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   You are taking gravity measurements (Δg_z) at the surface of the earth and you would like to generate a map of density anomalies ($\Delta \rho$) in the subsurface. Your data have been corrected for the background, so you are inverting for a density model with zero mean. Since we work in 2.5D, we can relate the vertical component of the gravitational acceleration to the density using the relation:

$$\Delta g_z^i = G\left(2y_{max}\right) \sum_{j=1}^{M} \Delta \rho^j \frac{\left(z^i - z^j\right)}{\left[\left(x^i - x^j\right)^2 + \left(z^i - z^j\right)^2\right]^{3/2}} \Delta x \Delta z \ , \tag{1}$$

where i is the station index and j is the model cell index and we assume that the structure under investigation is invariant in the y direction between ± 10 km. The gravitational constant is

$$G = 6.674 \ 10^{-2} \frac{km^3}{kg \ s^2} \ .$$

   Your measurements are the following, Figure 1:

- x_s: the station position (in km);

- z_s: the station elevation (in km);

- Δg_z: the vertical component of the gravitational acceleration (in km/s^2);

- σ_{g_z}: the measurement standard deviation (in km/s^2).

1. Find the model **m** representing the density anomaly relative to the background at all positions in the subsurface by solving a regularized least-squares INVERSE PROBLEM. Explain what INVERSE PROBLEM you solve and describe all variables involved. Make assumptions whenever necessary, but motivate your choices.

2. Plot the density anomaly map and the predicted gravity for three different values of the regularization parameter and discuss your results.

3. Compute the model posterior covariance $\widetilde{\mathbf{C}}_M$. Plot the posterior standard deviation as a function of space and explain its physical meaning.

4. Find the optimal value of the regularization parameter and explain the procedure you used for this.

   Attach all your codes written for this assignment.

N.B. This is an individual assignment – your work is subject to the Mines Student Honor Code.
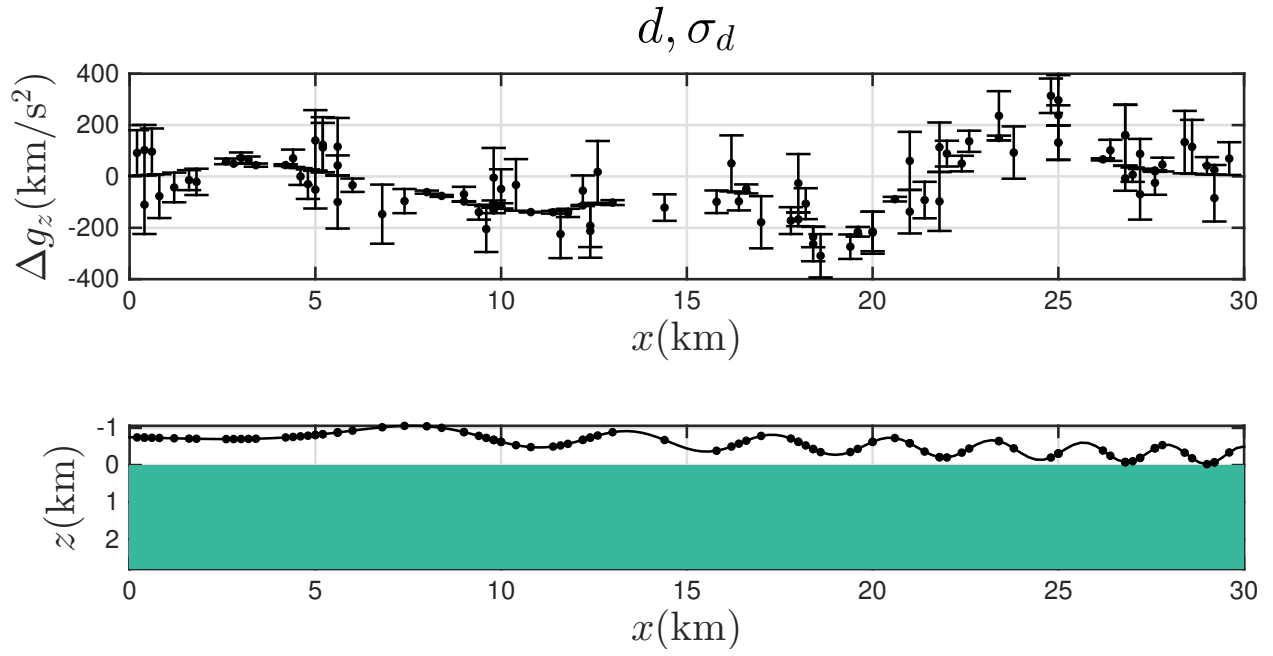
Figure 1: Corrected gravity data and associated uncertainty (top) and station coordinates (bottom).

xs	zs	dgz	sgz
0.2000	−0.7450	91.6149	177.0045
0.4000	−0.7400	−109.8304	228.5318
0.4000	−0.7400	102.4833	196.0956
0.6000	−0.7350	96.4567	180.5820
0.8000	−0.7301	−76.8139	170.4626
1.2000	−0.7207	−42.8302	115.5886
1.6000	−0.7122	−14.3638	77.8009
1.8000	−0.7086	−21.1879	102.5125
2.6000	−0.7005	58.7151	22.0641
2.8000	−0.7008	49.8647	0.3202
3.0000	−0.7022	72.0595	42.6201
3.2000	−0.7051	64.2218	26.6235
3.4000	−0.7094	44.0058	12.5333
4.2000	−0.7443	44.1902	6.6924
4.4000	−0.7578	70.7426	67.6066
4.6000	−0.7734	−0.2744	65.5082
4.8000	−0.7909	−30.0440	115.1165
5.0000	−0.8103	140.1506	236.0817
5.0000	−0.8103	−51.3577	146.9349
5.2000	−0.8314	123.5599	214.4174
5.2000	−0.8314	112.5457	192.3889
5.6000	−0.8778	−99.3200	206.9998
5.6000	−0.8778	115.9771	223.5944
5.6000	−0.8778	43.0191	77.6783
6.0000	−0.9273	−34.2293	52.0822
6.8000	−1.0179	−146.8102	230.1202
7.4000	−1.0556	−95.9845	94.0983
8.0000	−1.0445	−61.0387	11.9213
8.4000	−1.0027	−75.7924	7.4952
9.0000	−0.8885	−97.3661	1.7208
9.0000	−0.8885	−69.3348	57.7835

9.4000	−0.7864	−139.1599	58.2157
9.6000	−0.7318	−204.9499	178.6478
9.8000	−0.6772	−131.9487	22.1480
9.8000	−0.6772	−4.9525	231.8445
9.8000	−0.6772	−107.2201	27.3092
10.0000	−0.6250	−115.0564	21.2656
10.0000	−0.6250	−48.8740	153.6304
10.4000	−0.5365	−32.9985	200.8538
10.8000	−0.4839	−138.9012	2.8777
11.4000	−0.4982	−139.3968	10.4665
11.6000	−0.5292	−224.3244	187.5782
11.8000	−0.5715	−141.9524	32.1202
12.2000	−0.6798	−112.7885	3.7382
12.2000	−0.6798	−55.3769	118.5613
12.4000	−0.7389	−212.5816	207.8212
12.4000	−0.7389	−191.8740	166.4060
12.6000	−0.7957	17.5856	240.7968
13.0000	−0.8847	−101.8182	19.0652
14.4000	−0.6744	−121.1908	103.1434
15.8000	−0.3862	−98.6289	88.0629
16.2000	−0.4986	50.7715	218.2180
16.4000	−0.5764	−97.0370	70.4649
16.6000	−0.6568	−48.5650	35.0454
17.0000	−0.7847	−178.4157	203.0752
17.8000	−0.7140	−172.0145	104.4296
18.0000	−0.6273	−167.0238	53.3301
18.0000	−0.6273	−26.5933	227.5307
18.2000	−0.5288	−106.0412	120.4267
18.4000	−0.4317	−263.1027	134.4978
18.4000	−0.4317	−235.7457	79.7838
18.6000	−0.3496	−308.9705	168.4813
19.4000	−0.3512	−273.6306	95.2476
19.6000	−0.4347	−215.4867	37.7495
20.0000	−0.6250	−219.0592	163.9017
20.0000	−0.6250	−214.1573	154.0979
20.6000	−0.7285	−88.4447	17.9811
21.0000	−0.5885	60.5889	226.0560
21.0000	−0.5885	−137.2087	169.5392
21.4000	−0.3638	−91.9470	142.0725
21.8000	−0.2127	113.8176	192.6086
21.8000	−0.2127	−97.3351	229.6967
22.0000	−0.2055	88.8955	100.2893
22.4000	−0.3355	50.1684	60.7585
22.6000	−0.4449	136.9497	82.3942
23.4000	−0.6457	236.1828	191.5629
23.4000	−0.6457	149.6583	18.5139
23.8000	−0.4513	92.9966	203.5862
24.8000	−0.2036	314.1761	134.2139
25.0000	−0.3103	132.3884	133.7054
25.0000	−0.3103	296.9166	195.3511
25.0000	−0.3103	131.5522	135.3778
25.0000	−0.3103	238.1073	77.7324
26.2000	−0.3913	66.5743	8.2660
26.4000	−0.2509	101.7217	82.0770
26.8000	−0.0811	−7.0687	97.2888
26.8000	−0.0811	160.0161	236.8807
26.8000	−0.0811	160.7553	238.3591

27.0000	−0.1022	6.7711	55.7255
27.2000	−0.1932	−69.3299	197.5241
27.2000	−0.1932	87.7542	116.6442
27.6000	−0.4544	19.9645	4.2210
27.6000	−0.4544	−24.6292	93.4084
27.6000	−0.4544	23.3076	2.4652
27.8000	−0.5376	46.1960	53.8779
28.4000	−0.3348	133.7601	242.8951
28.6000	−0.1838	115.3055	210.0059
29.0000	−0.0253	41.3579	67.8251
29.2000	−0.0728	25.6243	37.8430
29.2000	−0.0728	−84.3866	182.1789
29.6000	−0.3392	69.5805	127.4367