ECSE 543A NUMERICAL METHODS IN ELECTRICAL ENGINEERING
Assignment 3

Set: 04-Nov-2021
Due: 02-Dec-2021

1. You are given a list of measured BH points for M19 steel (Table 1), with which to construct a continuous graph of B versus H.

   (a) Interpolate the first 6 points using full-domain Lagrange polynomials. Is the result plausible, i.e. do you think it lies close to the true B versus H graph over this range?

   (b) Now use the same type of interpolation for the 6 points at B = 0, 1.3, 1.4, 1.7, 1.8, 1.9. Is this result plausible?

   (c) An alternative to full-domain Lagrange polynomials is to interpolate using cubic Hermite polynomials in each of the 5 subdomains between the 6 points given in (b). With this approach, there remain 6 degrees of freedom – the slopes at the 6 points. Suggest ways of fixing the 6 slopes to get a good interpolation of the points.

B (T)	H (A/m)
0.0	0.0
0.2	14.7
0.4	36.5
0.6	71.7
0.8	121.4
1.0	197.4
1.1	256.2
1.2	348.7
1.3	540.6
1.4	1062.8
1.5	2318.0
1.6	4781.9
1.7	8687.4
1.8	13924.3
1.9	22650.2

Table 1: BH Data for M19 Steel

2. The magnetic circuit of Figure 1 has a core made of Ml9 steel, with a cross-sectional area 1 cm^2. L_c = 30 cm and L_a = 0.5 cm. The coil has N = 1000 turns and carries a current *1* = 8 A.

   (a) Derive a (nonlinear) equation for the flux ψ in the core, of the form $f(\psi) = 0$.

   (b) Solve the nonlinear equation using Newton-Raphson. Use a piecewise-linear interpolation of the data in Table 1. Start with zero flux and finish when

   $$| f(\psi) / f(0) | < 10^{-6}$$

   Record the final flux, and the number of steps taken.

   (c) Try solving the same problem with successive substitution. If the method does not converge, suggest and test a modification of the method that *does* converge.

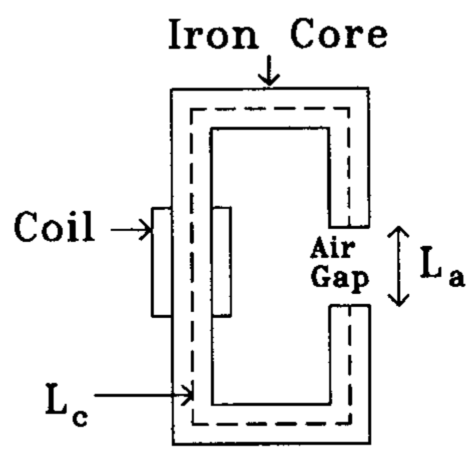


Figure 1

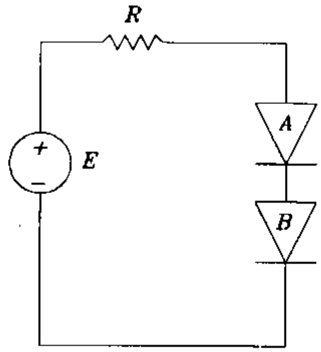
Continued on reverse …

NOTE: **ANSWER ONLY ONE OF THE TWO FOLLOWING QUESTIONS (EACH IS WORTH 10 MARKS)**

3. In the circuit shown below, the DC voltage E is 220 mV, the resistance R is 500 Ω, the diode A reverse saturation current I_{sA} is 0.6 μA, the diode B reverse saturation current I_{sB} is 1.2 μA, and assume kT/q to be 25 mV.

   (a) Derive nonlinear equations for a vector of nodal voltages, $\mathbf{v}_n$, in the form $\mathbf{f}(\mathbf{v}_n) = 0$. Give $\mathbf{f}$ explicitly in terms of the variables I_{sA}, I_{sB}, E, R and $\mathbf{v}_n$.

   (b) Solve the equation $\mathbf{f} = 0$ by the Newton-Raphson method. At each step, record $\mathbf{f}$ and the voltage across each diode. Is the convergence quadratic? [Hint: define a suitable error measure ε_k].



4.
   (a) Integrate the function $\cos(x)$ on the interval x=0 to x=1, by dividing the interval into N equal segments and using one-point Gauss-Legendre integration for each segment. Plot $log_{10}(E)$ versus $log_{10}(N)$ for N=1, 2,…, 20, where E is the absolute error in the computed integral. Comment on the result.

   (b) Repeat part (a) for the function $log_e(x)$, only this time plot for N=10, 20,…200. Comment on the result.

   (c) An alternative to dividing the interval into equal segments is to use smaller segments in more difficult parts of the interval. Experiment with a scheme of this kind, and see how accurately you can integrate $log_e(x)$ using only 10 segments.