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GPGN 303

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Assignment G1: Forward Modeling the Gravity Response of a Point Mass

Discussion

1. Gravity Response of a 10 Kg Dumbbell
    - Shallow (1 m) vs. Deep (10 m)
        - The shapes of the graphs for the deep dumbbell and the shallow dumbbell were significantly different.
        - The surface of the shallow dumbbell's gravity response graph was similar to a cone with a rounded top for the first 0.5 meters in the x and y directions. After 0.5 meters the surface of the graph asymptotically declines to the 1.0 m edge.
        - The surface of the deep dumbbell's gravity response graph is a gently sloping curved surface similar to a sphere for the first 0.5 meters in the x and y directions. After 0.5 meters the surface begins to linearly decline until the 1.0 m edge
    - High Resolution (0.01 m) vs. Low Resolution (0.20 m)
        - The shapes of the graphs for the high resolution dumbbell and the low resolution dumbbell were also significantly different, especially for the shallow dumbbell.
        - The surface of the low resolution graph for the deep dumbbell was a good representation of its high resolution version. It maintained the shape of the response graph at a much lower cost
        - The surface of the low resolution graph for the shallow dumbbell was not as good as a representation of the high resolution version compared to the deep dumbbell. The low resolution shallow dumbbell graph captured the maximum value directly over the mass but the low resolution lost much of the data in the immediate 0.5 m surrounding the maximum gravity response.
2. Gravity Response of a 1 kg Apple (It is an ultradense apple!)
    - In both cases, low resolution vs. high resolution and deep vs. shallow, altering the mass did not change the shape, precision, or accuracy of the gravity response graph. The only change occurred in the scaling of the gravity response measurement. This makes sense because in the gravity response equation the mass is multiplied in the denominator. In this case of the apple and the dumbell the gravity response was changed by a multiple of 10 and this change is observable on the z-scale of each graph.

Figures

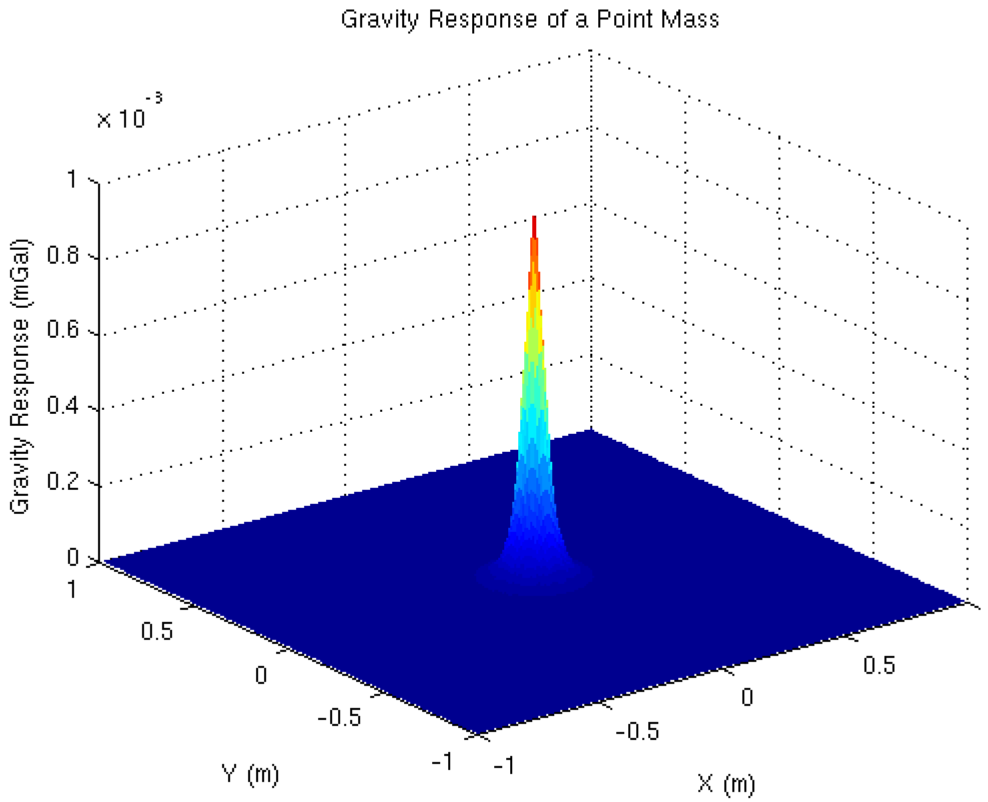

Figure 1: Gravity response of a 0.374 kilogram apple 0.05 meters below observation point with a measurement resolution of 0.01 meters

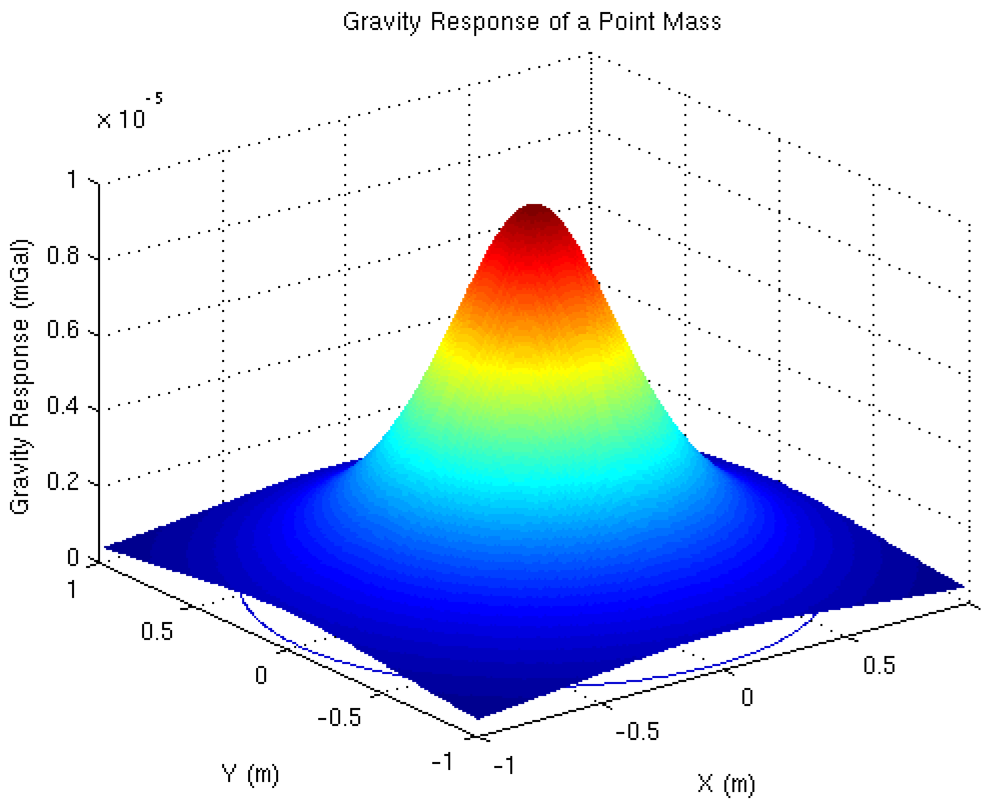

Figure 2: Gravity response of a 1 kilogram apple 1 meter below the surface with a measurement resolution of 0.01 meters

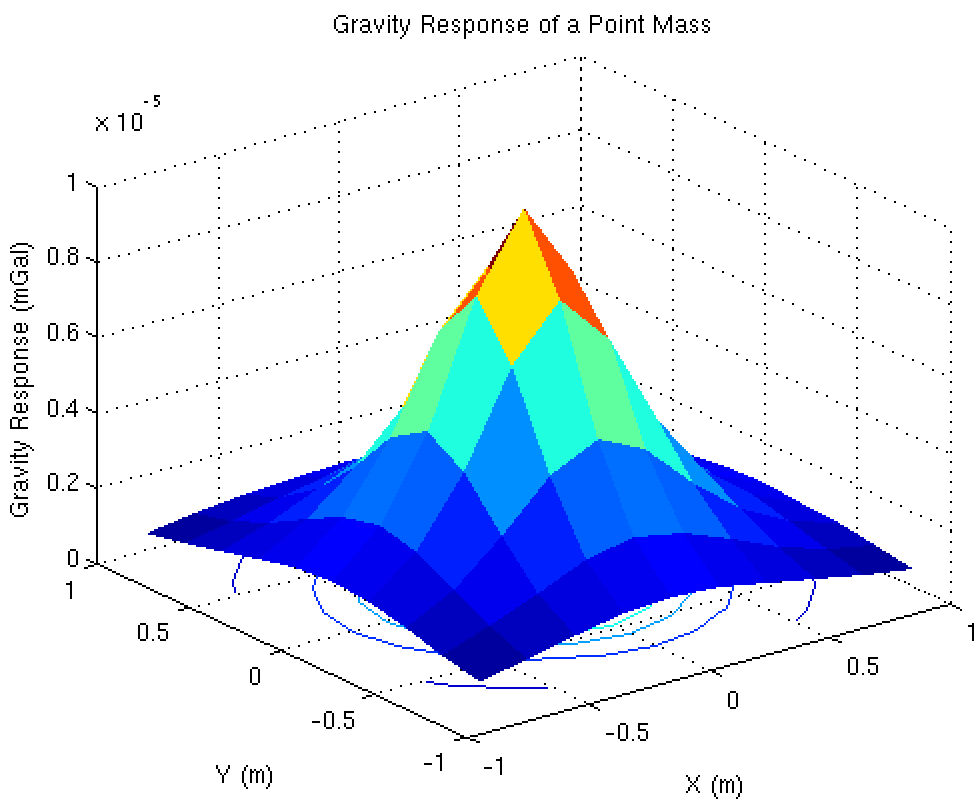

Figure 3: Gravity response of a 1 kilogram apple 1 meter below the surface with a measurement resolution of 0.2 meters

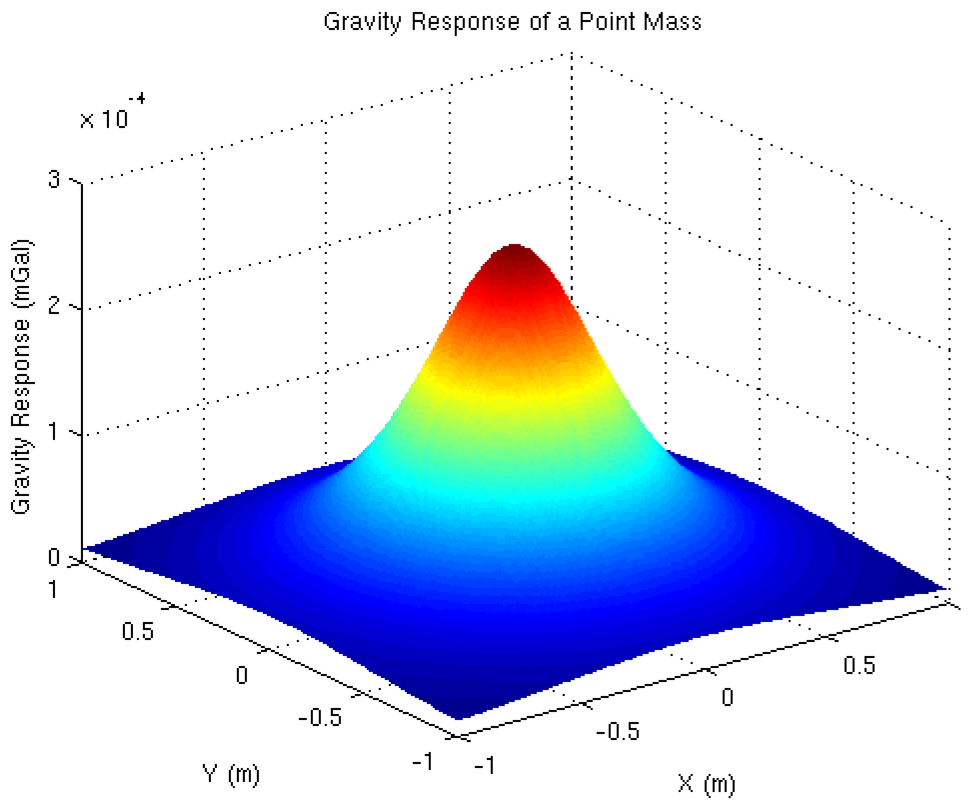

Figure 4: Gravity response of an 10 kilogram dumbbell 1 meter below the surface with a measurement resolution of 0.01 meters

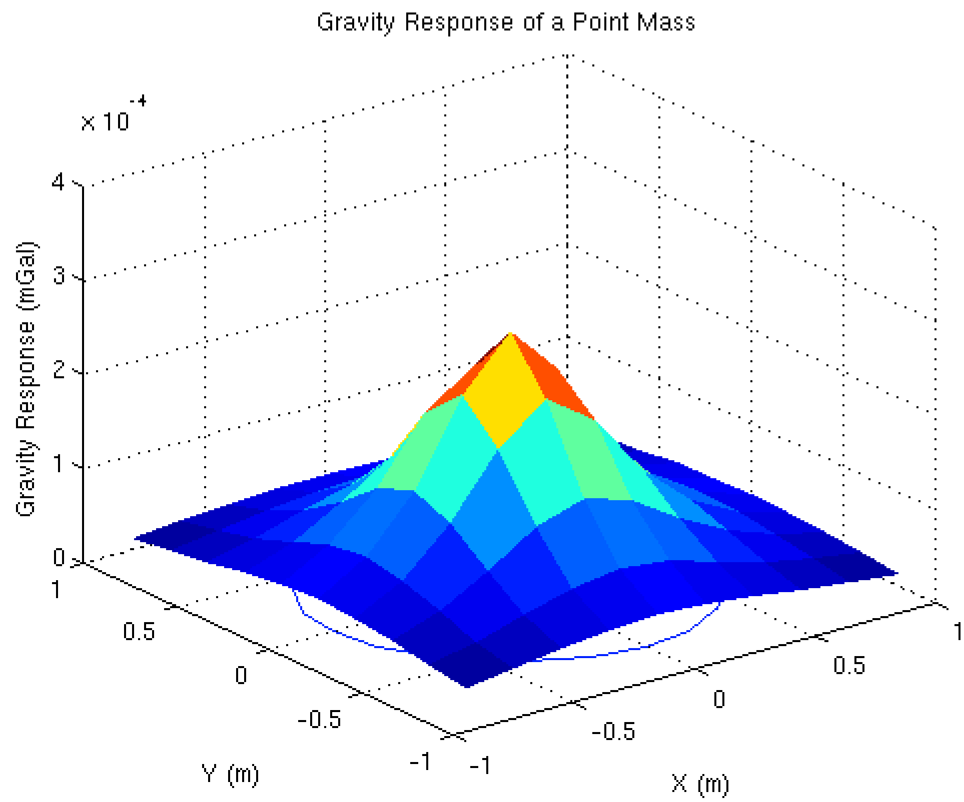

Figure 5: Gravity response of an 10 kilogram dumbbell 1 meter below the surface with a measurement resolution of 0.2 meters

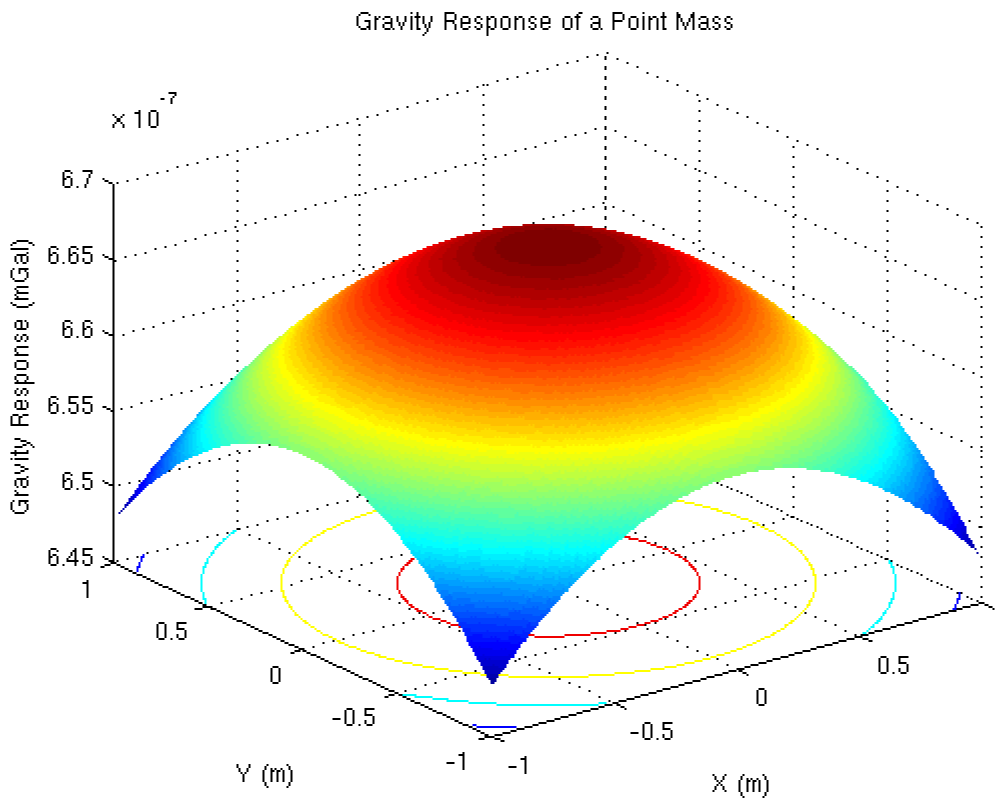

Figure 6: Gravity response of an 10 kilogram dumbbell 10 meters below the surface with a measurement resolution of 0.01 meters

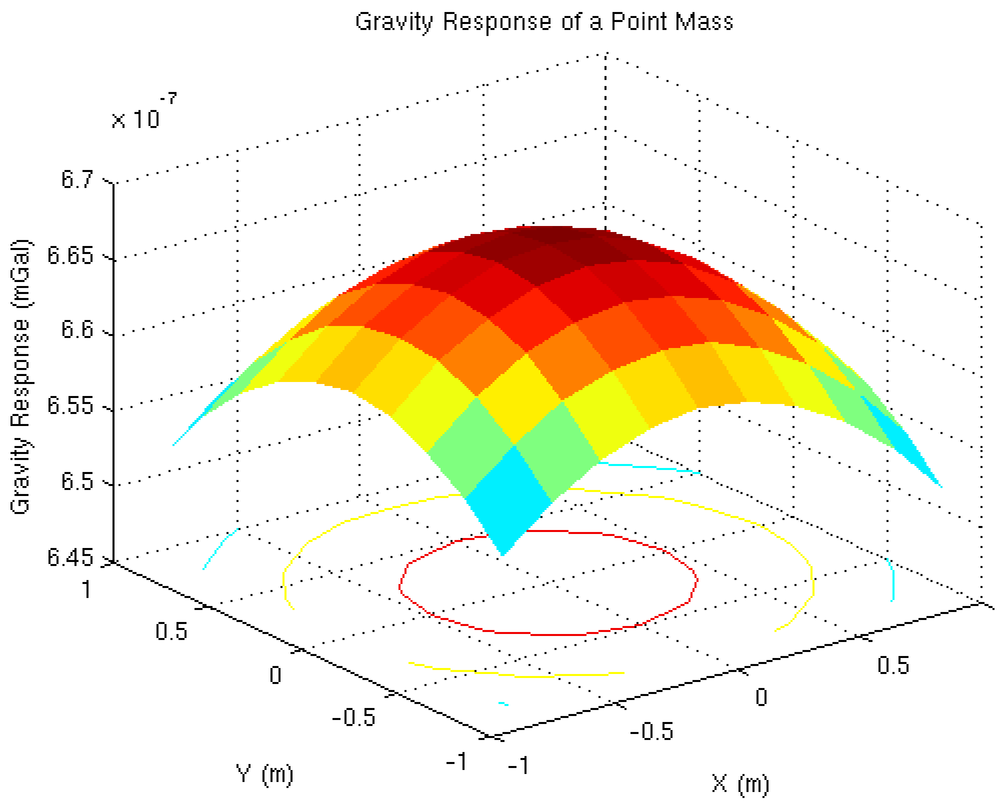

Figure 7: Gravity response of an 10 kilogram dumbbell 10 meters below the surface with a measurement resolution of 0.2 meters

Appendix

(I) Matlab Code for Gravity Response of a point mass

```
%Purpose:
%   The purpose of this function is to plot the gravity response of a point
%   mass below the surface as a function of your distance from the point
%   mass using the point mass' mass, location, and depth.

%Setup:
%   Coordinate System:
%       Cartesian
%           X
%               Positive    ->  Right
%               Negative    ->  Left
%           Y
%               Positive    ->  Out of Page
%               Negative    ->  Into Page
%           Z
%               Positive    ->  Down
%               Negative    ->  Up

%Input:
%   mass    ->  Mass at the point
%               Units: Kilograms
%   depth   ->  Magnitude of the depth of the point below the x-y plane
%               Units: Meters
%   x_mass  ->  Displacement of the mass in x direction (Positive Right)
%               Units: Meters
%   y_mass  ->  Displacement of the mass in y direction (Positive Out)
%               Units: Meters
%   spread  ->  Distance of axis extension from the origin
%               Units: Meters
%   res     ->  Distance between each gravity response measurement
%               Units: Meters

function gresponse(mass, depth, x_mass, y_mass, spread, res)

% Number of observations along a line of measurement
obs_total = 2 * spread / res;

% Setup arrays to store x and y coordinates and the corresponding gravity
% response
G_z = zeros(obs_total, obs_total);
x_obs = zeros(obs_total, obs_total);
```

```
y_obs = zeros(obs_total, obs_total);

% Iterate through every observation point on the observation grid and store
% data representing each measurement
for i = 1:obs_total
    for j = 1:obs_total
        % Calculate the x and y coordinates of each observation point
        x_obs(i, j) = -1.0 * spread + i * res;
        y_obs(i, j) = -1.0 * spread + j * res;

        % Calculate the gravity response at a given observation location,
        % (x,y)
        g = gresponse_observation(mass, depth, x_mass, y_mass, x_obs(i, j), y_obs(i, j), 0);
        G_z(i,j) = g(3);
    end;
end;

% Graph the data as a surface contour. Each (x, y, gravity response)
surfc(x_obs, y_obs, G_z, 'EdgeColor', 'none');
xlabel('X (m)');
ylabel('Y (m)');
zlabel('Gravity Response (mGal)');
title('Gravity Response of a Point Mass');

%Input:
%   mass    ->  Mass at the point
%               Units: Kilograms
%   depth   ->  Magnitude of the depth of the point below the x-y plane
%               Units: Meters
%   x_mass  ->  Displacement of the mass in the x direction
%               Units: Meters
%   y_mass  ->  Displacement of the mass in the y direction
%               Units: Meters
%   x_obs   ->  Displacement of the observation point in the x direction
%               Units: Meters
%   y_obs   ->  Displacement of the observation point in the y direction
%               Units: Meters
%   z_obs   ->  Displacement of the observation point in the z direction
%               Units: Meters

function g = gresponse_observation(mass, depth, x_mass, y_mass, x_obs, y_obs, z_obs)

%Constants
%   big_g   ->  The Universal Gravitational Constant
%               Units: (Meters)^(3) * (Kilograms)^(-1) * (Seconds)^(-2)
```

```
%   g_conv  ->  Conversion ratio from (Meter) * (Seconds)^(-2) to milliGals
%                  Units: None

big_g   = 6.67384E-11;
g_conv  = 100000;

coefficient = (g_conv * big_g * abs(mass)) / (((x_obs - x_mass)^2 + (y_obs - y_mass)^2 + (z_obs +
abs(depth))^2)^(3/2));

g(1) = coefficient * (x_obs - x_mass);
g(2) = coefficient * (y_obs - y_mass);
g(3) = coefficient * (z_obs + abs(depth));
```