You are investigating the lithology in a carbonate formation using several well logs measured at $0.5\ ft$ intervals, Figure 3:

- GR: Gamma ray log – can be used to evaluate the amount of shale mixed with the carbonates;
- CALI: Caliper log – can be used to characterize the condition of the borehole (normally this should be constant $6\ in$);
- ϕ: Porosity log – an indirect mearurement of the rock porosity;
- ρ: Density log - an indirect measurement of the rock density;

Your goal is to use all these data to construct a well of the P-wave velocity in the borehole. You know that the velocity is related to the porosity using the Wyllie equation

$$\frac{1}{v} = \frac{1-\phi}{v_M} + \frac{\phi}{v_F}\ , \tag{1}$$

where $v_M = 6.64$ km/s and $v_F = 1.5$ km/s are the matrix and fluid velocities, respectively, and that the density is linearly related to porosity:

$$\rho = (1-\phi)\,\rho_M + \phi\rho_F\ , \tag{2}$$

where $\rho_M = 2.71$ g/cm^3 and $\rho_F = 1.0$ g/cm^3 are the matrix and fluid densities, respectively. Your goal is to use Bayesian inversion to find at every depth level the distribution characterizing the P-wave velocity.

1. What are the model and data parameters?
2. Construct the prior joint probability density, Figures 1(a)-1(b), based on the observed values of ϕ and ρ and for a P-wave velocity related to the GR log by the relation

   $$v_P = 5.654 - 0.008\,GR\ , \tag{3}$$

   Specify what distribution you are using and justify your choice of parameters defining your chosen distributions. Assume that a-priori all variables are independent.
3. Construct the theoretical joint probability density assuming uncertainty relative to the theoretical prediction, Figures 1(c)-1(d). Specify what distribution you are using and justify your choice of parameters defining your chosen distributions.
4. Construct the posterior joint probability density based on the prior and theoretical PDFs, Figures 1(e)-1(f).
5. Compare the model prior and posterior PDFs and explain the observed differences, Figure 2.
6. Repeat this exercise at all depth levels and plot the P-wave velocity distributions as a function of depth, Figure 3.
7. Modify one assumption about the prior joint probability density and redo the inversion. Compare your results for the different assumptions made.
8. Repeat the inversion assuming that the logs have some correlation from one depth level to another.

N.B. This is an individual assignment – your work is subject to the Mines Student Honor Code.

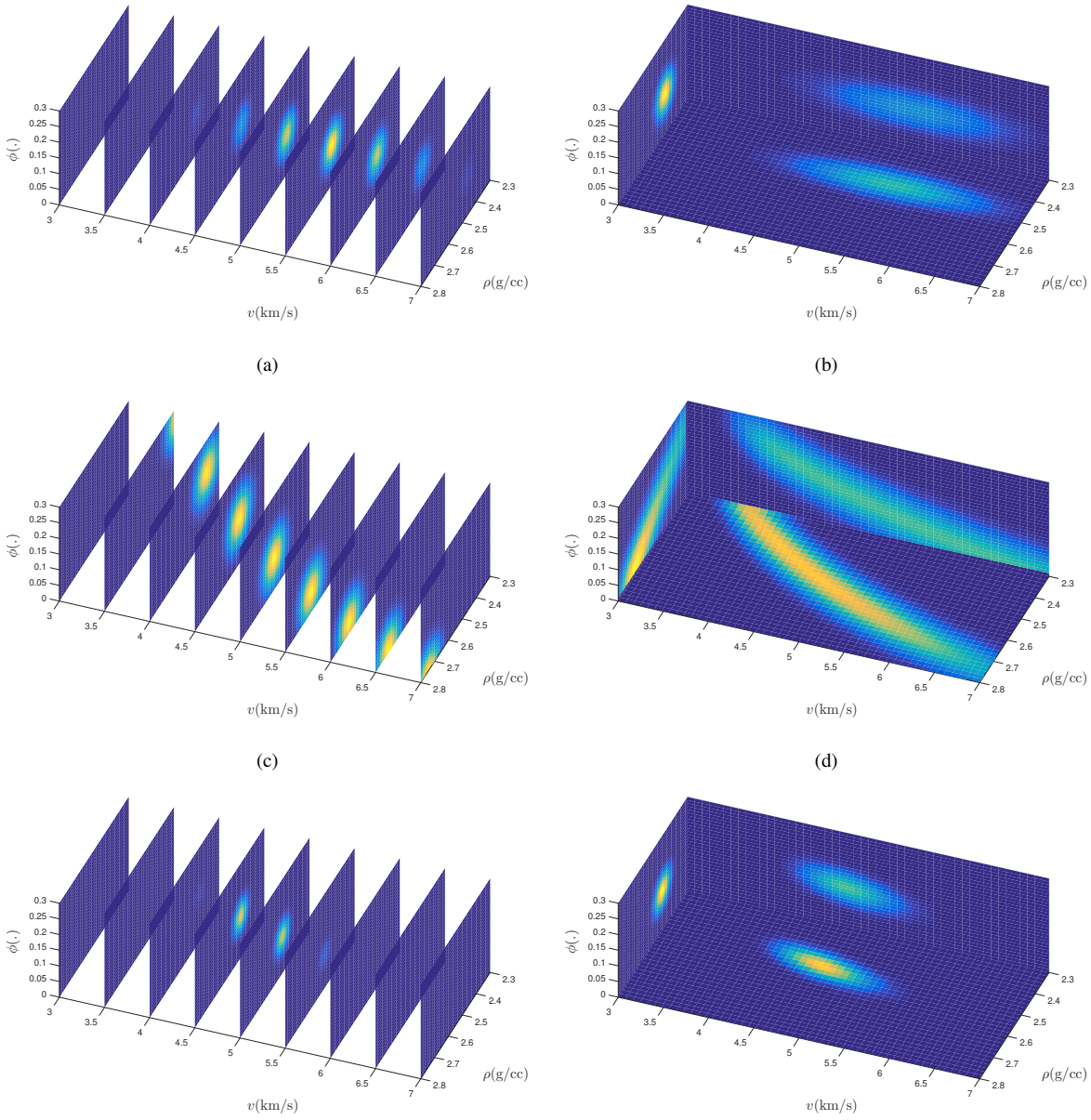

Figure 1: Joint (a)-(c)-(d) and marginal (b)-(d)-(f) PDFs: prior (top), theory (middle) and posterior (bottom).

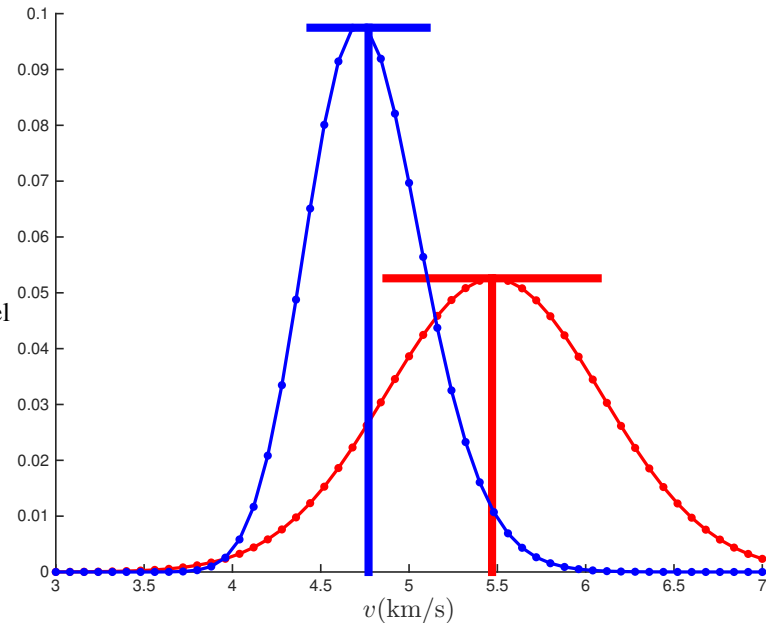

Figure 2: Prior (red) and posterior (blue) model PDFs.

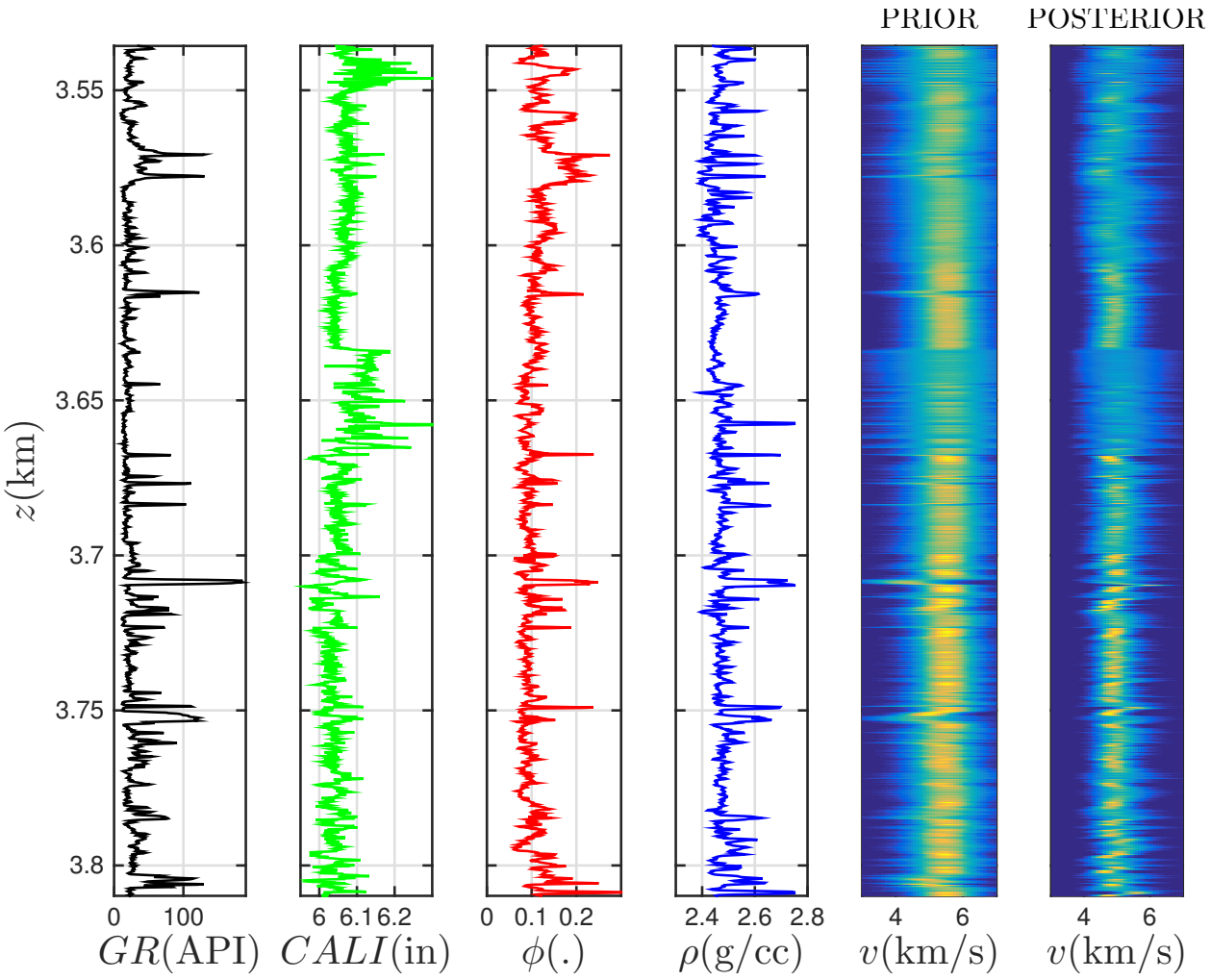

Figure 3: Well logs and the prior and posterior velocity PDFs.