ECSE 543A NUMERICAL METHODS IN ELECTRICAL ENGINEERING
Assignment 2

Set: 07-Oct-2021 Due: 04-Nov-2021

1) Figure 1 shows two first-order triangular finite elements used to solve the Laplace equation for electrostatic potential. Find a local **S**-matrix for each triangle and a global **S**-matrix for the mesh, which consists of just these two triangles. The local (disjoint) and global (conjoint) node-numberings are shown in Figure 1(a) and (b), respectively. Also, Figure 1(a) shows the (x, y)-coordinates of the element vertices in meters.

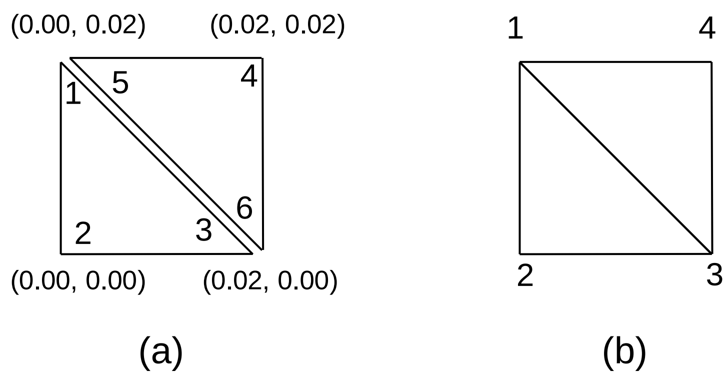

Figure 1

2) Figure 2 shows the cross-section of an electrostatic problem with translational symmetry: a rectangular coaxial cable. The inner conductor is held at 15 volts and the outer conductor is grounded. (This is similar to the system considered in Question 3, Assignment 1.)

(a) Use the two-element mesh shown in Figure 1(b) as a "building block" to construct a finite element mesh for one-quarter of the cross-section of the coaxial cable. Specify the mesh, including boundary conditions, in an input file following the format for the **SIMPLE2D** program as explained in the course notes. (Hint: Your mesh should consist of 46 elements.)

(b) Use the **SIMPLE2D** program with the mesh from part (a) to compute the electrostatic potential solution. Determine the potential at $(x,y) = (0.06, 0.04)$ from the data in the output file of the program.

(c) Compute the capacitance per unit length of the system using the solution obtained from **SIMPLE2D.**

Note: The **SIMPLE2D** program and related utility programs are available from the myCourses page for this course or on the World-Wide-Web at: http://www.cambridge.org/ca/academic/subjects/engineering/engineering-mathematics-and-programming/finite-elements-electrical-engineers-3rd-edition?format=PB?format=PB (Click on "Resources" tab.) Please read the file README.1ST before using the software.

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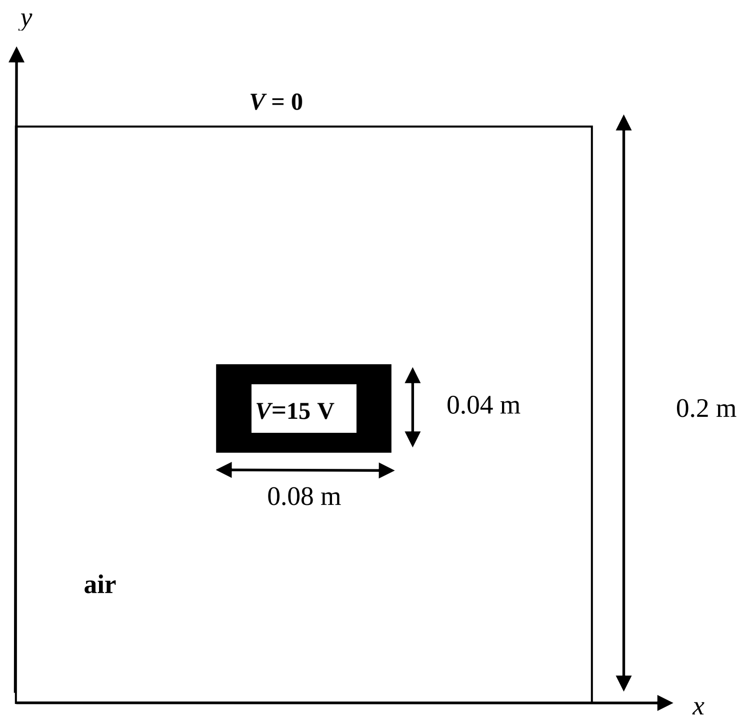

Figure 2.

3) Write a program implementing the conjugate gradient method (un-preconditioned). Solve the matrix equation corresponding to a finite difference node-spacing, h = 0.02m in x and y directions for the same one-quarter cross-section of the system shown in Figure 2 that you considered in Question 2 above. Use a starting solution of zero. (Hint: The program you wrote for Question 3 of Assignment 1 may be useful for generating the matrix equation.)

   (a) Test your matrix using your Choleski decomposition program that you wrote for Question 1 of Assignment 1 to ensure that it is positive definite. If it is not, suggest how you could modify the matrix equation in order to use the conjugate gradient method for this problem.

   (b) Once you have modified the problem, if necessary, so that the matrix is positive definite, solve the matrix equation first using the Choleski decomposition program from Assignment 1, and then the conjugate gradient program written for this assignment.

   (c) Plot a graph of the infinity norm and the 2-norm of the residual vector versus the number of iterations for the conjugate program.

   (d) What is the potential at (x,y) = (0.06, 0.04), using the Choleski decomposition and the conjugate gradient programs, and how do they compare with the value you computed in Question 2(b) above. How do they compare with the value at the same (x,y) location and for the same node spacing that you computed in Assignment 1 using SOR.

   (e) Suggest how you could compute the capacitance per unit length of the system from the finite difference solution.