser light.

## 6.1 Excel Analysis

**Chi-Squared Minimization**

Input the voltage, and uncertainty, data into excel the theoretical values could be produced and then compared to the experimental data using a chi-squared test. Below is the table exported from the excel file "Reflection Analysis":

| n1 | n2 | theta (deg) | volt_h (V) | volt_v (V) | sig_h (V) | sig_v (V) | theory_h (V) | theory_v (V) | chi_h | chi_v |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.53296691 | 15 | 3.43 | 1.1213 | 0.0081 | 0.0035 | 3.804672509 | 1.238612708 | 2139.605079 | 1123.450737 |
| | | 25 | 2.9867 | 1.251 | 0.0048 | 0.0029 | 3.131792882 | 1.45327441 | 913.7128665 | 4865.034106 |
| amp_v | amp_h | 35 | 2.1934 | 1.7184 | 0.0029 | 0.004 | 2.136240096 | 1.849142522 | 388.4963895 | 1068.350438 |
| 25.57996749 | 94.42155466 | 45 | 1.0042 | 2.3961 | 0.0027 | 0.0044 | 0.940258883 | 2.552631576 | 560.8321623 | 1265.606117 |
| | | 55 | 0.1835 | 3.5828 | 0.0032 | 0.0054 | 0.036734071 | 3.817820855 | 2103.538855 | 1894.197606 |
| diff_h | diff_v | 65 | 1.3975 | 4.3558 | 0.04 | 0.0196 | 1.123658524 | 6.154509891 | 46.86822137 | 8421.900439 |
| -0.374672509 | -0.117312708 | 75 | 9.61 | 9.0695 | 0.0125 | 0.0129 | 9.891703196 | 10.56969146 | 507.8828206 | 13524.27382 |
| | | | | | | | | | 6660.936395 | 32162.81327 |

Where volt_h and volt_v are the voltage readings for the horizontally and vertically polarized light respectively. The theory values theory_h and theory_v are computed by squaring equations 5 and 4 to give the TM and TE reflectance respectively. Notice that Figure 1 squares $r_s$ and $r_p$. Additionally, $n_1 = 1$ because the first medium is air, while $n_2$ is computed using a chi-squared minimization. Lastly, since the initial intensity of the laser was not recorded (it should have been for the normalization to match equations 4 and 5) so there are two quantities diff_h and diff_v, which were used to match the amplitude of the first measurement point (at $15°$) and the theory value for both horizontal and vertical polarizations.

Superimposing the plots of the theory and the experimentally gather data, with error bars given by the standard deviation, the following plot is generated:

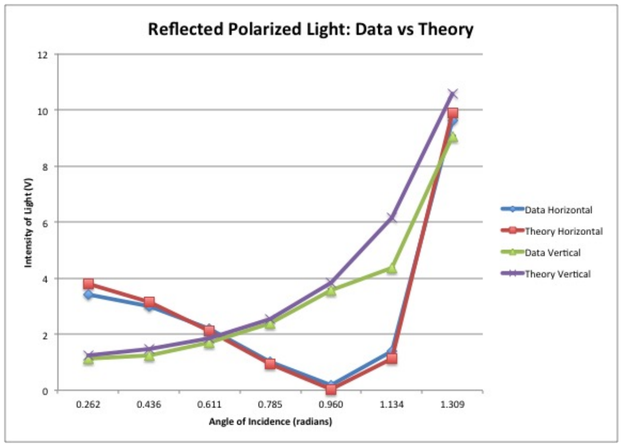


Figure 14: Reflection of Polarized Light: Theory vs Experiment

Despite the chi-squared values being very large even after the minimization, the plot of the theory verse experimental data is promising as the data does follow the general functional form the theory gives. Another issue with the excel analysis is that goal seeking (used for the chi-square minimization) can only be done one cell at a time. The implications of this is that the chi-squared value for the horizontal orientation had to be completed and then the chi-squared could be run on the vertical orientation; however, in doing the second minimization, the first minimization is ruined.

Nonetheless, the value calculated for the index of refraction of the glass tile is 1.53. While the type of glass used in the experiment is not known, it does match most glasses in that Wikipedia cites indices of refraction crown glass (pure) to be 1.50-1.54, crown glass (impure) to be 1.485-1.755, and flint glass (impure) to be 1.523-1.925.

## 6.2 Matlab Analysis

**Numerical Solver**

To be thorough, and improve upon the excel analysis, the data was also run through Matlab to determine the index of refraction of the glass tile. This was done by setting the Fresnel equations for reflectance (equations 4 & 5 squared) equal to the intensity measured amplified photodiode for each orientation. Below is the Matlab script used to find the index of refraction through numerical solving. Notice, that again the normalization intensity is missing, this is compensated using the same factors as in the excel analysis ($25.57996749$ and $94.42155466$).

```
angle=[15 25 35 45 55 65 75]*pi/180;
volt_h=[3.43 2.9867 2.1934 1.0042 0.1835 1.3975 9.61];
volt_v=[1.1213 1.251 1.7184 2.3961 3.5828 4.3558 9.0695];
sig_h=.1*volt_h;
sig_v=.1*volt_v;

syms  n
for i=1:7
    rs(i)=(cos(angle(i))-sqrt(n.^2-(sin(angle(i))).^2))/(cos(angle(i))+sqrt(n.^2-(sin(angle(i))).^2));
    rp(i)=(-n.^2*cos(angle(i))+sqrt(n.^2-(sin(angle(i))).^2))/(n.^2*cos(angle(i))+sqrt(n.^2-(sin(angle(i))).
    Rs(i)=25.57996749*rs(i)^2;
    Rp(i)=94.42155466*rp(i)^2;
    min_this_Rs(i)=volt_v(i)-Rs(i);
    min_this_Rp(i)=volt_h(i)-Rp(i);
end
n_Rs_15=min_this_Rs(1);
n_Rs_25=min_this_Rs(2);
n_Rs_35=min_this_Rs(3);
n_Rs_45=min_this_Rs(4);
n_Rs_55=min_this_Rs(5);
n_Rs_65=min_this_Rs(6);
n_Rs_75=min_this_Rs(7);
```

```
n_Rs_15=eval(solve(n_Rs_15==0,n));
n_Rs_25=eval(solve(n_Rs_25==0,n));
n_Rs_35=eval(solve(n_Rs_35==0,n));
n_Rs_45=eval(solve(n_Rs_45==0,n));
n_Rs_55=eval(solve(n_Rs_55==0,n));
n_Rs_65=eval(solve(n_Rs_65==0,n));
n_Rs_75=eval(solve(n_Rs_75==0,n));
n_Rs=[n_Rs_15(4), n_Rs_25(4),n_Rs_35(4),n_Rs_45(4),n_Rs_55(4),n_Rs_65(4),n_Rs_75(4)]

index=linspace(1,5,5000);
n_Rp_15=min_this_Rp(1);
n_Rp_25=min_this_Rp(2);
n_Rp_35=min_this_Rp(3);
n_Rp_45=min_this_Rp(4);
n_Rp_55=min_this_Rp(5);
n_Rp_65=min_this_Rp(6);
n_Rp_75=min_this_Rp(7);

[M,I]=min(abs(subs(n_Rp_15,{n},{index})));
n_Rp_15=index(I);
[M,I]=min(abs(subs(n_Rp_25,{n},{index})));
n_Rp_25=index(I);
[M,I]=min(abs(subs(n_Rp_35,{n},{index})));
n_Rp_35=index(I);
[M,I]=min(abs(subs(n_Rp_45,{n},{index})));
n_Rp_45=index(I);
[M,I]=min(abs(subs(n_Rp_55,{n},{index})));
n_Rp_55=index(I);
[M,I]=min(abs(subs(n_Rp_65,{n},{index})));
n_Rp_65=index(I);
[M,I]=min(abs(subs(n_Rp_75,{n},{index})));
n_Rp_75=index(I);
n_Rp=[n_Rp_15 n_Rp_25 n_Rp_35 n_Rp_45 n_Rp_55 n_Rp_65 n_Rp_75]
```

The first part of the code initializes the data recorded from the lab and inputs the reflectance equations that the data will be matched against. Next the program looks for the value of n, the index of refraction, that makes the difference between the theory and the experimental data go to zero. However, only data for the TE mode (Rs) can be solved with the difference exactly equal to zero. The TM mode data, when programmed like the TE mode data, repeated returned errors and negative numerical solutions. As a work around, the equations for the reflectance of the TM mode is fed a prescribed solution set of indices—namely 1 through 5—and then the minimum reflectance is solved for. This work around does add additional uncertainty as the set of indices is discrete. To compensate for this, the set of indices has 1000 values per unit 1 (5000 values total).

The results are summarized in the following table

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| n_Rs | 1.5000 | 1.4825 | 1.5059 | 1.5070 | 1.5026 | 1.3618 | 1.4053 |
| n_Rp | 1.5001 | 1.5185 | 1.5401 | 1.5473 | 1.6545 | 1.1664 | 1.5401 |

The propagated error for each of these is given in the following table, which will be described at length in the following section on error analysis:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| sig_n_Rs | .007 | .0074 | .0092 | .0138 | .0236 | .0356 | .077 |
| sig_n_Rp | .0071 | .0093 | .0103 | .0075 | .0044 | .0139 | .0722 |

Finally, with these numbers, a weighted average can be performed based on the uncertainty in each value of n. The values of n were weighted by $\frac{1}{\sigma_n}$ to give a weighted average of n where more accurate measurements contributed more to the final value of n. The equation for the weighted average is as follows:

$$\bar{n} = \frac{\sum_{i=1}^{5} n_i w_i}{\sum_{j=1}^{5} w_j} \tag{10}$$

The following is the code that was used to perform this weighted average. If any of the variables seem ill-defined see the error analysis in the following section as that was performed prior to the weighted average computation.

```
sig_n=[sig_n_Rs sig_n_Rp];
weights=[1./sig_n_Rs 1./sig_n_Rp];
n=[n_Rs n_Rp];

error_prop=[];
for k=1:14;
    error_prop(k)=(weights(k)/sum(weights)*sig_n(k))^2;
end
final_n=(weights*n')/sum(weights)
final_error=sqrt(sum(error_prop))
```

The final result of the Matlab analysis says that the index of refraction of the glass tile is ...

$$n = 1.5135 \pm 0.0028 \tag{11}$$

# 7 Error Analysis

## 7.1 Theory

**Error Propagation**

Traditional error propagation is done in accordance with the following method of error propagation: let f be a function of some random variables x,y,z. Then its uncertainty is derived from:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} * \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} * \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} * \sigma_z\right)^2} \tag{12}$$

## 7.2 Experiment

**Angle of Polarizers**

The error in the angle of the polarizers was determined to be .5°, since only integer degrees were marked. This meant that the last digit (the tenth) of the angle listed for the polarizers was interpolated for the solid degree markers.

However, the majority of the error in the angle of the polarizers was not due to the scale on the side, but rather due to the subjectivity of *dimmest*. As mentioned earlier, the polarizer angles in the first part of the lab were determined by noting the angle at which they transmitted the least light. This was a judgment call and it is safe to say that the angle recorded as the dimmest was probably within $\pm 15°$ of the *truly* dimmest angle.

**Index of Refraction Propagation**

The error for this experiment is not as tractable as the theory example. The reason being that the equation we are trying to match the data to has the variable we are interested in, n, is not solved for explicitly. This means the propagation laid out in equation 11 cannot be used.

So we try to solve the equations for n:

$$R_s = \left(\frac{cos(\theta) - \sqrt{n^2 - sin^2(\theta)}}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}}\right)^2 \quad \text{Solving for n} \quad n = \frac{-\sqrt{-2 * \sqrt{R_s} cos(2\theta) + R_s + 1}}{\sqrt{R_s + 2\sqrt{R_s} + 1}} \tag{13}$$

However $R_p$ does not really resolve. The output from wolframalpha, "explicitform.jpg", can be found on labarchives under Data & Lab Notes, Day 2, but the number of terms makes the equation unusable for error propagation.

Due to the limitation of computing power, the $R_p$ equation cannot be plugged into equation 11 for error propagation. Luckily equation 12 can. While it is a poor assumption that equation 12 holds for $R_p$ it is the best approximation that is computable. Below is that Matlab script used to propagate the error:

```
% let x=theta and y=Rs
angle=[15 25 35 45 55 65 75]*pi/180;
```

```
sig_x=pi/180;
sig_yh=.1*volt_h;
sig_yv=.1*volt_v;
n_Rs;
n_Rp;

syms x y
n=-(sqrt(-2*sqrt(y)*cos(2*x)+y+1))/(y+2*sqrt(y)+1);
dndx=diff(n,x);
dndy=diff(n,y);

sig_n_Rs=[];
for i=1:7;
    partialx=eval(subs(dndx,{x,y},{angle(i),n_Rs(i)}));
    partialy=eval(subs(dndy,{x,y},{angle(i),n_Rs(i)}));
    sig_n_Rs(i)=sqrt((partialx*sig_x).^2+(partialy*sig_yv(i)).^2);
end
sig_n_Rs

sig_n_Rp=[];
for i=1:7;
    partialx=eval(subs(dndx,{x,y},{angle(i),n_Rp(i)}));
    partialy=eval(subs(dndy,{x,y},{angle(i),n_Rp(i)}));
    sig_n_Rp(i)=sqrt((partialx*sig_x)^2+(partialy*sig_yh(i))^2);
end
sig_n_Rp
```

The first thing to note is that the uncertainty in the intensities is not given by the standard deviation anymore. Recognizing that the placement of the photodiode was paramount to the reading it gave, and having witnessed the fluctuation due to minor changes in location, the uncertainty was revised to be $10\%$ of the voltage reading since this captured about $66\%$ of the flucations due to minute movement.

With this stipulation, and the equation stipulation, the following table—which first appeared in the numerical analysis section—summarizes the uncertainty in the index of refraction for the glass tile as seen:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| sig_n_Rs | .007 | .0074 | .0092 | .0138 | .0236 | .0356 | .077 |
| sig_n_Rp | .0071 | .0093 | .0103 | .0075 | .0044 | .0139 | .0722 |

# 8 Discussion

## 8.1 Polarization of Laser Light

**Intensity vs Polarization Axis**

The measured intensity of the laser as a function of the linear polarizers' axis of polarization was well described by the theory (Malus' Law). The intensity was found to have a period of $\pi$ which perfectly matches the period of the expected functional form: $cos^2(\theta)$.

When the initial polarizer was added to the system, the characteristics of the curve still matched the theory. Indeed, the intensity of the beam decreased as portions of the beam not polarized along the initial polarizer's axis of polarization were blocked. Additionally, the period of $\pi$ remained as anticipated. The phase shift between the upper and lower plots in Figure 8 contain information regarding the laser's intrinsic polarization but aside from revealing that the laser's polarization axis was slightly more than $143.5 \pm .5°$ little could be extracted. The reason being that there was noticeable belt slippage.

## 8.2 Sequence of Polarizers

**Crossed Polarizers**

The first measurement, of the voltage after two crossed polarizer, verified both intuition and theory as the voltage reading was very close to zero at $.0053$V. This voltage indeed could have been exclusively due to background light since all of the background voltage data came in above $.0055$V. The computer screen was the likely culprit for all of this background light.

**Three Polarizers**

The measurements with all three polarizers again verified Malus' Law. The intensity of the light had a period of $\pi/2$ that matched the period of the theoretically predicted $cos^2(x)sin^2(x)$. While the phase was slightly off it can be understood as a misaligning of the motorized polarizer angle with the angle of the initial polarizer. Given that the angles of the axis of polarization had uncertainties around $.5°$ this phase discrepancy seems reasonable. As for the amplitude of the transmitted light, the experiment did not directly verify that the amplitude should have been .25 V the experimental amplitude of .15 V seemed reasonable under the assumption that the polarizers used in the lab were less than perfect. However, this would represent a loss of $40\%$ after three polarizers which does seem high.

**Corn Syrup Cells**

The corn syrup cells were admittedly poorer optical elements than other elements used but there was a distinct phase shift between Figure 8 (no corn syrup cells) and Figures 10 and 11 that had one and two corn syrup cells respectively. Using Jones Calculus, this is an expected result. When a linear polarizer acts on circularly polarized light there a factor of i that appears out front, which when traced back to the origin of Jones vectors represents a phase shift.

### 8.3 Polarization of Reflected Light

**Reflectance**

After two separate analyses, the index of refraction of the glass tile is certainly close to $n = 1.5135 \pm 0.0028$. While the excel analysis gave a slightly larger number (1.53) it was a less thorough analysis that hinged upon goal seeking one cell at a time. Additionally the chi-squared minimization was not all that successful as the chi-squared value for both orientations was in the thousands after minimization. Despite using error that based solely on standard deviation, which was revised in the Matlab analysis, the excel analysis was incomplete without the error of the index of refraction.

The Matlab analysis also had its limitation. Namely that the closed form for n in the $R_p$ reflectance equation was too extensive for a personal computer. This approximation certainly introduced error, but like the discreteness of the prescribed indices for the numerical solver for the $R_p$ case, these errors were minimal when compared to the error introduced by the alignment of the amplified photodiode. The error brought upon the calculation by the sensitivity of the voltage reading was unparalleled.

Despite the limitation of the analyses the results were comforting. The index of refraction given by the Matlab analysis ($n = 1.5135 \pm 0.0028$) could very well have fallen within the range of the uncertainty of the excel analysis had it yielded one. Furthermore both analyses gave an index of refraction that matched previous experimental results that Wikipedia provided.

## 9 Conclusion

Overall, the results from all sections of the experiment were at minimum roughly successful in verifying the respective theory under question. The polarization of the light from various sources was mapped and explained in the context of theory. If there was more time and more resources it would have been beneficial to use a amplified photodiode to determine when the transmitted light (through a polarizer) was dimmest. This would have eliminated the judgment call the human eye introduced.

The polarization of the laser was examined and then used to verify Malus' law governing intensities. While more measurements would have proved useful there was enough data to analyze. If this portion were to be further it would have been beneficial to spend more time tweaking the Matlab script that controlled the motorized polarizer so that the belt did not slip. This would have enabled a more careful analysis of the phase shifts in Figure 8.

Later, the intuiton of crossed polarizers was confirmed and then an extension of Malus' law was tested. Putting three polarizers together Malus' law was yet again confirmed albeit with reconcilable adjustments. If another experiment were to repeat this, it should carefully align the initial polarizer and the motorized polarizer to better than a fraction of a degree so the phase can be more carefully checked against theory. Another experiment should also utilize high quality polarizers—with characterized and lesser losses) so the amplitude of the transmitted light can be better tested against theory.

The corn syrup cells while poor optical elements proved their purpose and showed how the chiral structure of sugar turns linearly polarized light into circularly polarized light. This was evident by the signature phase shift, which is predicted under Jones Calculus. To further this portion of the experiment, better corn syrup cells should have been used. This would include more consistent corn syrup, so that the laser light is effected the same across different sections of the cell, and more consistent cells so that light doesn't get refracted off impurities in plastic/glass cell.

Lastly, while the index of refraction of $n = 1.5135 \pm 0.0028$ was satisfying when compared to accepted values for glass the analysis could have been better. With a more computer both equations of reflectance could have been solved explicitly for n so that the uncertainty propagation was perfect. More importantly there should have been a methodical way to find where the amplified photodiode should have been placed for each reflection. One way to do this would be to scan the area where the reflection falls. Ideally the scan would map three space so an exact point of highest intensity could be pin pointed.

# References


[1] Fowles, Grant R. *Introduction to Modern Optics*. New York: Holt, Rinehart and Winston, 1968. Print.

[2] ”List of Refractive Indices.” *Wikipedia*. Wikimedia Foundation, n.d. Web. 21 Mar. 2015.