

GPGN 303

Lab Exercise #7: Magnetism Forward Modeling

Handout Date: Tuesday, October 21, 2014

Due Date: Monday, October 27, 2014 - by 5PM to the TA in electronic form!

Task:

Building off of the code you developed for Assignments 1 and 2, create a program that calculates the total-field magnetic anomaly due to a buried spherical target with constant magnetic susceptibility. The susceptibility is assumed to be small ($K < 0.1$), i.e no self-demagnetization effect, and there is no remanent magnetization. Therefore, you are calculating the anomaly for a typical induced magnetization problem.

Useful Information:

Relation between susceptibility, magnetization and inducing field

$$\vec{M} = \kappa \vec{B}_0 / \mu_0$$

Magnetization is a dipole density, therefore the field due to a uniform sphere is the same as that due to a dipole!

$$\begin{aligned}\vec{B}_a &= \frac{\mu_0}{4\pi} \left(\frac{4\pi a^3}{3} \vec{M} \right) \cdot \nabla \nabla \frac{1}{|\vec{r}' - \vec{r}|} \\ &= \frac{\mu_0}{4\pi} \vec{m} \cdot \nabla \nabla \frac{1}{|\vec{r}' - \vec{r}|}\end{aligned}$$

Definition of the magnetic anomaly, related to magnetic dipole by tensor \mathbf{T}

$$\vec{B}_a = -\nabla \phi = \frac{\mu_0}{4\pi} \vec{m} \cdot \nabla \nabla \frac{1}{|\vec{r} - \vec{r}'|} = \frac{\mu_0}{4\pi} \mathbf{T} \vec{m}$$

\mathbf{T} is the same tensor you manually derived for gravity gradiometry that contains all position information for the target and observation points

$$\mathbf{T} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x \partial z} \\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y \partial z} \\ \frac{\partial^2}{\partial z \partial x} & \frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial z^2} \end{pmatrix} \frac{1}{|\vec{r} - \vec{r}'|}$$

Your total-field anomaly is the projection of the anomaly onto the direction of the earth's field. It is not merely the amplitude of the anomaly.

$$\Delta T \cong \vec{B}_a \cdot \hat{B}_0$$

$$\Delta T \neq |\vec{B}_a|$$

Run your algorithm with the following input parameters:

Target

Radius = 200m, susceptibility = 0.05, Location (East/North/Depth) = (0,0,250) meters

Earth's Inducing Field

Strength = 50000nT, Inclination = 45-deg, Declination = 25-deg

Observation Grid

Easting Min = -775; Easting Max = 800; Station Spacing = 25 [meters]

Northing Min = -775; Northing Max = 800; Station Spacing = 25 [meters]

Observation Station Height = 0.0 meters (constant, at the surface)

What to Turn In:

1. The code you created (documented clearly with comments) so it can be run.
2. Laboratory write-up as usual (Purpose, Description, Results, Discussion, etc.).
We would like to see the fully derived equations that you use to code your anomaly.
3. You should place your images directly into the lab write-up with appropriate captions.
4. Copy your code into the back of the lab write-up in addition to sending separately.
5. Code and write-up must be contained within zip-file when submitting.

Please submit in electronic form (pdf).