Gravity inversion

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force of gravity

$$\mathbf{F} = G\frac{mM}{r^2}\frac{\mathbf{r}}{r}$$

absolute gravitational acceleration

$$\mathbf{g} = G\ M\ \frac{\mathbf{r}}{r^3}$$

relative gravitational acceleration

$$\Delta\mathbf{g} = G\ \Delta M\ \frac{\mathbf{r}}{r^3}$$

$$\begin{aligned}\Delta g_z &= G\ \Delta M\ \frac{r_z}{r^3} \\ &= G\int\int \Delta\rho\frac{r_z}{r^3}\left(2y_{max}\right)dxdz\end{aligned}$$

the operator

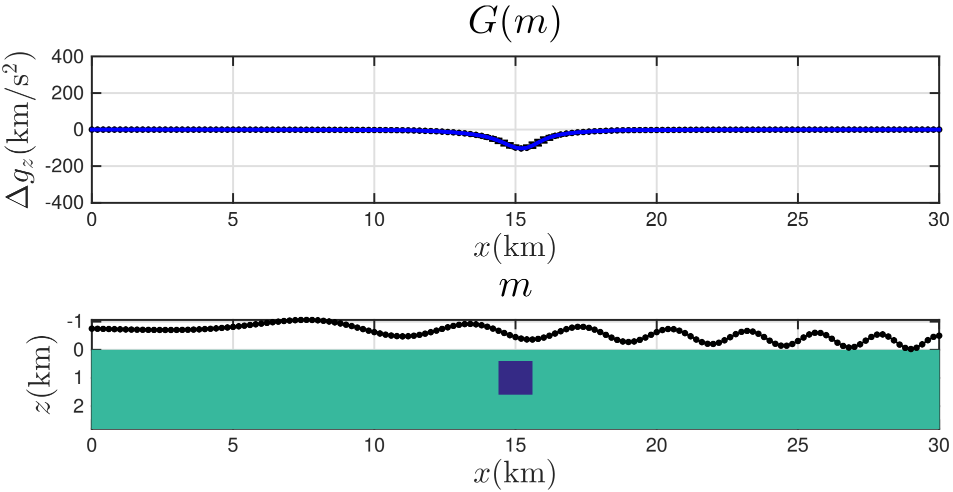
$$\Delta g_z^i = G\left(2y_{max}\right)\sum_{j=1}^{M}\Delta\rho^j \frac{\left(z^i - z^j\right)}{\left[\left(x^i - x^j\right)^2 + \left(z^i - z^j\right)^2\right]^{3/2}}\Delta x \Delta z$$

$i = 1 \ldots N$ *data index*

$j = 1 \ldots M$ *model index*

the topography

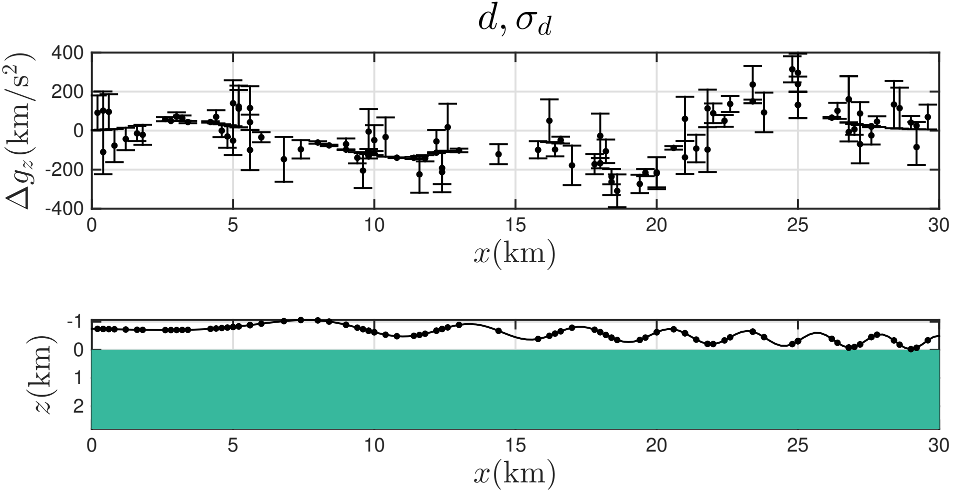
$$z = -1 + \frac{x}{40} + \frac{1}{4}\cos\left(\frac{\pi}{2}\frac{x^2}{x_{max} - x_{min}}\right)$$



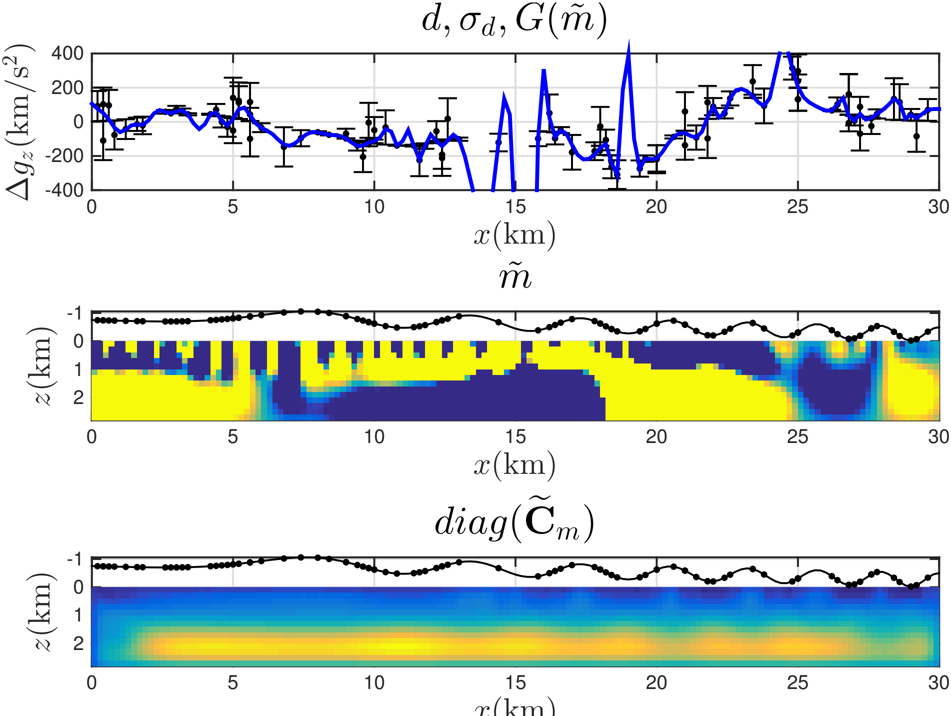
$$\Delta\rho = -100kg/km^3$$

$$y_{max} = 10km$$

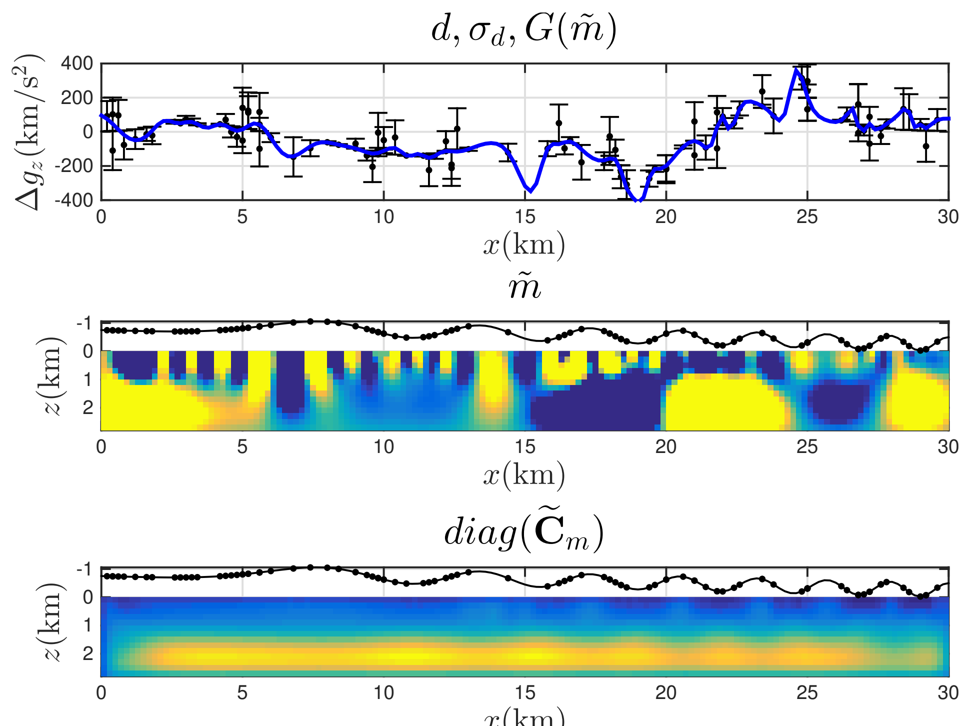
the data


d, σ_d
400
200
0
-200
-400
$\Delta g_z(\mathrm{km/s^2})$
0
5
10
15
20
25
30
$x(\mathrm{km})$
-1
0
1
2
$z(\mathrm{km})$
0
5
10
15
20
25
30
$x(\mathrm{km})$

inversion



$d, \sigma_d, G(\tilde{m})$
$\Delta g_z(\mathrm{km/s^2})$
400
200
0
-200
-400
0
5
10
15
20
25
30
$x(\mathrm{km})$
$\tilde{m}$
$z(\mathrm{km})$
-1
0
1
2
$x(\mathrm{km})$
$diag(\widetilde{\mathbf{C}}_m)$
$z(\mathrm{km})$
$x(\mathrm{km})$



$d, \sigma_d, G(\tilde{m})$
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400
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-400
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20
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$z(\mathrm{km})$
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$z(\mathrm{km})$
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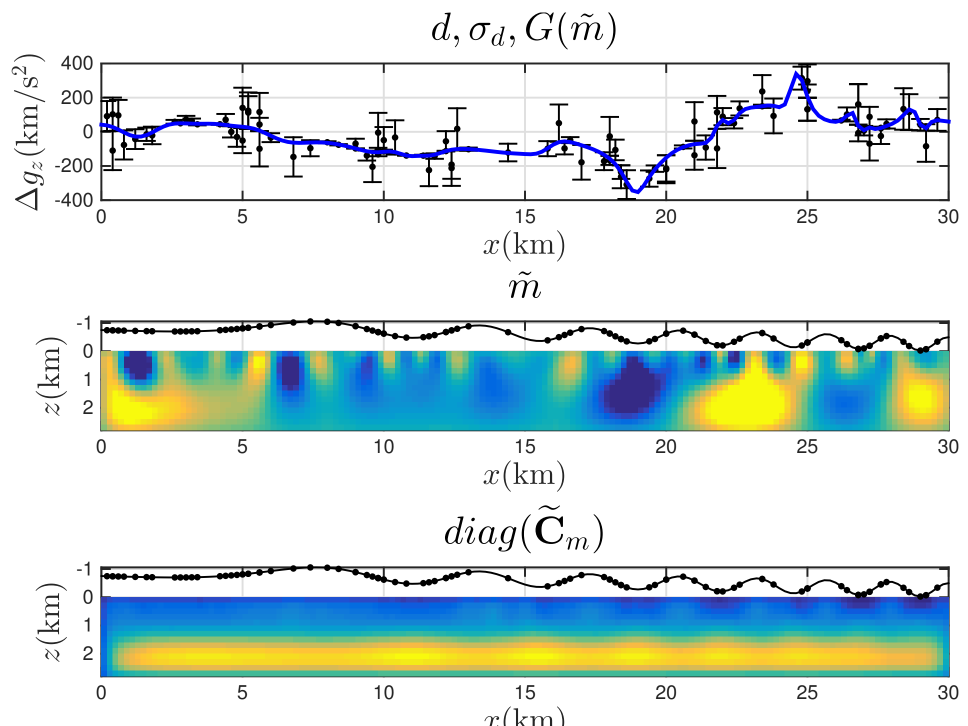
$d, \sigma_d, G(\tilde{m})$
$\Delta g_z(\mathrm{km/s^2})$
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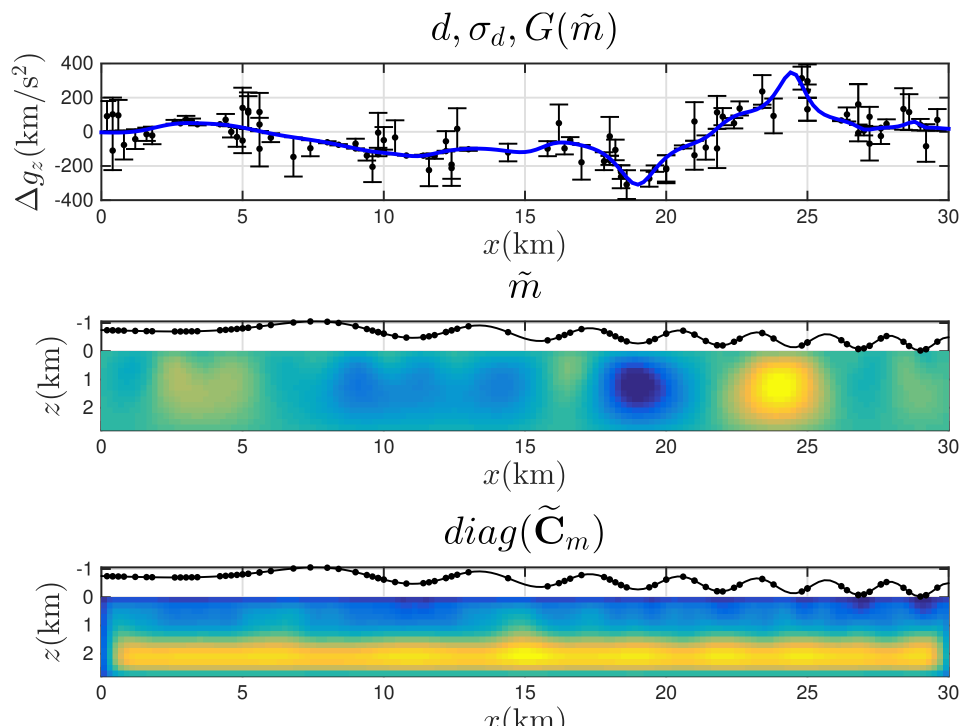
$x(\mathrm{km})$


$d, \sigma_d, G(\tilde{m})$
$\Delta g_z (\mathrm{km/s^2})$
$x(\mathrm{km})$
$\tilde{m}$
$z(\mathrm{km})$
$x(\mathrm{km})$
$diag(\widetilde{\mathbf{C}}_m)$
$z(\mathrm{km})$
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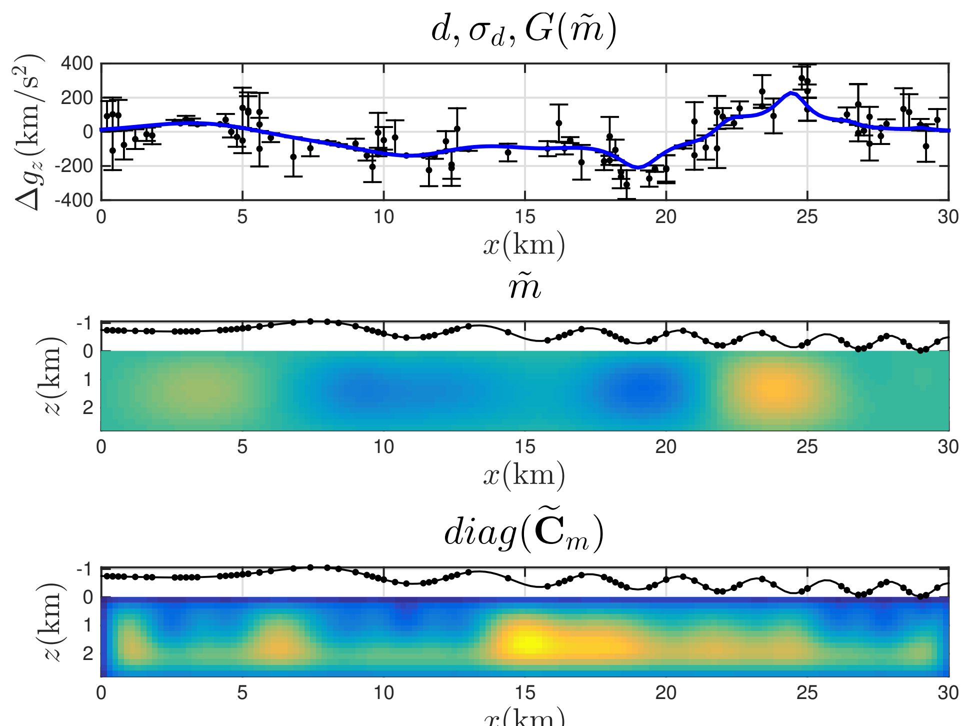

$d, \sigma_d, G(\tilde{m})$
$\Delta g_z(\mathrm{km/s^2})$
400
200
0
-200
-400
0 5 10 15 20 25 30
$x(\mathrm{km})$
$\tilde{m}$
$z(\mathrm{km})$
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1
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$x(\mathrm{km})$
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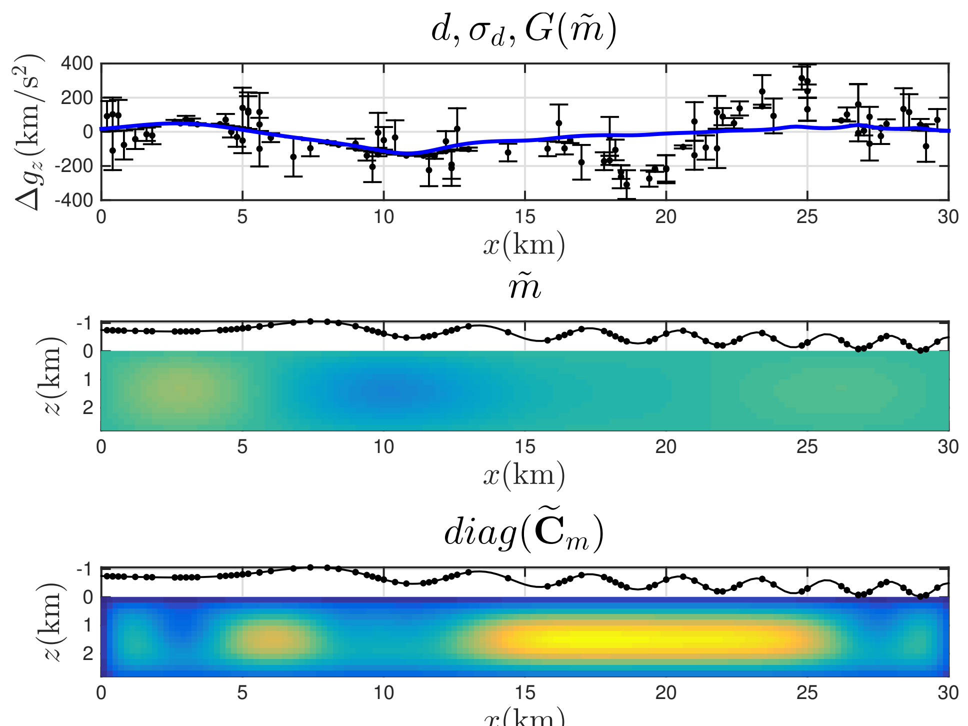

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$diag(\widetilde{\mathbf{C}}_m)$
$z(\mathrm{km})$
$x(\mathrm{km})$