Lab 1: Refraction of Light Summary Report
PHYS375

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Partners:	NA
Instructor:	Dr. Hill

Abstract

This experiment was performed to determine how light refracts through both a rectangluar glass slide and a semi-circular optical block. Using an amplified photodiode to visualize and measure the beam, one objective was to find the index of refraction of the glass slide in two different orientations using the dimensions of the glass slide, the angle between the normal of the glass slide and the beam, and the lateral displacement of the beam due to refraction. Using a screen to visualize the beam, another objective was to to find the index of refraction of a semi-circular optical block by finding its critical angle.

1 Objectives

First Objective

Determine the index of refraction for the glass slide measuring it in two orientations (see below for pictures). Using the angle of the glass compared to the beam (θ_1) and the lateral displacement of the beam due to refraction (d) along with some manipulation of the first of the following equations in combination with Snell's Law n_2 can be solved for.

$$d * cos(\theta_2) = L * sin(\theta_1 - \theta_2) \tag{1}$$

L is the width of the glass when the its normal is anti-parallel to the direction of the beam propagation. Solving explicitly for n_2, as we did in our homework, we get the following equation

$$n_2 = \frac{n_1 sin\theta_1}{sin\theta_1 - \left(\frac{d}{L}\right)} \sqrt{1 + \left(\frac{d}{L}\right)^2 - 2\left(\frac{d}{L}\right) sin\theta_1} \tag{2}$$

Second Objective

Determine the index of refraction of the semi-circluar optical block by finding the critical angle (θ_C) which is when the refraction angle is 90 degrees and thus none of the beam passes through the semi-circular block and all the beam is internally reflected.

$$\theta_C = sin^{-1}\frac{n_2}{n_1} \tag{3}$$

2 Apparatus

Here is a short list of the materials needed for this experiment

- Optical Breadboard
- Amplified Photodiode
- LabJack Interface
- Diode Laser
- Computer with Matlab
- Stepper Motor

- Rotary Stage
- Motor Controller
- Glass Plate
- Ruler
- Translation Stage
- Calipers

This lab utilized three different appartus setups. The first and second setup was used for objective 1 and utilized the circular table, the glass side, and the amplified photodiode. The third setup was used to determine the 2nd objective, and it made use of the semi-circular block and a screen. All the setups should be assembled consistant with the following descriptions and diagrams.

2.1 Experimental Setups

Setup 1:

Align the diode laser beam with a line of holes in the breadboard so that one direction of the laser is well defined. This can be done by placing a ruler on half of each hole in the breadboard and ensuring it blocks half the beam. It's good practice to check both halves of each hole and move down the length of the breadboard. If the laser beam does not fall in the middle of this line of holes it must be adjusted side to side using the fine adjustment knob on the side of the laser diode mount (or if the beam is grossly out of line then one should loosen the mount, twisting the optical post, and retighten the mount). Once this is aligned, the beam should be checked for flatness in the sense it does not propagate upwards towards the ceiling or down towards the floor. This can done by using a ruler to check the height of the beam when it leaves the diode laser and when it gets to the end of the breadboard. It is good practice to measure it beyond the breadboard but it should be noted that the height of the breadboard itself must then be taken into account.

With the beam aligned the amplified photodiode should setup an inch off the breadboard so that the vibration from the motor is not transferred to the breadboard. The amplified photodiode should also be aligned so that the beam enters the middle of the ring that holds the aperature in a square manner. The aperture itself is elliptical with the top of the major axis falling in the middle of the detector and the bottom of the major axis falling below that. The aperature should be shut so that the minor axis of the aperture ellipse is a fraction of the laser beam.

Next the rotary stage should be placed in between the laser and the amplified photodiode so that its pivot point coincidences with the beam. This means the rotary stage should be mounted along the line of breadboard holes, which was previously used to align the laser. It should also be set at a height where the glass slide, when mounted in the rotary stage vertical (see below), will be in the path of the beam. The rotary stage scale must be set so the zeros coincide, while simultaneously the glass slide (or the normal to its face) is made square to beam. This can be done by aligning the reflection off the first face of the glass slide to the spot of emission of the diode laser.

Setup 2:

The major alignment done in setup one should still be accurate. The only change that is needed is to flip the glass slide on its side so that the clear (not frosted) edges face the beam. This new face again will have to be in the path of the beam, so the rotary stage might need to be lifted. Lastly the alignment of the scale of the rotary stage must be redone. Again this can be achieved by matching the reflection of the laser beam off the first face of the glass slide back with the spot of emission from the diode laser.

Setup 3:

This time the amplified photodiode and glass slide will not be used. Rather a semi-circular block will sit in the rotary stage, and a screen will be used so the transmitted laser light can be visualized. Once again the rotary stage scale will need to be reset and aligned so the zeros coincide which can be achieved using the same technique as previously.

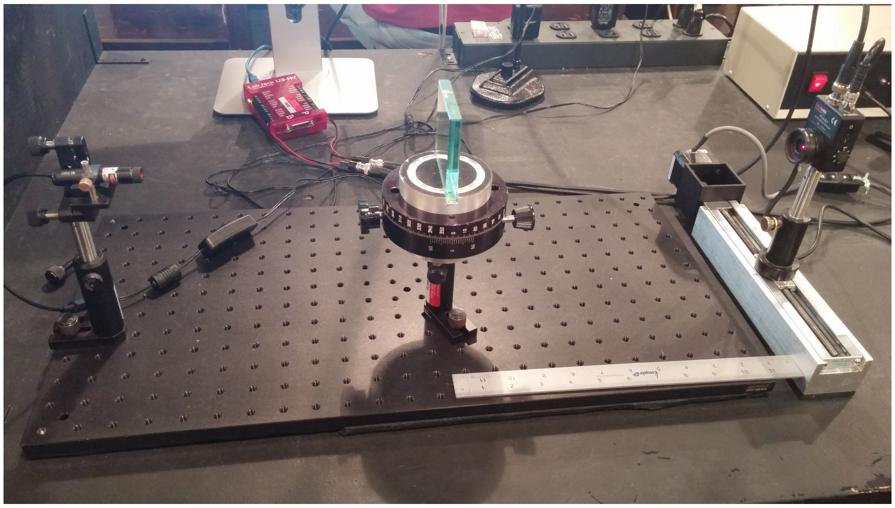


Figure 1: First Setup

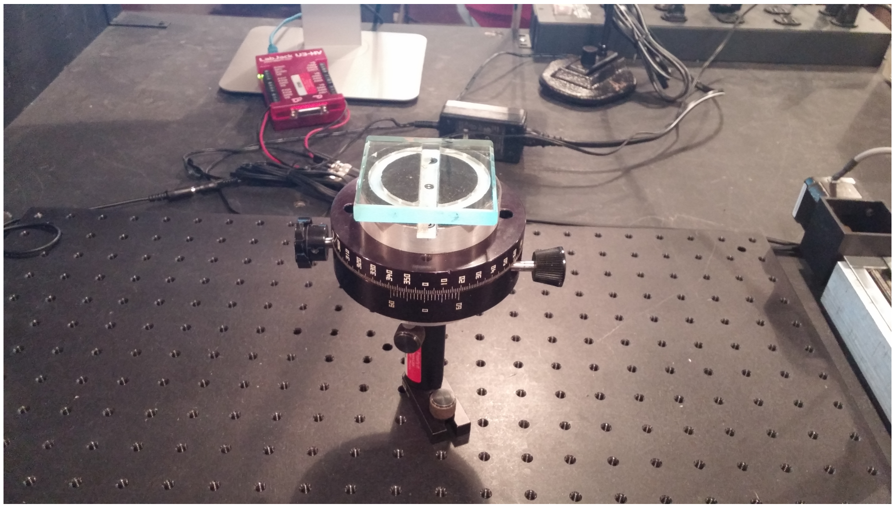


Figure 2: Second Setup

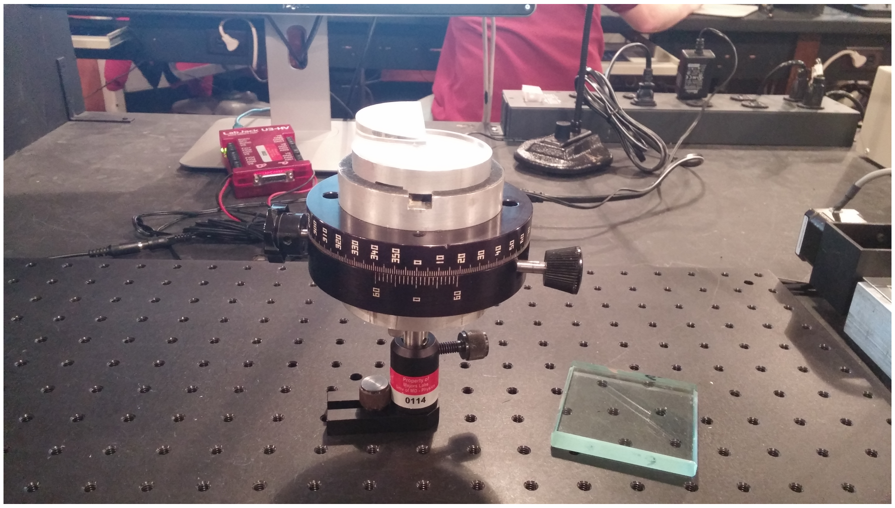


Figure 3: Third Setup

3 Procedure

The basic procedure for collecting data is as follows . . .

Initial Measuring:

Some initial measurements, and their associated error, should be completed before the following procedures. A couple such measurements are the dimensions of the glass slide. While it should be uniform it will probably not be perfect. Using a set of calipers measure the height, width, and thickness of the glass slide in at lease two places (noting an appropriate uncertainty in each measurement). The calipers I used gave measurements in which the last digit varied by three so I used the uncertainty of .03mm.

Determining the uncertainty with which you can measure the angle of the rotary table can also be estimated at this time. I found it to be about a twentieth of a degree given the scale the rotary table I used provided.

Setup 1:

Once setup 1 has been erected according to previous instructions, the first step is to open Matlab so the script that moves the motor and records the data can be run. This script was called `Scan_Laser.m` and it would step the photodiode 4mm in one direction (which depends on the position of the switch on the controller). Then the switch on the controller is flipped, and a script it would send the motor back 4mm to the initial position. This way all the scans are done in the same direction and the positioning of the photodiode across different measurements would be highly similar.

With Matlab open and script ready, move the motor (using `test_motor.m`) so the beam (without the glass slide in the path) falls to the one side of the photodiode. This way the four mm scan will span the beam for all the angles that will be tested. Now record the initial position of the photodiode. The scale on the stepper motor was cm and with estimating the last digit the precision was to two decimal places.

With everything in place, take the glass slide out of the path of the beam so that an initial characterization of the beam can be done. This will also show whether or not the zero degree measurement was truly at zero degrees (if it isn't the beam will move laterally). Run the scan, flip the controller switch, send the photodiode back to the initial position, and save all the Matlab data in recognizable filenames and plot the data by creating a location vector using linspace(initial position, initial position + 4mm, 1600). The 1600 is because the motor has 1600 steps and measurements as it moves 4mm (i.e. 400 per mm). Plot the voltage data against this location vector. Label, name and save this plot something recognizable, as it will be needed for analysis.

With the characterization scan done, place the glass slide back into the aligned rotary table so the beam goes through the largest faces. Repeat the measure in the last paragraph at zero, five, ten, fifteen, twenty, and twenty-five degrees. The direction in which you rotate the glass slide should not matter. Be sure to save all the data so it can be analyzed. Be sure to flip the controller switch at the proper times to ensure the measurement are made over the same positions and thus can be compared.

Setup 2:

Once setup 1 has been erected according to previous instructions, the steps for setup procedure can almost be followed exactly. There is one issue that must be accommodated for: the lateral displacement due to fraction will be significantly larger this time. In all likelihood a four mm span will not be enough to reach angles of even ten degrees. So the Matlab script should be modified to step 3200 times thus spanning eight mm on both the scanning pass and the pass to reset the photodiode to the initial location. This also means a new location vector will have to be made using linspace(initial position, initial position + 8mm, 3200). Besides these changes the procedure should be the same as in setup 1. Taking measurements at 3 degree intervals will allow for 5 angle measurements as in setup 1.

Setup 3:

Setup the equipment as described previously. Starting from an angle of zero slowly rotate the semi-circular block by using the knobs of the rotary table and follow the transmitted light on the screen behind it. At a certain angle the screen will need to be moved as the angle of the semi-circular block refracts the light farther than the edge of the screen. After moving the screen again start rotating the semi-circular block slowly. At a certain angle of the rotary stage, the semi-circular block will refract light at a ninety degree angle which means no light will be transmitted through the block and onto the screen. This angle is call the critical angle and is the point when all light is internally reflected. Approach this angle from both the low and high side to get the best record of the critical angle. Do this same process again but rotate the semi-circular block in the opposite direction of the first time. After this, there should be two angles (though one will be 360-critical angle) at which the semi-circular block just starts to have total internal reflection. The uncertainty in the critical angle can be estimated as the uncertainty in the angle of the rotary stage propagated to the average of the two critical angles found.

4 Experimental Data

The raw voltage data the photodiode took for both the vertical and flat orientations of the glass slide can be found on labarchives, under "Data & Lab Notes", under page "Day 2", and they are the .mat files.

Below are Matlab figures with all of my data superimposed for both orientation of the glass slide (vertical and horizontal). Notice that the voltage reading for the horizontal orientation is significantly weaker than the characterization reading (without the glass in place). This is because the glass in the horizontal orientation significantly distorted the beam in the vertical direction, which meant the photodiode was registering a small portion of the beam.

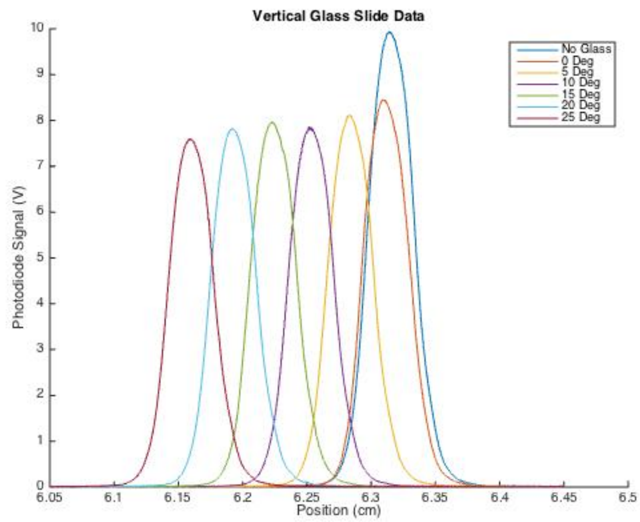


Figure 4: Vertical Glass Slide Data

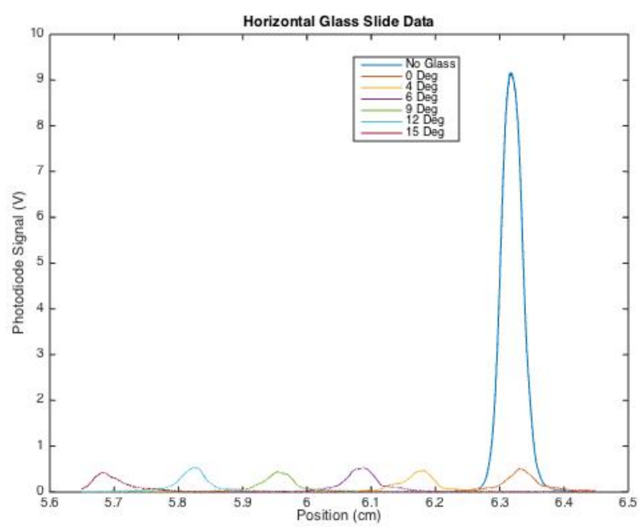


Figure 5: Horizontal Glass Slide Data

Additionally there was the data from measuring the critical angle of the semi-circular optical block. Below are the two measurements that were made (one for each direction).

$$\theta_C = 45.1 \pm .05^\circ \qquad \theta_C = 325.0 \pm .05^\circ \tag{4}$$

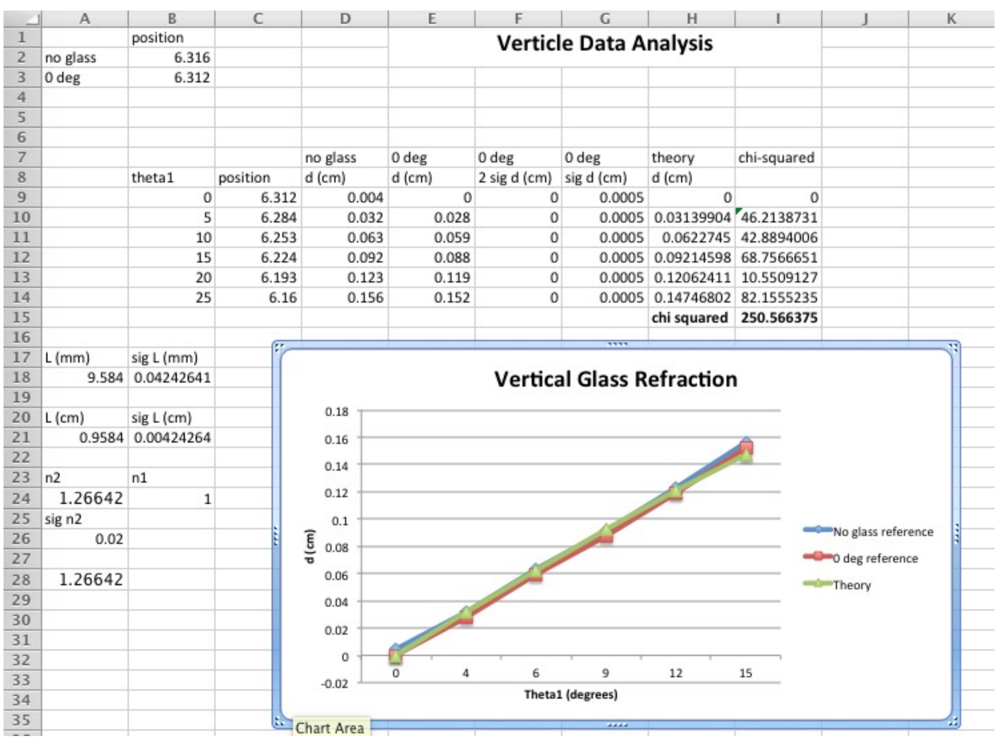
The second reading should be normalized as it is a result taken starting an angle of 360° and moving backwards. So angle the semi-circular optical block was moved through was actually ...

$$\theta_C = 360 - 325.0 = 45.0 \pm .05^\circ \tag{5}$$

5 Numerical Analysis

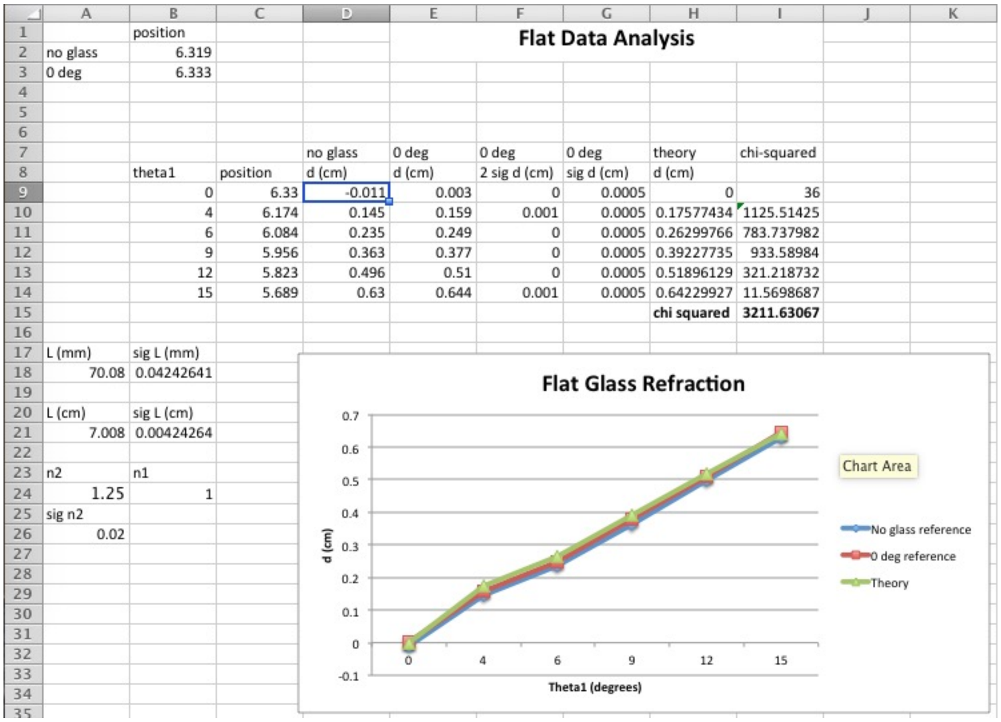
The first step in analyzing the data was to use Matlab's Gaussian fit to get a position for the beam at each angle a measurement was made. From here fits, the positions and the error were determined. The results from the Gaussian fits can be found in a word document called Gaussian_Fit_Parameters on labarchives under Data & Lab Notes, Lab 1, and finally Day 2. These positions were then put into an excel spreadsheet to determine d, the lateral displacement for each measurement. The Gaussian fits for the vertical orientation all had r-square values, a measure of goodness of fit, above .995, and for the horizontal orientation, despite the vertical beam distortion, all had r-square values above .95. This meant the 95% confidence intervals for the position and hence d were extremely low. In a lot of cases there was no interval but rather the same point down to three decimal places. Thus an error of .0005 cm was used for the excel analysis.

While excel was used for the graphs and some of the data analysis the results did not seem correct as the chi-squared minimization put the index of refraction for the glass around 1.25 in both orientations. Additionally the error in the index of refraction was estimated by seeing when the graph for theory fell outside the error bars (which are there but extremely small for reasons already stated). Thus the vectors for d and θ_1 for both orientations were exported to Matlab for more thorough data analysis. Below are images of the excel data analysis (which can be found under Data & Lab Notes, Day 2, called Flat_Glass_Analysis):



	A	B	C	D	E	F	G	H	I
1		position			Verticle Data Analysis				
2	no glass	6.316							
3	0 deg	6.312							
7				no glass	0 deg	0 deg	0 deg	theory	chi-squared
8		theta1	position	d (cm)	d (cm)	2 sig d (cm)	sig d (cm)	d (cm)	
9		0	6.312	0.004	0	0	0.0005	0	0
10		5	6.284	0.032	0.028	0	0.0005	0.03139904	46.2138731
11		10	6.253	0.063	0.059	0	0.0005	0.0622745	42.8894006
12		15	6.224	0.092	0.088	0	0.0005	0.09214598	68.7566651
13		20	6.193	0.123	0.119	0	0.0005	0.12062411	10.5509127
14		25	6.16	0.156	0.152	0	0.0005	0.14746802	82.1555235
15								chi squared	250.566375
17	L (mm)	sig L (mm)							
18	9.584	0.04242641							
20	L (cm)	sig L (cm)							
21	0.9584	0.00424264							
23	n2	n1							
24	1.26642	1							
25	sig n2								
26	0.02								
28	1.26642								

Figure 6: Vertical Orientation Excel Analysis



	A	B	C	D	E	F	G	H	I
1		position			Flat Data Analysis				
2	no glass	6.319							
3	0 deg	6.333							
7				no glass	0 deg	0 deg	0 deg	theory	chi-squared
8		theta1	position	d (cm)	d (cm)	2 sig d (cm)	sig d (cm)	d (cm)	
9		0	6.33	-0.011	0.003	0	0.0005	0	36
10		4	6.174	0.145	0.159	0.001	0.0005	0.17577434	1125.51425
11		6	6.084	0.235	0.249	0	0.0005	0.26299766	783.737982
12		9	5.956	0.363	0.377	0	0.0005	0.39227735	933.58984
13		12	5.823	0.496	0.51	0	0.0005	0.51896129	321.218732
14		15	5.689	0.63	0.644	0.001	0.0005	0.64229927	11.5698687
15								chi squared	3211.63067
17	L (mm)	sig L (mm)							
18	70.08	0.04242641							
20	L (cm)	sig L (cm)							
21	7.008	0.00424264							
23	n2	n1							
24	1.25	1							
25	sig n2								
26	0.02								

Figure 7: Vertical Orientation Excel Analysis

Using the vectors for d and θ_1 (in radians) for the first and second setup, the evaluation of d had to be computed for each measurement point and then weighted by $\frac{1}{\sigma_d}$. The error in d, σ_d, had to also be computed at each measurement point and is discussed in the next section. For each measurement, d is computed using equation 2 via Matlab. Note the Matlab script includes the error and must be run twice, once for each orientation of the glass slide. The script with d and θ_1 for the flat orientation is shown below (and can be found under Data & Lab Notes, Day 2, called data_analysis_flat.m along with data_analysis_vertical.m):

```
syms n1 theta1 d L;
f=(n1*sin(theta1))/(sin(theta1)-(d/L))*(1+(d/L).^2-2.*(d/L).*sin(theta1)).^(1/2);

val_theta1=pi/180*[4 6 9 12 15];
val_d=[.159 .249 .377 .51 .644];
val_L=7.008;
val_n1=1;
sig_theta1=.05*pi/180;
sig_d=.0005;
sig_L=.00424264;

dd=diff(f,d);
dL=diff(f,L);
dtheta1=diff(f,theta1);

index_refraction=[];
for j=1:5;
    n2=eval(subs(f,{n1,d,L,theta1},{val_n1,val_d(j),val_L,val_theta1(j)}));
    index_refraction(j)=n2;
end
index_refraction

sig_n2=[];
for i=1:5;
    partiald=eval(subs(dd,{n1,d,L,theta1},{val_n1,val_d(i),val_L,val_theta1(i)}));
    partialL=eval(subs(dL,{n1,d,L,theta1},{val_n1,val_d(i),val_L,val_theta1(i)}));
    partialtheta1=eval(subs(dtheta1,{n1,d,L,theta1},{val_n1,val_d(i),val_L,val_theta1(i)}));
    sig_n2(i)=sqrt((partialtheta1*sig_theta1)^2+(partiald*sig_d)^2+(partialL*sig_L)^2);
end
sig_n2
```

```
weights=1./sig_n2;

error_prop=[];
for k=1:5;
    error_prop(k)=(weights(k)/sum(weights)*sig_n2(k))^2;
end
final_index=(weights*index_refraction')/sum(weights)
final_error=sqrt(sum(error_prop))
```

This matlab script yielded the following results for the flat and (changing out the d vector, θ_1 vector, and L) vertical orientations:

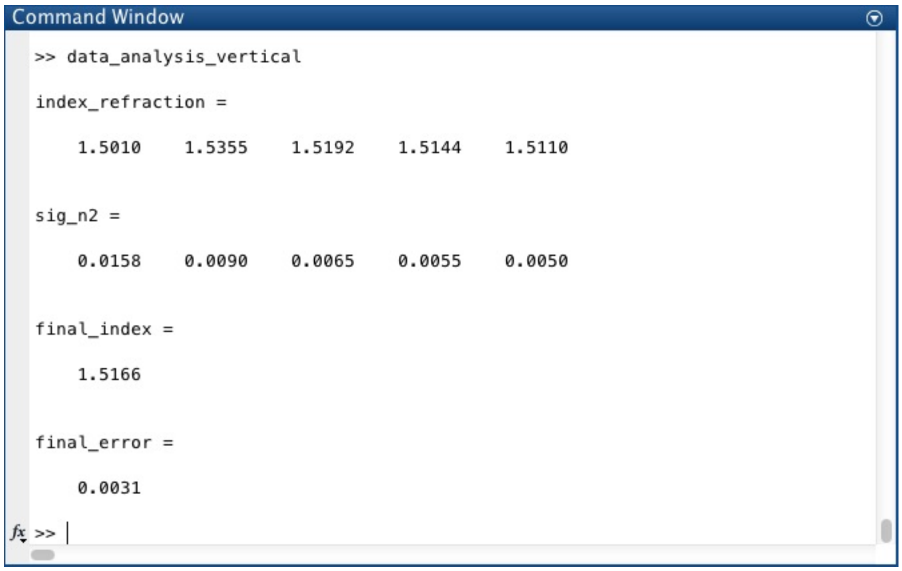


Figure 8: Vertical Orientation Matlab Analysis

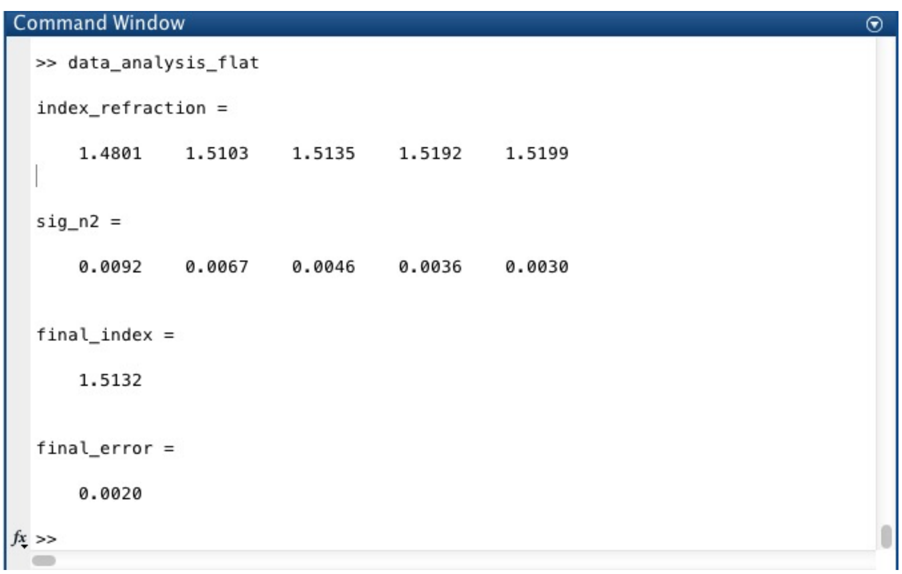


Figure 9: Vertical Orientation Matlab Analysis

This method of data analysis proved significantly better than the excel analysis as the index of refraction for the vertical and flat orientations of the glass slide were respectively as follows:

$$n_2 = 1.517 \pm .003 \tag{6}$$

$$n_2 = 1.513 \pm .002 \tag{7}$$

These indices of refraction match the index of refraction for crown glass (soda-lime) as stated by "http://physics.info/refraction/" very well as they quote the index of refraction as 1.512

For the third setup where the critical angle was measured the numerical analysis was fairly simple. Since the angle was measured twice it was averaged and the error was propagated in the familiar way, which will be further discussed in the next section.

$$\theta_C = \frac{45.0 + 45.1}{2} = 45.05 \pm .07^\circ \tag{8}$$

Thus using equation three from the objectives section, but taking the sine of both sides and recognizing that $n_2 = 1$ because the second medium, which the beam never makes it to, is air. Thus the index of refraction for the semi circular block with error propagation that will be described in the next section is . . .

$$n_{semi-circular} = \frac{1}{sin(\theta_C)} = 1.41 \pm .10 \tag{9}$$

6 Error Analysis

First let f be a function of some random variables x,y,z. Then its uncertainty is derived from:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} * \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} * \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} * \sigma_z\right)^2} \tag{10}$$

For the first and second setup, the evaluation of d must be computed for each measurement point and then weighted by $\frac{1}{\sigma_d}$. The error in d, σ_d, must also be computed at each measurement point. For each measurement, d is computed using equation 2 and the error in d is computed by . . .

$$\sigma_{n_2} = \sqrt{\left(\frac{\partial n_2}{\partial L} * \sigma_L\right)^2 + \left(\frac{\partial n_2}{\partial \theta_1} * \sigma_{\theta_1}\right)^2 + \left(\frac{\partial n_2}{\partial d} * \sigma_d\right)^2} \tag{11}$$

Since these are both d and σ_d are very complex, Matlab was used to evaluate both. To see how this was implemented see the matlab script above in the numerical analysis section.

With the values of d and σ_d for each measurement for each orientation, the values of d were weighted by $\frac{1}{\sigma_d}$ to give a weighted average of d where more accurate measurements contributed more to the final value of d. The equation for the weighted average is as follows:

$$\bar{n_2} = \frac{\sum_{i=1}^{5} n_{2_i} w_i}{\sum_{j=1}^{5} w_j} \tag{12}$$

Thus the error in $\bar{n_2}$ was computed by using equation 10 on equation 12.

The error in computing θ_C was computed as follows since it was derived as an average of the two values:

$$\sigma_{\theta_C} = \sqrt{.05^2 + .05^2} = .0707 \approx .07 \tag{13}$$

As for the error in the index of refraction for semi-circular optical block, the error propagation equation 8 just needed to be applied to equation 7. The result was . . .

$$\sigma_{n_{semi-circular}} = \sqrt{\left(\frac{cos(\theta_C)}{sin^2(\theta_C)}\sigma_{\theta_C}\right)^2} = .0997 \approx .10 \tag{14}$$

7 Discussion

The results from the Matlab data analysis were significantly closer to the index of refraction given for glass by "http://physics.info/refraction/" compared to the results of the excel analysis. Ultimately there are multiple kinds of glass and the type used in our experiment was not determined which makes comparing our numbers to a baseline index of refraction impossible.

Comparing the results of the vertical and horizontal orientation it is clear they were very consistent with one another despite the error introduced by the vertical distortion of the beam introduced in the horizontal orientation. While this increased the error in the measurement of d, because the Gaussian fits were not quite as good (they had a lower r-square value) as the fits for the vertical orientation, the index of refraction match given the uncertainty in each.

Despite the index of refractions in both orientations matching, there is the possibility that a systematic error compromised both sets measurements. The most obvious would be the systematic error introduced by the faces of the glass slide not being parallel. This would cause the incident angle and the exit angle to be different, meaning the beam would be directed along a different path which would in turn cause d to be measured in correctly.

The width of the glass was measured to be different by $.01$ which did fall into the range of uncertainty, and the length of the glass was measured different by $.04$ which fell slightly outside the range of uncertainty. If these differences were real they would introduce an angle for the exiting beam which would shift the beam more the farther the photodiode was from the glass slide. Estimating these angles:

$$\theta_{error-vertical} = arccot\left(\frac{70.08}{.01}\right) = .0082^{\circ} \tag{15}$$

$$\theta_{error-horizontal} = arccot\left(\frac{9.584}{.04}\right) = .2391^{\circ} \tag{16}$$

While these angles propagated over the distance between the glass slide and the photodiode (around 12 inches) may lead to small deviations in d, these errors would only make a difference if the no glass measurement was used as the reference to which d was measured. For this reason, the reference for d was the zero degree measurement with the glass slide in place. Looking back to figures 4 and 5 it can be seen there is in fact a difference in the beams of the no glass and the zero degree measurements and this could be due to the faces of the glass slide not being parallel, because the reflection off the front face (and not the back) was used for alignment.

To improve the measurements of the first two setups, more angles should have been measured to minimize random errors. Additionally, the angles that were measured should have been measured both in positive and negative degrees from zero to ensure that the same d was measured which again would reduce random errors. To minimize systematic errors, the substance of the glass slide should be known so systematic errors could be recognized. From there, a glass slide with more parallel sides could prove useful in reducing systematic errors.

As for the semi-circular optical block, again the actual substance and hence the index of refraction was unknown. Given the simplicity of the procedure for this measurement the only real problem is whether there was complete internal reflection. To improve this, perhaps a photodiode could be used as the critical angle is approached to ensure, beyond the capabilities of the human eye, there was no light transmitted.