Radially Averaged Power Spectra and Source Depth GPGN 411/511

LAB 4

October 29, 2015

Lab Due: Nov 12, 2015

1 Objectives

- Examine numerically calculated radially averaged power spectra and gain basic understanding of their behaviour
- Perform a simple exercise to calculate the source depth from the slope of the radial power spectrum on a semilog plot

2 Background

Radially averaged power spectra of magnetic or gravity data decays exponentially as

$$P_r(\omega_r) \alpha e^{-2h\omega_r} \tag{1}$$

in the intermediate to high wavenumber bands. When plotted on a semilog (y axis) scale, the power spectrum appears as a straight line with a slope of $-2h$ where h is the depth to the source. Given a set of data, the radial power spectrum can be calculated numerically through the Fast Fourier Transform (FFT). It then provides a quick and effective way to estimate the general depth of the sources.
A simple way of performing the depth calculation would be to identify segments of straight lines and compute their slope. A more rigorous approach would involve an ensemble fitting using a nonlinear least squares approach. This lab focuses on the former.

2.1 Concepts

- Radially averaged power spectrum
- Ensemble fitting
- Source depth

2.2 Tasks

- Using lab 3 or whatever means you prefer, construct a code to compute the 2D FFT and ultimately the power spectrum and radially averaged power spectrum for data in a similar format to Lab 3.
  - Import data

  - Perform FFT (keep all issues such as folding in mind)
  - Estimate the power spectrum by squaring the modulus of the FFT result
  - Calculate the raw radial power spectrum (radially averaged power spectrum) at the grid nodes given by ω_x and ω_y
  - Perform a smoothing of the radial power spectrum by integrating over constant intervals of your choice

- Apply this code to a dataset from lab one or two (your choice), and estimate the depth to the source from the slope.
- Apply your code to the two supplied datasets and compare the depths. Plot the original data as well (this will illuminate the differences)

2.3 Radial power spectrum

In order to compute the radial power spectrum, follow these general guidelines. First, compute the power spectrum. Then, using the ω_x and ω_y meshes, compute a new grid, ω_r. Then, vectorize your power spectrum– that is, 'unwrap' your grids into a vector. Thus you'll have a single column of w_r and a single column of power. This is similar to how you output your forward model from Labs 1 and 2 for lab 3, except instead of x and y you have ω_r. Then sort both vectors by ω_r. (Hint: construct a 4096x2 matrix where the first column is ω_r and the second is the power spectrum. Then use Matlab's sort function.)

2.4 Smoothing

Because we're dealing with the power spectrum, we need to keep the area under the curve unchanged. In order to smooth the power spectrum, we will interpolate the data onto a fine grid, and then implement trapezoidal integration. First, generate a fine vector of wavenumbers, and use one of Matlab's interpolators to construct an interpolated power spectrum on a fine, regular interval. In order to avoid oscillations, use a linear or quadratic interpolator only. Then, decide on a larger interval. Integrate over this interval using the trapezoidal rule in each of these intervals and divide by the interval length. Each of these larger intervals should contain at least several of the interpolated sub-intervals you used in the previous step. The result will be an averaged point, centered in the larger interval, that will serve as your estimate of the radial power spectrum at that wavenumber.

3 Deliverables

Submit a lab report containing

- Your code (if you used code that you didn't write, please detail where you got it)
- A plot of at least one of the power spectra before and after smoothing, and commentary on why smoothing is necessary
- Estimates of the source depth from labs 1 or 2, and associated commentary on how the power spectrum does or does not differ from what you expected. Include all relevant plots of data and power spectra.
- The power spectra, data plots, and depth estimates for the two included datasets
- Discuss the major difference between the two provided datasets
- Any other insights