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# A Tutorial on Gassmann Fluid Substitution: Formulation, Algorithm and Matlab Code

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## Introduction

Fluid substitution is an important part of the seismic rock physics analysis (e.g., AVO, 4D analysis), which provides a tool for fluid identification and quantification in reservoir. This is commonly performed using Gassmann's equation (Gassmann, 1951). Many authors (Batzle and Wang, 1992; Berryman, 1999; Wang, 2001; Smith et al., 2003; Russell et al., 2003; Han and Batzle, 2004) have discussed the formulations, strength and limitations of the Gassmann fluid substitution. However comprehensive discussions on this subject have been rather scant. In this paper I present an explicit formulation, algorithm and a matlab code (computer program) for the Gassmann fluid substitution.

The objective of fluid substitution is to model the seismic properties (seismic velocities) and density of a reservoir at a given reservoir condition (e.g., pressure, temperature, porosity, mineral type, and water salinity) and pore fluid saturation such as 100% water saturation or hydrocarbon with only oil or only gas saturation. Seismic velocity of an isotropic material can be estimated using known rock moduli and density. P- and S-wave velocities in isotropic media are estimated as,

$$V_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \quad (1)$$

and

$$V_s = \sqrt{\frac{\mu}{\rho}} \quad (2)$$

respectively, where  $V_p$  and  $V_s$  are the P- and S-wave velocity,  $K$  and  $\mu$  are the bulk and shear moduli, and  $\rho$  is the mass density. Density of a saturated rock can be simply computed with the volume averaging equation (mass balance). Other parameters required to estimate seismic velocity after fluid substitution are the moduli  $K$  and  $\mu$ , which can be computed using the Gassmann's equations.

## Gassmann's equations

Gassmann's equations relate the bulk modulus of a rock to its pore, frame, and fluid properties. The bulk modulus of a saturated rock is given by the low frequency Gassmann theory (Gassmann, 1951) as

$$K_{sat} = K_{frame} + \frac{\left(1 - \frac{K_{frame}}{K_{matrix}}\right)^2}{\frac{\phi}{K_{fl}} + \frac{(1-\phi)}{K_{matrix}} - \frac{K_{frame}}{K_{matrix}^2}} \quad (3)$$

where  $K_{frame}$ ,  $K_{matrix}$ , and  $K_{fl}$  are the bulk moduli of the saturated rock, porous rock frame (drained of any pore-filling fluid), mineral matrix, and pore fluid, respectively, and  $\phi$  is porosity (as fraction). In the Gassmann formulation shear modulus is independent of the pore fluid and held constant during the fluid substitutions. Bulk modulus ( $K_{sat}$ ) and shear modulus ( $\mu$ ) at in-situ (or initial) condition can be estimated from the wireline log data (seismic velocities and density) by rewriting equations 1 and 2 as

$$K_{sat} = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right) \quad (4)$$

and

$$\mu = \rho V_s^2 \quad (5)$$

Many times density log is not reliable; in that case  $\rho_{sat}$  density can be calculated from the porosity log using the mass balance equation.

To estimate the saturated bulk modulus (equation 3) at a given reservoir condition and fluid type we need to estimate bulk moduli of frame, matrix and pore fluid. In the following section, I will discuss formulations to calculate bulk modulus and density of mineral matrix, pore fluid and rock frame. Then I will present an algorithm for the Gassmann fluid substitution followed by an example and discussions. For hands-on-practice a matlab code for the fluid substitution is included in the appendix.

## Formulations

### 1) Matrix properties:

To calculate the bulk modulus of mineral matrix, we need to know the mineral composition of rock, which can be found from the laboratory examination of core samples. In the absence of the laboratory data, lithology can be assumed to be a composition of quartz and clay minerals. The clay percentage can be derived from the volume shale ( $V_{sh}$ ) curve, which is typically derived from the wireline log data (Gamma-ray log). Typical shale contains about 70% of clay and 30% of other minerals (mostly quartz). Once the mineral abundances are determined,  $K_{matrix}$  can be calculated via the application of Voigt-Reuss-Hill (VRH) averaging (Hill, 1952)

of the mineral constituents. Input for  $K_{matrix}$  calculation are  $V_{sh}$ ,  $K_{clay}$  (bulk modulus of clay), and  $K_{qtz}$  (bulk modulus of quartz). The  $K_{matrix}$  can be calculated by the VRH averaging as

$$K_{matrix} = \frac{1}{2} \left( [V_{clay} K_{clay} + V_{qtz} K_{qtz}] + \left[ \frac{V_{clay}}{K_{clay}} + \frac{V_{qtz}}{K_{qtz}} \right] \right), \quad (6)$$

where  $V_{clay}$  and  $V_{qtz}$  are

$$V_{clay} = 70\% V_{sh} \quad (\text{this is an assumption}) \quad (7)$$

and

$$V_{qtz} = 1 - V_{clay}. \quad (8)$$

Density of the mineral matrix  $\rho_{matrix}$  can be estimated by arithmetic averaging of densities of individual minerals as

$$\rho_{matrix} = V_{clay} \rho_{clay} + V_{qtz} \rho_{qtz} \quad (9)$$

where  $\rho_{clay}$  and  $\rho_{qtz}$  are the density of the clay and quartz minerals. Bulk moduli of clay (20.9 GPa) and quartz (36.6 GPa), and densities of clay (2.58 g/cm<sup>3</sup>) and quartz (2.65 g/cm<sup>3</sup>) can be found in text books (e.g., Mavko, Mukerji and Dvorkin, 1998) or from the core analysis in the laboratory. The values of  $K_{matrix}$  and  $\rho_{matrix}$  remain constant during the Gassmann fluid substitution.

## 2) Fluid properties:

Bulk modulus and density of the pore fluid (brine, oil, and gas) are estimated by averaging the values of individual fluid type. Let's first calculate properties of each fluid type (brine, gas, and oil).

### i) Bulk modulus and density of brine

Bulk modulus of brine can be estimated from known seismic velocity and density of brine as

$$K_{brine} = \rho_{brine} V_{brine}^2 \times 10^{-6} \quad (10)$$

where  $K_{brine}$  (GPa),  $\rho_{brine}$  (g/cm<sup>3</sup>) and  $V_{brine}$  (m/s) are the bulk modulus, density and P-wave velocity in brine. Units are given in the brackets (also in Table 1). Brine composition can range from almost pure water to saturated saline solution (characterized by the salinity). Density and velocity in brine can be calculated following Batze and Wang (1992). Density of brine is given by

$$\rho_{brine} = \rho_w + 0.668S + 0.44S^2 + 10^{-6} S [300P - 2400PS + T(80 + 3T - 3300S - 13P + 47PS)] \quad (11)$$

where  $P$ (MPa) and  $T$ (°C) are the in-situ pressure and temperatures,  $S$ (as weight fraction) is the salinity of brine, and  $\rho_w$  (g/cm<sup>3</sup>) is density of water given by

$$\rho_w = 1 + 10^{-6} (-80T - 3.3T^2 + 0.00175T^3 + 489P - 2TP + 0.016T^2P - 1.3 \times 10^{-5} T^3P - 0.333P^2 - 0.002TP^2) \quad (12)$$

P-wave velocity of brine,  $V_{brine}$  (m/s) is given as

$$V_{brine} = V_w + S(1170 - 9.6T + 0.055T^2 - 8.5 \times 10^{-5} T^3 + 2.6P - 0.0029TP - 0.0476P^2) + S^{1.5}(780 - 10P + 0.16P^2) - 1820S^2 \quad (13)$$

where  $V_w$  (m/s) is P-wave velocity in pure water, which can be estimated by

$$V_w = \sum_{i=1}^5 \sum_{j=1}^4 w_{ij} T^{i-1} P^{j-1} \quad (14)$$

where constants  $W_{ij}$  are given in Table 2.

### ii) Bulk modulus and density of gas

Bulk modulus and density of gas in a reservoir depend on the pressure, temperature and the type of gas. Hydrocarbon gas can be a mixture of many gases, and they are characterized by specific gravity  $G$ , the ratio of the gas density to air density at 15.6°C and atmospheric pressure. Following Batzle and Wang (1992) density of gas can be estimated as

$$\rho_{gas} \cong \frac{28.8GP}{ZR(T + 273.15)} \quad (15)$$

where  $G$  is the specific gravity of gas (API),  $R$  is the gas constant (8.314) and  $Z$  is the compressibility factor given by

$$Z = [0.03 + 0.00527(3.5 - T_{pr})^3] P_{pr} + (0.642T_{pr} - 0.007T_{pr}^4 - 0.52) + E \quad (16)$$

and

$$E = 0.109(3.85 - T_{pr})^2 \exp \left\{ - \left[ 0.45 + 8 \left( 0.56 - \frac{1}{T_{pr}} \right)^2 \right] \frac{P_{pr}^{1.2}}{T_{pr}} \right\} \quad (17)$$

In the above equation,  $T_{pr}$  and  $P_{pr}$  are the pseudo-reduced temperature and pressure, respectively, and are given by

$$T_{pr} = \frac{T + 273.15}{94.72 + 170.75G} \quad (18)$$

and

$$P_{pr} = \frac{P}{4.892 - 0.4048G} \quad (19)$$

The bulk modulus of gas (GPa) is given by (Batzle

**Table 1.** Symbols and units used in this paper

| Symbols        | Definitions                            | Units             | Equations        |
|----------------|--|-------------------|------------------|
| $K_{sat}$      | Bulk modulus of saturated rock         | GPa               | 3,4              |
| $K_{frame}$    | Bulk modulus of rock frame             | GPa               | 36               |
| $K_{matrix}$   | Bulk modulus of mineral matrix         | GPa               | 6                |
| $K_{ft}$       | Bulk modulus of pore fluid             | GPa               | 30               |
| $K_{brine}$    | Bulk modulus of brine                  | GPa               | 10               |
| $K_{gas}$      | Bulk modulus of gas                    | GPa               | 20               |
| $K_{oil}$      | Bulk modulus of oil                    | GPa               | 29               |
| $\mu$          | Shear modulus of rock                  | GPa               | 5                |
| $\rho$         | Density of in-situ rock                | g/cm <sup>3</sup> | Input / log      |
| $\rho_{brine}$ | Density of brine                       | g/cm <sup>3</sup> | 11               |
| $\rho_{gas}$   | Density of gas                         | g/cm <sup>3</sup> | 15               |
| $\rho_{oil}$   | Density of oil                         | g/cm <sup>3</sup> | 24               |
| $\rho_0$       | Reference density of oil               | g/cm <sup>3</sup> | Input            |
| $G$            | Specific gravity of gas                | API               | Input            |
| $p$            | In-situ pressure                       | MPa               | Input            |
| $T$            | In-situ temperature                    | °C                | Input            |
| $S$            | Salinity                               | Wt. fraction      | Input            |
| $WS$           | Water saturation                       | fraction          | Input            |
| $R_G$          | Gas-to-oil ratio (GOR)                 | Litre/litre       | Input            |
| $V_p$          | P-wave velocity of initial rock        | Km/s              | Input / log      |
| $V_s$          | S-wave velocity of initial rock        | Km/s              | Input / log      |
| $V_p^{sat}$    | P-wave velocity after fluid saturation | Km/s              | Output (step 10) |
| $V_s^{sat}$    | S-wave velocity after fluid saturation | Km/s              | Output (step 10) |
| $\rho^{sat}$   | Density of saturated rock              | g/cm <sup>3</sup> | Output (step 8)  |

**Table 2.** Coefficients for water velocity computation (from Batzle and Wang, 1992)

|                                  |                                   |
|----------------------------------|-----------------------------------|
| $w_{11} = 1402.85$               | $w_{13} = 3.437 \times 10^{-3}$   |
| $w_{21} = 4.871$                 | $w_{23} = 1.739 \times 10^{-4}$   |
| $w_{31} = -0.04783$              | $w_{33} = -2.135 \times 10^{-6}$  |
| $w_{41} = 1.487 \times 10^{-4}$  | $w_{43} = -1.455 \times 10^{-8}$  |
| $w_{51} = -2.197 \times 10^{-7}$ | $w_{53} = 5.23 \times 10^{-11}$   |
| $w_{12} = 1.524$                 | $w_{14} = -1.197 \times 10^{-5}$  |
| $w_{22} = -0.0111$               | $w_{24} = -1.628 \times 10^{-6}$  |
| $w_{32} = 2.747 \times 10^{-4}$  | $w_{34} = 1.237 \times 10^{-8}$   |
| $w_{42} = -6.503 \times 10^{-7}$ | $w_{44} = 1.327 \times 10^{-10}$  |
| $w_{52} = 7.987 \times 10^{-10}$ | $w_{54} = -4.614 \times 10^{-13}$ |

and Wang, 1992)

$$K_{gas} \cong \frac{P}{\left(1 - \frac{P_{pr}}{Z} \frac{\partial Z}{\partial P_{pr}}\right)_T} \frac{\gamma_0}{1000} \quad (20)$$

where

$$\gamma_0 = 0.85 + \frac{5.6}{P_{pr} + 2} + \frac{27.1}{(P_{pr} + 3.5)^2} - 8.7 \exp[-0.65(P_{pr} + 1)] \quad (21)$$

$$\left(\frac{\partial Z}{\partial P_{pr}}\right)_T = 0.03 + 0.00527(3.5 - T_{pr})^3 + 0.109(3.85 - T_{pr})^2 F \quad (22)$$

$$F = -1.2 \frac{P_{pr}^{0.2}}{T_{pr}} \left[ 0.45 + 8 \left( 0.56 - \frac{1}{T_{pr}} \right)^2 \right] \exp \left\{ - \left[ 0.45 + 8 \left( 0.56 - \frac{1}{T_{pr}} \right)^2 \right] \frac{P_{pr}^{1.2}}{T_{pr}} \right\} \quad (23)$$

### iii) Bulk modulus and density of oil

Oil contains some dissolved gas characterized by the GOR (gas-to-oil ratio) value. Similar to gas, oil density and bulk modulus depend on the temperature, pressure, GOR and the type of oil. Density and velocity in oil can be written following Batzle and Wang (1992) and Wang (2001) as,

$$\rho_{oil} = \frac{\rho_s + (0.00277P - 1.71 \times 10^{-7} P^3)(\rho_s - 1.15)^2 + 3.49 \times 10^{-4} P}{0.972 + 3.81 \times 10^{-4} (T + 17.78)^{1.175}} \quad (24)$$

and

$$V_{oil} = 2096 \sqrt{\frac{\rho_{ps}}{2.6 - \rho_{ps}}} - 3.7T + 4.64P + 0.0115 \left[ \sqrt{\frac{18.33}{\rho_{ps}}} - 16.97 - 1 \right] TP \quad (25)$$

where  $\rho_{oil}$  (g/cm<sup>3</sup>) and  $V_{oil}$  (m/s) are the density and P-wave velocity in oil (containing some dissolved gas). In the above equations,  $\rho_s$  and  $\rho_{ps}$  are the saturation density and pseudo density given as

$$\rho_s = \frac{\rho_0 + 0.0012R_G G}{B_0} \quad (26)$$

and

$$\rho_{ps} = \frac{\rho_0}{(1 + 0.001R_G)B_0} \quad (27)$$

where  $R_G$  is the GOR (litre/litre),  $\rho_0$  (g/cm<sup>3</sup>) is the reference density of oil measured at 15.6°C and atmospheric pressure and is called the formation volume factor given as

$$B_0 = 0.972 + 0.00038 \left[ 2.495R_G \sqrt{\frac{G}{\rho_0}} + T + 17.8 \right]^{1.175} \quad (28)$$

Once velocity and density are known, bulk modulus of oil,  $K_{oil}$  (GPa) can be calculated as

$$K_{oil} = \rho_{oil} V_{oil}^2 \times 10^{-6} \quad (29)$$

Fluid in the pore spaces consists of brine and hydrocarbon (oil and/or gas). Bulk modulus and density of the mixed pore fluid phase can be estimated by inverse bulk modulus averaging (also known as Wood's equation) and arithmetic averaging of densities (i.e., mass balance) of the separate fluid phases, respectively. Bulk modulus ( $K_{fl}$ ) and density ( $\rho_{fl}$ ) of the fluid phase are given as

$$\frac{1}{K_{fl}} = \frac{WS}{K_{brine}} + \frac{HS}{K_{hyc}} \quad (30)$$

and

$$\rho_{fl} = WS\rho_{brine} + HS\rho_{hyc} \quad (31)$$

where  $WS$  is the water saturation (as fraction) and  $HS (=1-WS)$  is the hydrocarbon saturation,  $K_{hyc}$  and  $\rho_{hyc}$  are the bulk modulus and density of hydrocarbon, respectively. In the case of oil as hydrocarbon,

$$K_{hyc} = K_{oil} \quad (32)$$

and

$$\rho_{hyc} = \rho_{oil} \quad (33)$$

and in the case of gas as hydrocarbon,

$$K_{hyc} = K_{gas} \quad (34)$$

and

$$\rho_{hyc} = \rho_{gas} \quad (35)$$

### 3) Frame properties:

Frame bulk modulus can be derived from the laboratory measurement, empirical relationship, or wireline log data. When working with wireline data,  $K_{frame}$  can be determined by rewriting the Gassmann equation (equation 3) for  $K_{frame}$  (Zhu and McMechan, 1990) as

$$K_{frame} = \frac{K_{sat} \left( \frac{\phi K_{matrix}}{K_{fl}} + 1 - \phi \right) - K_{matrix}}{\frac{\phi K_{matrix}}{K_{fl}} + \frac{K_{sat}}{K_{matrix}} - 1 - \phi} \quad (36)$$

All the parameters in the above equation (36) are known from the previous formulations:  $K_{sat}$  (equation 4),  $K_{matrix}$  (equation 6), and  $K_{fl}$  (equation 30). The  $K_{frame}$  value remains unchanged during fluid substitution.

### Algorithm

Now I present an algorithm using the above formulations for a depth point in reservoir (see appendix for actual matlab code). The same process can be repeated for multi-points.

- 1) Evaluate bulk modulus  $K_{matrix}$  (equation 6) and density of matrix  $\rho_{matrix}$  (equation 9)
- 2) Estimate bulk modulus  $K_{brine}$  (equation 10) and density of brine  $\rho_{brine}$  (equation 11)
- 3) Estimate bulk modulus  $K_{hyc}$  (equation 32 or 34) and density,  $\rho_{hyc}$  (equation 33 or 35) of the initial hydrocarbon (oil or gas)
- 4) Evaluate bulk modulus  $K_{fl}$  (equation 30) and density  $\rho_{fl}$  (equation 31) of initial fluid (using bulk moduli and densities of brine and hydrocarbon from steps 2 and 3). Initial fluid is defined by initial water saturation ( $WS$ ) and the type of hydrocarbon. If it is oil, it contains some dissolved gas defined by the GOR value  $R_G$ .
- 5) Evaluate initial saturated bulk modulus  $K_{sat}$  (equation 4) and shear modulus  $\mu$  (equation 5)
- 6) Estimate frame bulk modulus  $K_{frame}$  (equation 36)
- 7) Estimate bulk modulus  $K_{hyc}$  (equation 32 or 34) and density  $\rho_{hyc}$  (equation 33 or 35) of the desired hydrocarbon (oil or gas).
- 8) Evaluate bulk modulus,  $K_{fl}$  (equation 30) and density,

$\rho_{fl}$  (equation 31) of desired fluid (use bulk moduli and densities of brine and hydrocarbon from steps 2 and 7). Desired fluid is defined by the target water saturation (WS) and the type of hydrocarbon. Estimate density of saturated rock  $\rho_{sat}$  (g/cm<sup>3</sup>) after fluid substitution using

$$\rho^{sat} = \phi \rho_{fl} + (1 - \phi) \rho_{matrix} \quad (37)$$

- 9) Estimate bulk modulus of the saturated rock after fluid substitution (using fluid bulk modulus from step 8) using equation 3.
- 10) Estimate seismic velocities (km/s) (equations 1 and 2) after fluid substitution using density from step 8, bulk modulus from step 9, and shear modulus from step 5.

## Example

For an example of Gassmann fluid substitution, I wrote a Matlab (a programming language) program with the graphical user interface (GUI) following the algorithm given in the previous section. Main matlab code is given in the appendix, however GUI part of the program can not be included in text since it is a matlab figure type file (a binary low level encrypted file). The action performed by the GUI part of code can be done by entering input parameters in the matlab code (appendix) and running it from the matlab prompt. Figure 1 shows the GUI page, which contain all the input parameters including rock properties, initial hydrocarbon (oil or gas) and wanted (or desired) fluid type (brine, oil or gas).

The GUI window is titled "gassmann\_gui (Author: Dhananjay Kumar)". The main title is "Gassmann Fluid Substitution". Under the "INPUT" section, there are several input fields and radio buttons:

| oil Gravity (API) | PHI (fraction) | Vp (ft/s) | GOR (l/l) |
|-------------------|----------------|-----------|-----------|
| 42                | 0.20           | 11000     | 160       |

| Gas gravity (API) | IWS (fraction) | Vs (ft/s) | Initial Hyc<br><input checked="" type="radio"/> Oil<br><input type="radio"/> Gas |
|-------------------|----------------|-----------|--|
| 0.9               | 0.40           | 6500      |  |

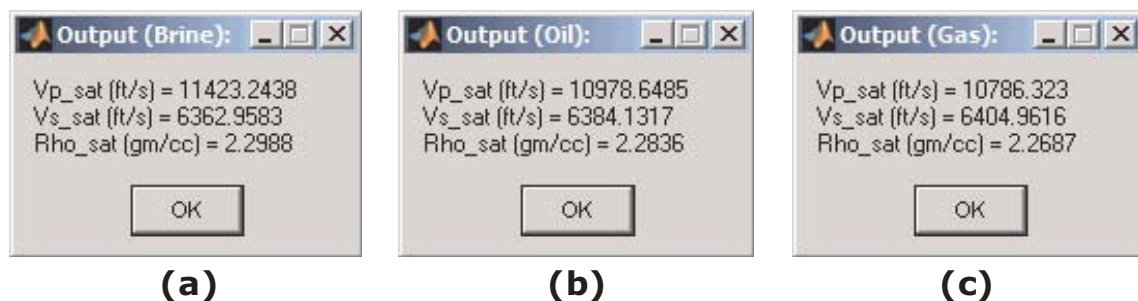
| P (psi) | TWS (fraction) | Density (gm/cc) | Fluid Wanted<br><input checked="" type="radio"/> Brine<br><input type="radio"/> Oil<br><input type="radio"/> Gas |
|---------|----------------|-----------------|--|
| 3200    | 1.00           | 2.20            |  |

| T (deg C) | Salinity (ppm) | VSH (fraction) |
|-----------|----------------|----------------|
| 150       | 3800           | 0.20           |

A "RUN" button is located at the bottom right of the input section.

**Fig. 1.** Matlab GUI page showing inputs used in Gassmann fluid substitution as described in the text. It uses bulk moduli and densities of clay and quartz minerals as given in text. If the hydrocarbon is oil it contains some dissolved gas (characterized by the GOR value). Some new notations used here are: PHI for porosity, IWS for initial water saturation, TWS for target water saturation and VSH for volume shale.



**Fig. 2.** Matlab GUI page showing outputs if you press "RUN" button on Figure 1 for oil as initial hydrocarbon and for three different desired fluids: a) brine with target water saturation (TWS) of 100%, b) oil with TWS of 80%, and c) gas with TWS of 80%. As output we get  $V_p$ ,  $V_s$ , and density after fluid substitutions. Input selection in Figure 1 exactly corresponds to output in Figure 2(a).



It uses bulk moduli and densities of clay and quartz mineral as given in matrix formulation section. You notice the differences in the units of velocity, pressure, salinity and reference oil gravity between GUI page and actual calculation. Units for velocity, pressure, salinity and oil gravity shown on the GUI page are ft/s, PSI, PPM and API, respectively, and units used in calculation are km/s, MPa, weight fraction and g/cm<sup>3</sup>, respectively due to availability of data in the units shown on the GUI page (Figure 1). These units conversion is trivial (see appendix). This completes the fluid substitutions for a depth sample point.

To do fluid substitutions for all depth points of log data (say for each 0.5 ft depth sample interval point) repeat the same process in a loop over depth points. Each depth point can have different reservoir properties and fluid type that means there is no dependence on the previous or future calculations. This is a simple algorithm, which can be changed according to the needs. Also some assumptions like mineral types (here assuming as clay and quartz only), percentage of clay in volume shale (here considering that 70% of shale is clay), and bulk moduli and densities of clay and quartz can be changed according to the availability of the data from your reservoir.

## Discussions and Conclusions

I have discussed explicit formulations, an algorithm, and a matlab code for the Gassmann fluid substitution. This algorithm can be coded in any programming language including Excel sheet. There are three elements of the fluid substitution formulation: fluid, rock frame and mineral matrix. Evaluating the fluid properties is most difficult, especially estimating initial fluid properties. There are many assumptions inherent in the Gassmann formulation which should be studied first in any reservoir before using this formulation; however it works well in most of the cases and is most widely used for fluid substitution. Accuracy in the Gassmann fluid substitution depends on the quality of the well logs,

derivation of porosity and volume shale from logs, mineral properties, initial fluid properties estimation and validity of different assumptions.

## Acknowledgements

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## Appendix

## Matlab code for Gassmann fluid substitution

Below is a matlab code for Gassmann fluid substitution. One can simply copy the same code in a file and save as gassmann.m (for example) and on the matlab prompt enter gassmann, it will give you the P- and S-wave velocity and density after fluid substitution according to the input parameters. The following code is for a single depth point in log, performed the same in a loop for multi-points. Unfortunately the GUI part of the matlab for input (as in Figure 1) can not be included in text, but one can do the same by entering the input parameters below and running from the matlab prompt. For any question write to Dhananjay Kumar (email: [dhku@chevron.com](mailto:dhku@chevron.com)).

```
% Matlab program for "Gassmann fluid substitution"
% Author: Dhananjay Kumar (September 8, 2005)
% Chevron Energy Technology Company, California, USA
% References: Wang (2001), Batzle and Wang (1992),
% Geophysics
% AIM: Model fluid properties for brine sand, oil sand
% and gas sand
% Desired properties: P- and S-wave velocities, and
% density
% Assumptions: mineral is a mixture of quartz and Clay
% And as in the Gassmann theory (e.g., homogeneous
% fluid, isotropy)
% input: rho_o = reference density of oil ( 42 deg API)
% GOR = gas-to-oil ratio (160 l/l)
% rho_g = specific gravity of gas (0.9 API)
% T= Temperature (150 deg C)
% P= Pressure (3200 psi)
% S= salinity (3800 ppm)
% phi = porosity (0.20)
% VSH= volume shale log (0.20)
% isw = SWT: initial water saturation from log (0.40)
% tsw = target water saturation (1.00)
% ifluid = type of initial hydrocarbon (Gas, Oil)
% fluid = type of output fluid (Brine, Gas, Oil)
% vp = P-wave velocity from log (ft/s) - insitu / original
% vs = S-wave velocity from log (ft/s) - insitu / original
% rho = Bulk density from log (gm/cc) - insitu / original
% Output: vp_sat = P-wave velocity after fluid subs (ft/s)
% vs_sat = S-wave velocity after fluid subs (ft/s)
% rho_sat = Density after fluid substitution (gm/cc)
% How to run: check all input and enter file name on the
% matlab prompt
% NOTE: if hydroc. is oil, it contains some dissolved gas
% (defined by GOR)
% if desired fluid is oil or gas, it contains water (defined
% by tws)
%%%%%%%%%%%%%%
%%%%%%%%%%%%%% %%%%%%%%%%
% Input parameters (use defined)
%
```

```

rho_o = 42;           % Oil gravity (deg API)
GOR = 160.0;          % GOR (L/L)
rho_g = 0.9;          % Gas gravity (API)
T = 150.00;           % Temperature (0 C)
P = 3200.00;          % Pressure (psi)
S = 3800;              % water salinity (ppm)
phi = 0.20;           % porosity (in fraction)
vsh = 0.20             % Vsh (volume shale in fraction)
isw = 0.40;           % initial water saturation (SW)
tsw = 1.00;           % target water saturation (in fraction)
ifluid = 1;           % initial hydrocarbon is 1(oil), 2(gas)
fluid = 1;            % Desired fluid is 1(brine), 2(oil) 3(gas)
vp = 11000.0;         % ft/s - from log (initial value)
vs = 6500.0;          % ft/s - from log (initial value)
rho = 2.2;            % gm/c - from log (initial value)
%
% Fixed parameters (e.g., Mavko et al., 1998)
%
k_clay = 20.9;         % Bulk mod (GPa)
k_qtz = 36.6;
rho_clay = 2.58;      % gm/cc
rho_qtz = 2.65;
%
% some applied properties
%
div_mill = 1/1000000; % factor used to divide by million
fs2kms = 0.000305;   % factor for ft/s to km/s conversion
kms2fs = 3280.84;    % factor for km/s to ft/s conversion
v_clay = vsh*0.70;    % Assumption: V_clay = 70% of VSH
v_qtz = 1-v_clay;     % quartz fraction in mineral
ish = 1-isw;           % initial hydrocarbon saturation
tsh = 1-tsw;           % final hydrocarbon saturation
rho_o = 141.5/(rho_o+131.5); % oil gravity in gm/cc (from API)
P = P*6.894757*0.001; % Press in MPa (from Psi)
S = S*div_mill;        % salinity as weight fraction
vp = vp*fs2kms;        % ft/s to km/s
vs = vs*fs2kms;        % ft/s to km/s
%
% Step 1: Matrix properties (using VRH averaging, equation 6)
%
k_voigt = v_clay*k_clay + v_qtz*k_qtz;
k_reuss = 1/(v_clay/k_clay + v_qtz/k_qtz);
k_matrix = 0.5*(k_voigt + k_reuss); % GPa
rho_matrix = v_clay*rho_clay+v_qtz*rho_qtz; % gm/cc
%
% Step 2: water/brine properties (Equations 10 and 11)
%
w(1,1) = 1402.85;      w(1,3) = 3.437*10^(-3); % Table 2
w(2,1) = 4.871;        w(2,3) = 1.739*10^(-4);
w(3,1) = -0.04783;     w(3,3) = -2.135*10^(-6);
w(4,1) = 1.487*10^(-4); w(4,3) = -1.455*10^(-8);
w(5,1) = -2.197*10^(-7); w(5,3) = 5.230*10^(-11);
w(1,2) = 1.524;        w(1,4) = -1.197*10^(-5);
w(2,2) = -0.0111;      w(2,4) = -1.628*10^(-6);

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w(3,2)=2.747*10^(-4); w(3,4)=1.237*10^(-8);
w(4,2)=-6.503*10^(-7); w(4,4)=1.327*10^(-10);
w(5,2)=7.987*10^(-10); w(5,4)=-4.614*10^(-13);
sum = 0;
for i=1:5
    for j=1:4
        sum = sum+w(i,j)*T^(i-1)*P^(j-1);
    end
end
v_water = sum;
v1 = 1170-9.6*T+0.055*T*T-8.5*10^(-5)*T*T*T+2.6*P-
0.0029*T*P-0.0476*P*P;
v_brine = v_water+S*v1+S^1.5*(780-10*P+0.16*P*P)-
1820*S*S; % m/s
r1 = 489*P-2*T*P+0.016*T*T*P-1.3*10^(-5)*T*T*T*P-
0.333*P*P-0.002*T*P*P;
rho_water=1+10^(-6)*(-80*T-3.3*T*T+0.00175*T*T*T+r1);
r2 = 300*P-2400*P*S+T*(80+3*T-3300*S-13*P+47*P*S);
rho_brine = rho_water+0.668*S+0.44*S*S+10^(-6)*S*r2; %
gm/cc (held const)
k_brine = rho_brine*v_brine*v_brine/div_mill; %
GPa (held const)
%
% Step 3: Initial Hydrocarbon properties (Equations 32 to 35)
%
if ifluid == 1 % 'Oil' Oil by default contains gas also
    B0 = 0.972+0.00038*(2.495*GOR*sqrt(rho_g/
rho_o)+T+17.8)^1.175;
    rho_ps = rho_o/((1+0.001*GOR)*B0);
    rho_s = (rho_o+0.0012*GOR*rho_g)/B0;
    r1 = rho_s+(0.00277*P-1.71*0.0000001*P*P*P)*(rho_s-
1.15)^2+3.49*0.0001*P;
    rho_hyc = r1/(0.972+3.81*0.0001*(T+17.78)^1.175); % gm/
cc (will change)
    v = 2096*sqrt(rho_ps/(2.6-rho_ps))-
3.7*T+4.64*P+0.0115*(sqrt(18.33/rho_ps-16.97)-1)*T*P;
    k_hyc = rho_hyc*v*v/div_mill; % GPa (will
change)
else % 'gas' : means no OIL only gas is present
    R = 8.314; % gas constant (eqn, same as in step7 for fluid
== 3)
    Ta = T+273.15;
    Ppr = P/(4.892-0.4048*rho_g);
    Tpr = Ta/(94.72+170.75*rho_g);
    E1 = exp(-Ppr^1.2/Tpr*(0.45+8*(0.56-1/Tpr)^2));
    E = 0.109*(3.85-Tpr)^2*E1;
    Z1 = 0.03+0.00527*(3.5-Tpr)^3;
    Z = Z1*Ppr+0.642*Tpr-0.007*Tpr^4-0.52+E;
    rho_hyc = 28.8*rho_g*P/(Z*R*Ta);
    dz_dp = Z1+0.109*(3.85-Tpr)^2*E1*(-1.2*Ppr^0.2/
Tpr*(0.45+8*(0.56-1/Tpr)^2));
    yo = 0.85+5.6/(Ppr+2)+27.1/(Ppr+3.5)^2-8.7*exp(-
0.65*(Ppr+1));
    k_hyc = P*yo/1000*1.0/(1-Ppr/Z*dz_dp); % GPa
end
%
% Step 4: Fluid properties(initial insitu model, equations 30

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and 31)
%
k_fl = 1/(isw/k_brine+ish/k_hyc);
rho_fl = isw*rho_brine+ish*rho_hyc;
%
% Step 5: Insitu original moduli (for saturated – insitu rock,
equations 4 and 5)
%
dens_poros = 0; % 1 (use porosity to est initial density), 0
(use input log)
if dens_poros == 1
    rho = phi*rho_fl + (1-phi)*rho_matrix;
end
k_sat = rho*(vp*vp-vs*vs*4/3); % GPa (will change in step 9)
g = rho*vs*vs; % GPa (held constant)
%
% Step 6: Porous frame properties (rewrite Gassmann eqn,
equation 36)
%
k1 = k_sat*(phi*k_matrix/k_fl+1-phi)-k_matrix;
k2 = phi*k_matrix/k_fl+k_sat/k_matrix-1-phi;
k_frame = k1/k2; % GPa (held constant)
%
% Step 7: select the type of output fluid, cal hyc/fluid prop
(equations 32 to 35)
%
if fluid == 1 % 'Brine'
    % disp('Changing fluid to brine')
elseif fluid == 2 % 'Oil'
    % disp('Changing fluid to Oil [with dissolved gas] with
TWS brine')
    B0 = 0.972+0.00038*(2.495*GOR*sqrt(rho_g/
rho_o)+T+17.8)^1.175;
    rho_ps = rho_o/((1+0.001*GOR)*B0);
    rho_s = (rho_o+0.0012*GOR*rho_g)/B0;
    r1 = rho_s+(0.00277*P-1.71*0.0000001*P*P*P)*(rho_s-
1.15)^2+3.49*0.0001*P;
    rho_hyc = r1/(0.972+3.81*0.0001*(T+17.78)^1.175); % gm/
cc (will change)
    v = 2096*sqrt(rho_ps/(2.6-rho_ps))-
3.7*T+4.64*P+0.0115*(sqrt(18.33/rho_ps-16.97)-1)*T*P;
    k_hyc = rho_hyc*v*v/div_mill; % GPa (will
change)
elseif fluid == 3 % 'Gas'
    % disp('Changing fluid to Gas with TWS brine')
    R = 8.314; % gas constant
    Ta = T+273.15;
    Ppr = P/(4.892-0.4048*rho_g);
    Tpr = Ta/(94.72+170.75*rho_g);
    E1 = exp(-Ppr^1.2/Tpr*(0.45+8*(0.56-1/Tpr)^2));
    E = 0.109*(3.85-Tpr)^2*E1;
    Z1 = 0.03+0.00527*(3.5-Tpr)^3;
    Z = Z1*Ppr+0.642*Tpr-0.007*Tpr^4-0.52+E;
    rho_hyc = 28.8*rho_g*P/(Z*R*Ta);
    dz_dp = Z1+0.109*(3.85-Tpr)^2*E1*(-1.2*Ppr^0.2/
Tpr*(0.45+8*(0.56-1/Tpr)^2));
    yo = 0.85+5.6/(Ppr+2)+27.1/(Ppr+3.5)^2-8.7*exp(-
0.65*(Ppr+1));

```

