Gaussian Beam with Lenses
Lab 2 Summary Report
PHYS375

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Partners:	NA
Instructor:	Dr. Hill

Abstract

This experiment was performed to determine how a Gaussian beam propagates through various lenses. First we used a plano-convex lense in combination with a bi-convex lense to expand and, approximately, collimate our diode laser beam. Next this expanded beam was propagated through a 100mm converging lense and then a 100mm diverging lense so the focal lengths of the each could analyzed. For each lense lateral scans of the beam were taken at multiple distances away from each lense so the waist, radius of curvature and focal length could be analyzed. The focal length of the 100mm convergent lense was measured to be 98.3 ± 1.7mm where as the focal length of the 100mm divergent lense was measure to be 106 ± 28mm.

1 Objectives

First Objective

Use the plano-convex lense in conjunction with a bi-convex lense to create a telescope for the diode laser beam. This requires the proper placement of the bi-convex in relation to the plano-convex lense.

Second Objective

Determine the focal length of both the 100mm converging lense and then a 100mm diverging lense from the scans of the beam at multiple z locations after each lense.

2 Apparatus

See figure 1 on the next page

Materials			
Optical Breadboard	Diode Laser	Amplified Photodiode	Computer with Matlab
LabJack Interface	Stepper Motor	Motor Controller	15mm plano-convex lense
40mm bi-convex lense	100mm convex lense	100mm concave lense	Screen

2.1 Experimental Setup

Align the diode laser beam with a line of holes in the breadboard so that one direction of the laser is well defined. This can be done by placing a ruler on half of each hole in the breadboard and ensuring it blocks half the beam along the whole length of the breadboard. Adjust the laser beam laterally using the fine adjustment knob on the side of the laser mount. Next aligned the beam so it is flat (i.e. it does not propagate upwards towards the ceiling or down towards the floor). Using a ruler to check the height of the beam when it leaves the diode laser and when it gets to the end of the breadboard adjust the laser up or down as necessary using the fine adjustment knob on the top of the laser mount. The beam being level as it propagates is very important, as it will help ensure the same section of the beam is measured later on.

Next, place the 15mmplano-convex lense roughly 7cm after the diode laser along its path of propagation such that the beam hits the curved side first. The beam must hit the plano-convex lense so that it's normal to the surface so make sure the beam is centered on the lense. Place the screen after this lense and make sure that the beam is still propagating along the same row of holes in the breadboard and is centered at the same height as before the plano-convex lense. With this lense, aligned place the bi-convex lense about 55mm after the pano-convex lense ($55 = 15 + 40$ the sum of the lenses focal lengths). Roughly align this bi-convex lense so the beam can be looked at using the screen. First ensure that the after the bi-convex lense there is no focus in the beam. If there is a focus the bi-convex must be moved closer to or farther from the plano-convex lense. When there is no focus in the beam, adjust the lense's position so that the beam after the bi-convex is again aligned with the original row of holes in the breadboard and it centered at the same height as originally. Next, using a piece of paper mark the size of the beam immediately after the bi-convex lense and carefully increase or decrease distance between the bi-convex and plano-convex lense so that the beam far away is the same size.

The first couple measurements of the collimated beam should be taken now before extra lenses are put into the system.

Next place the 100mm convergent lense at the end of the breadboard and in the path of the collimated beam. Align this lense so that the beam after it is parallel to the original row of holes and the height of the center of the beam is the same as originally. Using the screen or paper inspect the resulting beam and take note of the approximate distance from the breadboard of the focus and distances where the beam grows to a couple mm (before and after the focus). This is the z axis range where scans of the beams cross-section should be taken. A picture of this setup can be seen in Figure 1.

Scans of the convergent lense should be taken now before the convergent lense is swapped out for the divergent lense.

Next place the 100mm divergent lense in the optical post mount that previously held the convergent lense. Align this lense so the beam comes out parallel to the original beam and is still level. Next use the screen to make a rough estimate of the beam size at the z axis locations where the scans will be taken so the Matlab code can be adjusted to accommodate the beam at all z axis locations. Note that the beam will grow rapidly and measurement distances suitable for the convergent lense may not be feasible for the divergent lense.

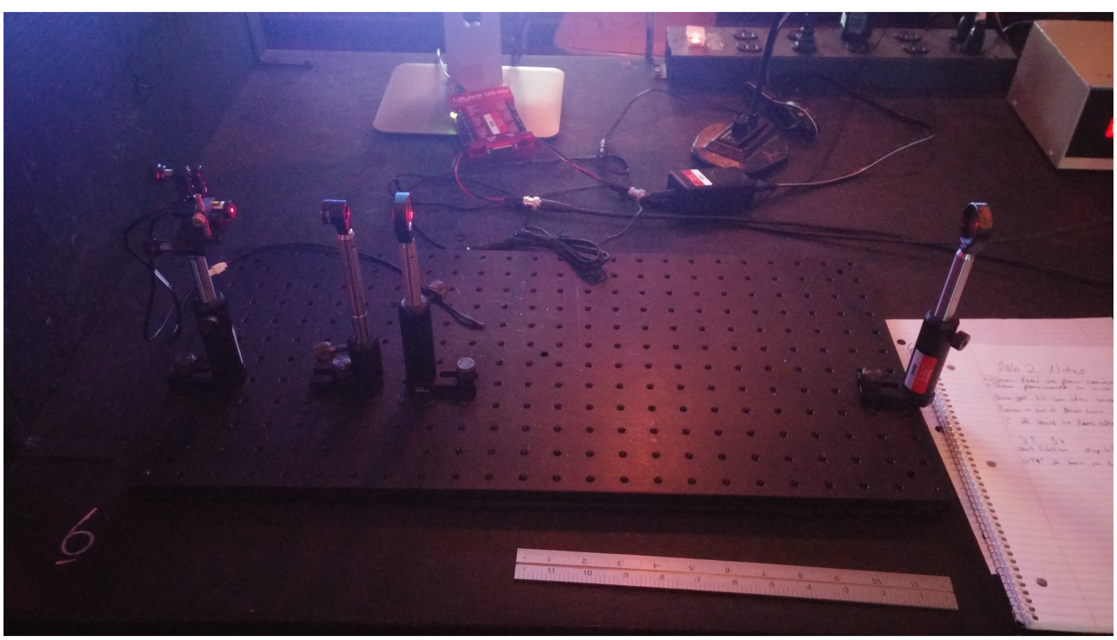

Figure 1: Experimental Setup

3 Procedure

Characterize Collimated Beam:

After the telescope is setup, the resulting collimated beam should be scanned with the gain of the photodiode set at 60. The photodiode should scan across the full profile of the collimated beam at two different z locations. The scans can be performed using the Matlab script previously written called `Scan_Laser.m`. The distance from the end of the telescope to the photodiode where the scans were taken must be recorded along with their uncertainty. This way the divergence or convergence of the *collimated* beam can be analyzed later.

Next, with the 100mm convergent lense in place, five scans of the beam after the convergent lense at distances two, three, four, five, and six inches from the convergent lense should be performed. Note that the gain of the photodiode gain should be set at 30; lower than for the collimated beam characterization since the beam is more focused now. The distance, and uncertainty in the distance, between the convergent lense and the housing of the photodiode should be recorded for each scan as should the lateral start and stop position of the stepper motor.

Lastly, with the 100mm divergent lense in place, four scans of the beam after the convergent lense at distances one, one and a half, two, and two and a half inches from the divergent lense should be performed using the Matlab script from before. Note the Matlab script for the scan should be modified to accommodate a beam of width $\approx$ 10mm. This can be done by changing the number of steps the motor takes from 1600 to 4800, which will scan 12mm

4 Experimental Data

The raw voltage data (.mat files) from the photodiode for the collimated beam characterization, the convergent and divergent lense can be found on labarchives, under "Data & Lab Notes", Lab 2.

Below are Matlab figures with all of the data for each lense setup superimposed. The features for the convergent lense plot is that the beam width narrows then grows as the scan locations move through the lense's focus. The divergent lense plot on the other hand is characterized by a steadily increasing beam width. As of now width of the beam is ill-defined but it will become more clear and quantified in the numerical analysis section, which is to follow. It should be noted that the lateral locations do not match up in the scans because the stepper motor was moved between each scan. It is only the widths of the beam at each scan that is useful to look at.

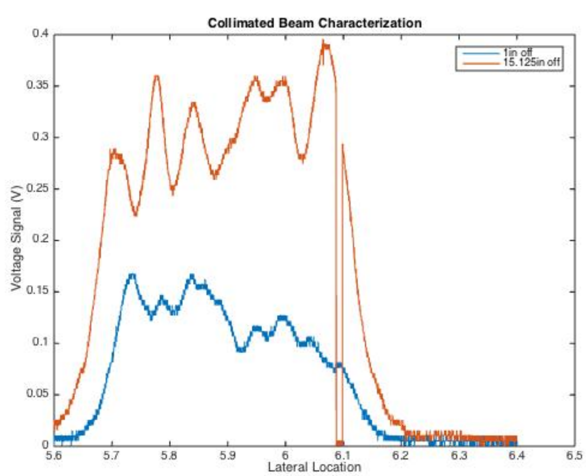

Figure 2: Collimated Beam Scans

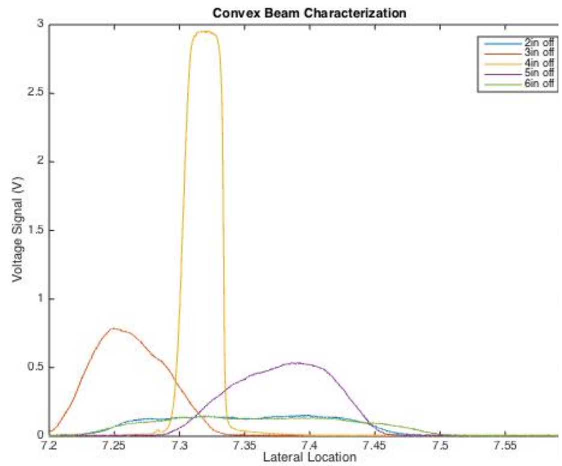

Figure 3: Convex Beam Scans

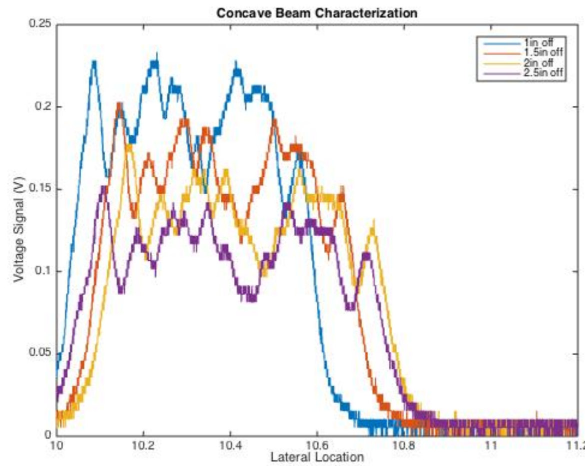

Figure 4: Concave Beam Scans

5 Numerical Analysis

The first step in analyzing the data was to determine the width of the beam for each scan. With the width of each scan determined, further analysis of the beam's waist, radius of curvature and angle of divergence can be done.

The widths of the beams where determined using Matlab code to find the indices where the voltage signal were half the max value. The difference in these indices were then multiplied by the conversion factor that took them from a step count to a mm distance. The Matlab code used is as follows:

```
%% Find Width of Beam
coll1=voltage_reading_afterscope1inoff;
coll2=voltage_reading_afterscope15_125off;
convex2=voltage_reading_convex2inout;
convex3=voltage_reading_convex3inout;
convex4=voltage_reading_convex4inout;
convex5=voltage_reading_convex5inout;
convex6=voltage_reading_convex6inout;
concave1=voltage_reading_concave1inout;
concave1_5=voltage_reading_concave1_5inout;
concave2=voltage_reading_concave2inout;
concave2_5=voltage_reading_concave2_5inout;

%Find indices of close values to half max
coll1_ind=find(abs(coll1-max(coll1)/2)<.004);
coll2_ind=find(abs(coll2-max(coll2)/2)<.004);
convex2_ind=find(abs(convex2-max(convex2)/2)<.004);
convex3_ind=find(abs(convex3-max(convex3)/2)<.004);
convex4_ind=find(abs(convex4-max(convex4)/2)<.004);
convex5_ind=find(abs(convex5-max(convex5)/2)<.004);
convex6_ind=find(abs(convex6-max(convex6)/2)<.004);
concave1_ind=find(abs(concave1-max(concave1)/2)<.004);
concave1_5_ind=find(abs(concave1_5-max(concave1_5)/2)<.004);
concave2_ind=find(abs(concave2-max(concave2)/2)<.004);
concave2_5_ind=find(abs(concave2_5-max(concave2_5)/2)<.004);

step_conversion=1/400;

%After Running the above to find the 2 spots at Half Max
coll1_wid=(1915-400)*step_conversion
coll2_wid=(2069-323)*step_conversion
convex2_wid=(997-207)*step_conversion
convex3_wid=(390-97)*step_conversion
convex4_wid=(536-410)*step_conversion
convex5_wid=(925-504)*step_conversion
```

```
convex6_wid=(1065-210)*step_conversion
concave1_wid=(2346-146)*step_conversion
concave1_5_wid=(2737-371)*step_conversion
concave2_wid=(2878-452)*step_conversion
concave2_5_wid=(2824-209)*step_conversion
```

The widths of the beams after the convex and concave lenses are summarized by the plots below:

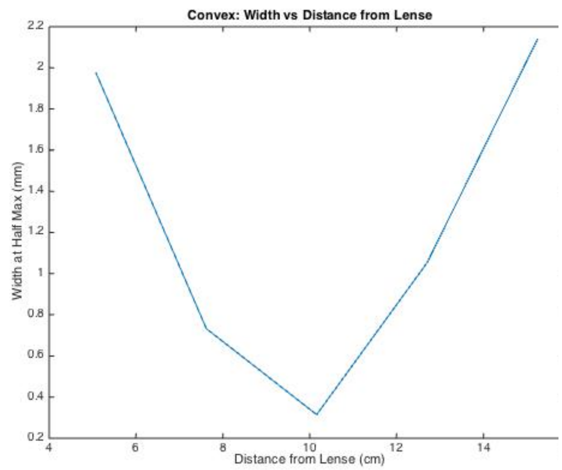

Figure 5: Beam Width after Convex

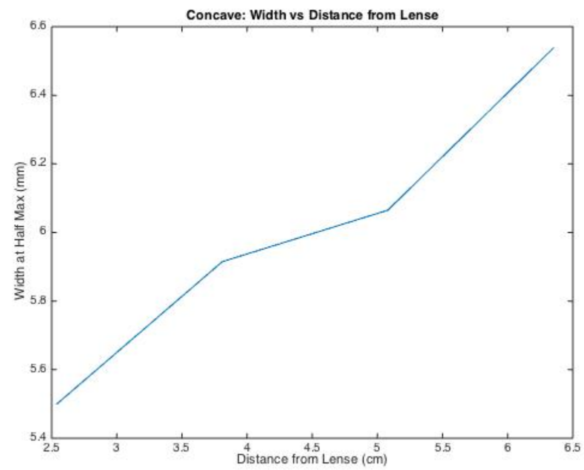

Figure 6: Beam Width after Concave

The uncertainty in the widths of the beams of all scan is estimated to be 20 motor steps (this is estimated by the number of indices the Matlab width script returns) which is equivalent to $20/400 = .05$mm

The collimated beam had width 3.7875mm and 4.365mm at 1 inch and 15.125 inches off the breadboard respectively. So, the collimated beam was actually diverging at an angle given by...

$$\theta_{divergence} = arctan\left(\frac{\frac{4.365}{2} - \frac{3.7875}{2}}{(15.125-1)*25.4}\right) = 8.05*10^{-4} \text{ radians} \tag{1}$$

The error in this angle follows from propagating the error for each step, where the uncertainty in the distance measurements was 1/32 of an inch

$$\Delta h = \frac{4.365}{2} - \frac{3.7875}{2} = .2888 \text{ mm} \qquad \sigma_{\Delta h} = \sqrt{\left(\frac{.05}{2}\right)^2 + \left(\frac{.05}{2}\right)^2} = .00125 \text{ mm} \tag{2}$$

$$\Delta z = 15.125 - 1 = 14.125 \text{ in} = 358.78\text{mm} \qquad \sigma_{\Delta z} = \sqrt{\left(\frac{1}{32}\right)^2 + \left(\frac{1}{32}\right)^2} = .0442 \text{ in} = 1.12\text{mm} \tag{3}$$

So the total error for $\theta_{divergence}$ is given by:

$$\sigma_{\theta_{divergence}} = \sqrt{\left(\frac{358.78*.00125}{358.78^2+.2888^2}\right)^2 + \left(\frac{.2888*1.12}{358.78^2+.2888^2}\right)^2} = 4.3*10^{-6} \text{ radians} \tag{4}$$

Overall, for the collimated beam the angle of divergence is given by ...

$$\theta_{divergence} = (8.05 \pm .04)*10^{-4} \text{ radians} \tag{5}$$

Using Matlab to do the same process for the data of convex lense the location of the waist and hence to location of the focus can be found. Below is the Matlab code for the convex case and the output in mm for both the convex and concave:

```
widths=[1.975 .7325 .3150 1.0525 2.1375];
sig_widths=.05;
waist=widths/2;
sig_waist=sig_widths/2;

del_waist1=waist(5)-waist(3);
sig_delwaist=sqrt(2*sig_waist^2);

val_d2=25.4*[2 3 4 5 6];
val_d2=val_d2+5.5; %housing to iris 5.5
sig_d2=25.4/32;

del_z1=val_d2(5)-val_d2(3);
sig_delz=sqrt(2*(25.4/32)^2);

angle_divergence1=tan(del_waist1/del_z1)
sig_angle=sqrt(((del_z1*sig_delwaist)/(del_z1^2+del_waist1^2))^2+((del_waist1*sig_delz)/(del_z1^2+del_waist1^2))^

%find where waist is
x_waist1=val_d2(3)-waist(3)/tan(angle_divergence1)
sig_waist=sqrt((sig_d2)^2+(sig_waist/tan(angle_divergence1))^2+(waist(3)*sig_angle/(sin(angle_divergence1))^2)^2)
```

Figure 7: Convex Focus Location

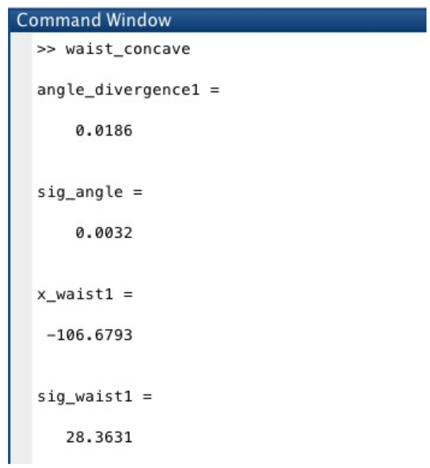

Figure 8: Concave Focus Location

6 Error Analysis

The error propagation done throughout the numerical analysis section was done in accordance with the following method of error propagation: let f be a function of some random variables x,y,z. Then its uncertainty is derived from:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} * \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} * \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} * \sigma_z\right)^2} \tag{6}$$

7 Discussion

The focal lengths determined by the Matlab analysis of the convex and concave lenses were consistent with the nominal focal length of 100mm for each. The absolute value of the negative location of the concave lense's focus location gave the focal length. With the focal lengths of the convex and concave lenses being 98.3 ± 1.7mm and 106 ± 28mm respectively, both focal lengths agreed with what the manufacturer quoted. For the convex lense 100 mm just barely fell inside the uncertainty, but for the concave lense 100mm was almost in the middle of its uncertainty.

The concave lense had a very large uncertainty in its focal length, which was due to the z locations of the scans being close together. The main reason these scans were taken so close together was because the diverging beam grew rapidly in size and would have required large scan ranges, which would have taken significantly longer to run. Additionally, looking at figure 4, it is evident that the beam underwent some distortion or aberration since the beam profile has extra peaks and troughs.

Some systematic errors may have come from the poor alignment of the telescope and lenses. This portion of the experiment was rushed to some degree and as the analysis showed the *collimated* beam was not truly collimated but was rather diverging from the beginning. This, in turn, could have created some aberration that skewed the data such that the angle of divergence in both the convex and concave lense was less than it should have been.

8 Conclusion

Overall, the results for the focal lengths of the convex and concave lenses were precisely what they were expected to be. However the uncertainty in focal length of the divergent lense was less than ideal to say without question the focal length was 100mm. To improve upon these results, the scans of the divergence lense should be taken at z locations, which vary over a larger distance than just 1.5 inches.

To reduce possible systematic errors, the alignment of the telescope should be analyzed before the convex and concave lenses are put in place. Analyzing the collimated beam using Matlab and the techniques in the numerical analysis section would allow for better collimation, which should reduce the aberrations in the beam and reduce systematic error.