Signal Analysis
Lab 4: Fourier Analysis

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Due: Oct 16, Group A - 9:00am, Group B - 1.00pm

General instruction: If you need to use Matlab's or Python's fft routine, use the length of the signal for the $nfft$ parameter.

1) Using the time-series in the file $Lab4\_t\_xt.dat$, answer the following questions:
   a) What is the sampling frequency f_s of the signal?
   b) What is the DC component X_0 of the signal?
   c) What is the time period T_0 of the signal? Give the value in seconds rounded to 3 decimal places.

2) Consider a continuous-time signal $x(t) = \sin(10\pi t)$
   a) Sketch the two-sided amplitude specturm of $\mathcal{F}[x(t)]$.
   b) Plot $x[n]$ obtained using $f_s = 20$ Hz for the time interval $t = [0s, 2s)$. Plot the amplitude spectrum of $\mathcal{F}[x[n]]$ obtained using Matlab's or Python's fft routine. Does the amplitude spectrum match with that in (a)? If not, why? Reconstruct $x(t)$ from the Fourier coefficients. Plot the reconstructed $x(t)$ and the original $x(t)$ on the same figure. Are they the same?
   c) Plot $x[n]$ obtained using $f_s = 9$ Hz for the time interval $t = [0s, 2s)$. Plot the amplitude spectrum of $\mathcal{F}[x[n]]$ obtained using Matlab's or Python's fft routine. Does the amplitude spectrum match with that in (a)? If not, why? Reconstruct $x(t)$ from the Fourier coefficients. Plot the reconstructed $x(t)$ and the original $x(t)$ on the same figure. Are they the same?

3) Recall that the synthesis formula is given as
$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{i(2\pi/T_0)kt},$$
where c_k are the Fourier coefficients. Consider the square wave
$$x(t) = \begin{cases} 1, & 0 \leq t < T_0/2 \\ 0, & T_0/2 \leq t < T_0 \end{cases}$$
   a) Solve for the Fourier coefficients using the Fourier integral.
   b) Plot the $x(t)$ for $T_0 = 1s$ within the interval $t = [0s, 4s]$.
   c) Plot the two-sided amplitude spectrum for the first 20 harmonics and the DC component using the solution in (a).
   d) Plot the two-sided phase spectrum for the first 20 harmonics and the DC component using the solution in (a).
   e) Synthesize an approximate square wave by using the first 3 harmonics (and the DC component) and plot it for the interval $t = [0s, 4s]$.
   f) Synthesize an approximate square wave by using the first 10 harmonics (and the DC component) and plot it for the interval $t = [0s, 4s]$.
   g) Synthesize an approximate square wave by using the first 20 harmonics (and the DC component) and plot it for the interval $t = [0s, 4s]$.