A Discrete Büchi Automata Distance for Formal Methods Based Control

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Outline

1. Problem and Motivation
   - Formal Methods Based Control
   - The Product Automaton
   - Execution: Path Finding
2. Our Contribution
   - Büchi Distance and Algorithm
3. Performance on Common Formulas
   - Reachability While Avoiding Regions
   - Sequencing
   - Coverage
   - Recurrence (Liveness)
4. More Complex Formulas
   - Study of Various Formulas
5. Conclusions

Outline

Linear Temporal Logic

- We will be using **Linear Temporal Logic (LTL)**, defined recursively as $\varphi ::= \top|\alpha|\neg\varphi_1|\varphi_1 \vee \varphi_2|\mathbf{X}\varphi_1|\varphi_1\boldsymbol{\mathcal{U}}\varphi_2$

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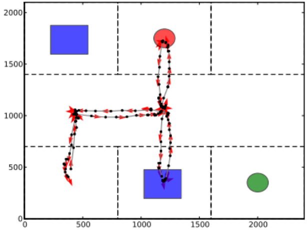
Ex. from [Guo15a]

$\varphi = \diamond(\text{rball} \wedge \diamond\text{basket}) \wedge \diamond\square\text{r1}$

"Eventually pick up the red ball and put it in one of the baskets. Then go home to r1"



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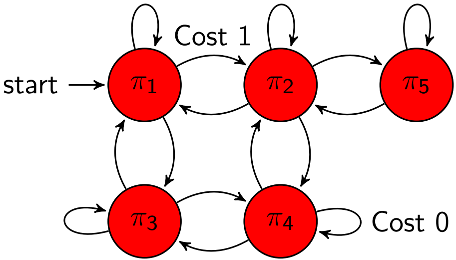
Finite-State Transition System (FTS)

An FTS is a tuple $\mathcal{T} = (\Pi, \rightarrow, \Pi_0, AP, L_D)$ where Π is the set of states, $\rightarrow \subseteq \Pi \times \Pi$ is the transitions, $\Pi_0 \subseteq \Pi$ is the initial state(s), AP is the set of atomic propositions, and $L : \Pi \rightarrow 2^{AP}$ is the labelling function (goes from a state to the set of atomic propositions that are true in that state).



Büchi Automaton

Büchi Automaton

A Büchi automaton is a tuple $\mathcal{A}_\varphi = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$ where $\mathcal{Q}$ is a finite set of states, $\mathcal{Q}_0 \subseteq \mathcal{Q}$ is the set of initial states, 2^{AP} is the alphabet, $\delta : \mathcal{Q} \times 2^{AP} \to 2^{\mathcal{Q}}$ is a transition relation, and $\mathcal{F} \subseteq \mathcal{Q}$ is the set of accepting states.

- A path on a Büchi automaton is accepting if it passes through an accepting state infinitely many times.
- For any LTL formula φ over AP, there exists a Büchi automaton over 2^{AP} corresponding to φ [BKL08]

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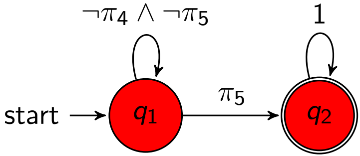
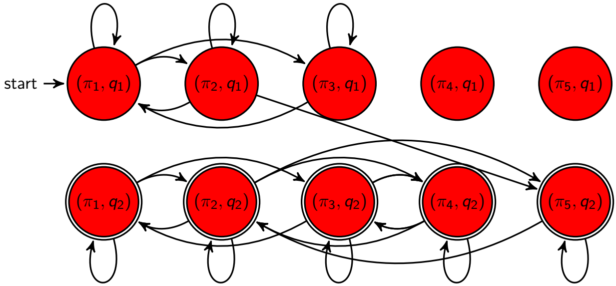
Reachability while avoiding regions $\varphi = \neg\pi_4 \boldsymbol{\mathcal{U}} \pi_5$



Product Automaton

$\mathcal{A}_p = \mathcal{T}_w \otimes \mathcal{A}_\varphi = (Q', \delta', Q'_0, \mathcal{F}')$, where
$Q' = \Pi \times Q = \{\langle \pi, q \rangle \in Q' | \forall \pi \in \Pi, \;\; \forall q \in Q\}$; $\delta' : Q' \rightarrow 2^{Q'}$.
$\langle \pi_j, q_n \rangle \in \delta'(\langle \pi_i, q_m \rangle)$ iff $(\pi_i, \pi_j) \in \rightarrow_c$ and $q_n \in \delta(q_m, L_d(\pi_j))$;
$Q'_0 = \{\langle \pi, q \rangle | \pi \in \Pi_0, \;\; q_0 \in Q_0\}$, $\mathcal{F}' = \{\langle \pi, q \rangle | \pi \in \Pi, q \in \mathcal{F}\}$

- Also a Büchi automaton



Accepted Algorithm

- We prefer a prefix, suffix structure
  $R = \langle R_{pre}, R_{suf} \rangle = q'_0 q'_1 \dots q'_f [q'_{f+1} q'_{f+2} \dots q'_n q'_f]^\omega$

Here is the algorithm currently used in the literature [Guo15a],[FGKGP09],[KB08],[STBR10]

Procedure 1 OptRun() [Guo15a]

Input: Input $\mathcal{A}_p$

Output: R_{opt}

1. For initial state $q'_0 \in \mathcal{Q}'_0$, find the optimal path to each $q'_f \in \mathcal{F}$.
2. For each accepting state $q'_f \in \mathcal{F}'$, calculate the optimal path back to q'_f.
3. Find the pair of $(q'_{0,opt}, q'_{f,opt})$ that minimizes the total cost
4. Optimal accepting run R_{opt}, prefix: shortest path from q'_{0*} to q_{f*}; suffix: the shortest cycle from q'_{f*} and back to itself.

State-Space Explosion Problem

- State-space explosion problem is the combinatorial explosion of the number of states in the product automaton.
- Number of states in Büchi automaton can be exponential in the size of the LTL formula [GL02] and $|\mathcal{Q}'| = |\Pi| \cdot |\mathcal{Q}|$
- State-space explosion problem is the bottle neck of formal methods based control synthesis.

Outline

Büchi Distance Measure

- We introduce a discrete Büchi distance measure, which is the least number of transitions possible to get to an accepting state

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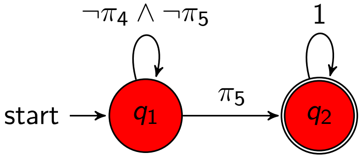
Ex. Reachability While Avoiding Regions $\neg\pi_4 \mathcal{U} \pi_5$



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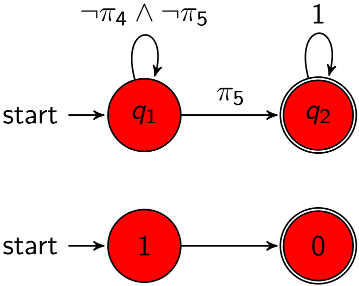


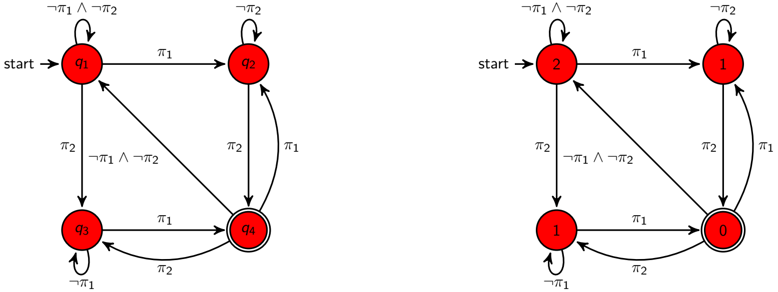
- Denoted $d_p : \mathcal{Q} \to \mathbb{Z}$, e.g. $d_p(q_2) = 1$

Note: Transitions with && are removed from LTL2BA because regions do not overlap

Büchi Distance Measure Cont.

Büchi Automaton with Distance

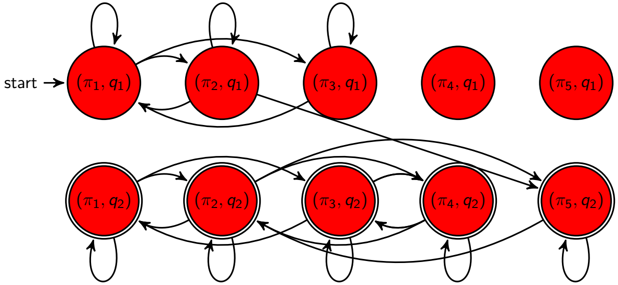
A Büchi automaton with distance $\mathcal{A}_{\varphi,d} = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F}, d)$ where $\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F}$ are defined as before and $d : \mathcal{Q} \to \mathbb{Z}$ is defined as $d(q_n) = \min_i\{|q_i \in \mathcal{F}$ and $q_k \in \delta(q_{k-1}, S_{k-1})$ for some $S_k \in 2^{AP}$ and $k = 0, 1, \ldots, i-1\}$



Büchi Distance in Product Automaton

Product Automaton with Distance

Product automaton with distance, $\mathcal{A}_{p,d} = \mathcal{T} \otimes \mathcal{A}_\varphi = (Q', \delta', Q'_0, \mathcal{F}', d_p)$, defined similarly, with $d_p(q') = d(q'|\mathcal{Q})$



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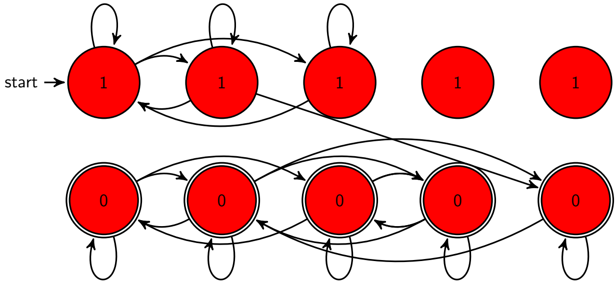


Greedy Algorithm

- Motivation for our algorithm: $\mathcal{F}' = \{\langle \pi, q \rangle | \pi \in \Pi, q \in \mathcal{F}\}$
- We greedily find the optimal path which reduces the Büchi distance at each step

Procedure 2 GreedyRun()

Input: Input $\mathcal{A}_{p,d}$
Output: R_g

1: LEVEL $= d_p(q'_0 \in \mathcal{Q}'_0)$
2: **while** LEVEL > 0 **do**
3: find optimal path down to q'_n s.t. $d_p(q'_n) == LEVEL - 1$
4: Level = Level - 1
5: Find optimal path from q'_n back to itself
6: Accepting run R_g, prefix: the optimal paths calculated in the while loop concatenated together; suffix: optimal path from q'_n back to itself.

Why?

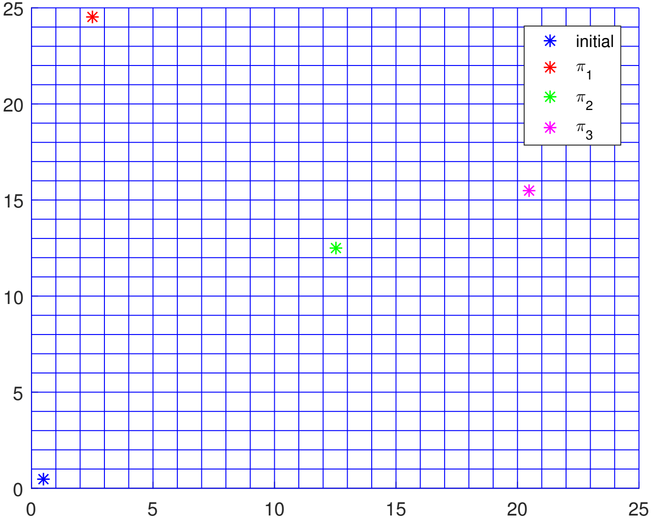
- We approximate the globally optimal path with a series of locally optimal paths.
- We sacrifice a degree of optimality for easier computation!

Code

- The code for the accepted algorithm is from the P_MAS_TG GitHub Repository [Guo15b].
- The code for the greedy algorithm is a modified version of code from P_MAS_TG.
- All computations were done on a 2.5 GHz MacBook Pro and used Python 2.7.5.

Outline

Workspace



Reachability While Avoiding Regions

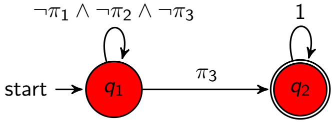


Figure: Büchi automaton corresponding to $\neg(\pi_1 \vee \pi_2)\mathcal{U}\pi_3$

$$d_p(q_1) = 1 \text{ and } d_p(q_2) = 0$$

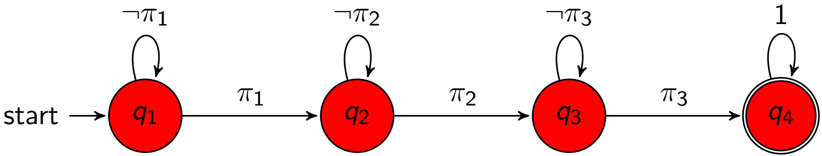
```
Accepted Algorithm
=====================
plan done within 0.02s: precost 37.00, sufcost 0.00
...
full construction and synthesis done within 0.11s

Greedy Algorithm
=====================
plan done within 0.01s: precost 37.00, sufcost 0.00
...
full construction and synthesis done within 0.10s
```

Animation

Sequencing

- We look at the sequencing formula $\diamond(\pi_1 \wedge \diamond(\pi_2 \wedge \diamond\pi_3))$



- $d_p(q_1) = 3,\ d_p(q_2) = 2,\ d_p(q_3) = 1,\ d_p(q_4) = 0$
- Only one path down $\rightarrow$ Both algorithms calculate the same path!

Simulation

```
Accepted Algorithm
==================
plan done within 0.04s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.19s
```

Our algorithm computed the same path, with an output of

```
Greedy Algorithm
==================
plan done within 0.02s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.17s
```

Nodes Searched

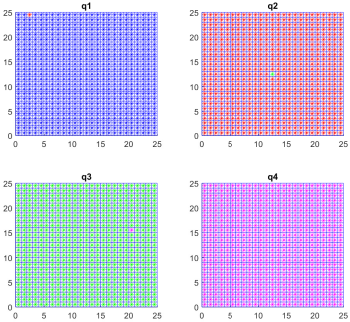


Figure: Nodes with accepted algorithm

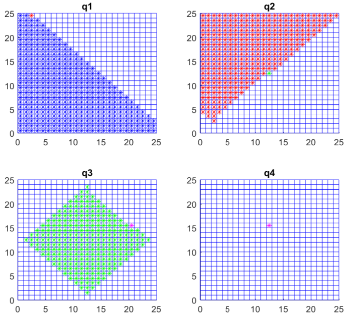


Figure: Nodes with greedy algorithm

Coverage

- $\varphi = \diamond\pi_1 \wedge \diamond\pi_2 \wedge \cdots \wedge \diamond\pi_n$.
- A coverage formula represents the statement visit $\pi_1, \pi_2, \ldots, \pi_n$ in any order. Ex. $\varphi = \diamond\pi_1 \wedge \diamond\pi_2 \wedge \diamond\pi_3$

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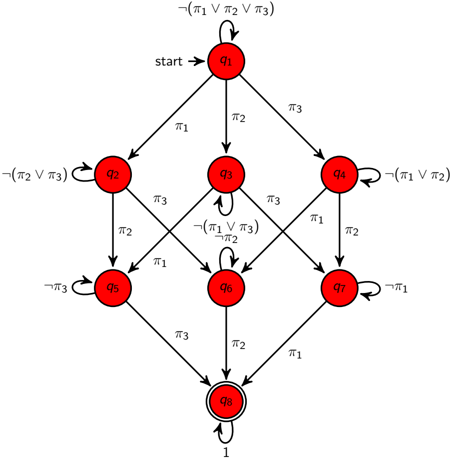

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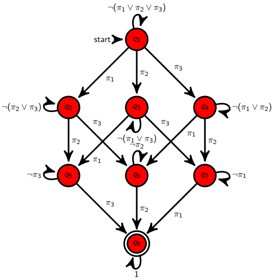

- Now we have "choices" that have to be made!
- Choices → we lose optimality. The path our algorithm finds will likely cost more than the path calculated by the accepted algorithm. However our path is computed faster.

Coverage

```
Accepted Algorithm
==================
plan done within 0.08s: precost 59.00, sufcost 0.00
...
full construction and synthesis done within 0.43s
```

and our algorithm is

```
Greedy Algorithm
================
plan done within 0.02s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.38s
```

Cost Bound

- The travelling salesperson problem: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?"

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Greedy Algorithm Cost Bound

$$\text{GREEDY} + 2\max_{i,j} c_{i,j} \leq \left(\frac{1}{2}\lceil \log(n) \rceil + \frac{1}{2}\right)(\text{ACCEPT} + 2\max_{i,j} c_{i,j})$$

Recurrence (Liveness)

- Recurrence: "Visit π_1, π_2, ..., π_n infinitely many times."

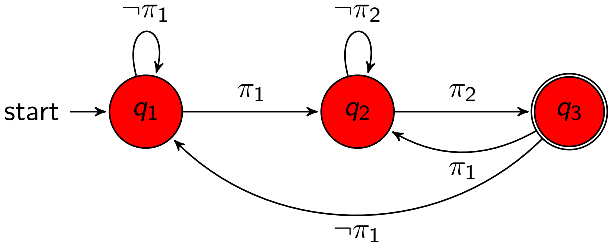

Figure: Büchi Automaton for $\Box(\Diamond\pi_1 \land \Diamond\pi_2)$ 1

- Automaton from LTL2BA. Note: Not tight.

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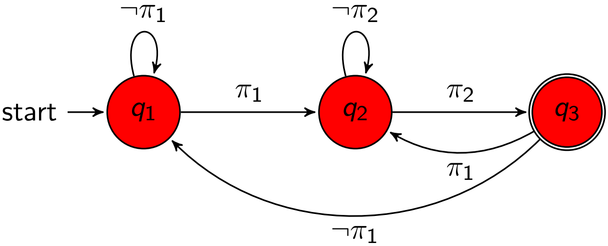

Figure: Büchi Automaton for $\Box(\Diamond\pi_1 \wedge \Diamond\pi_2)$ 1

- Automaton from LTL2BA. Note: Not tight.
- For the first time we have a formula that has a **non-trivial suffix**.

Case Study

- Remember that the accepted algorithm computes the suffix from **every** accepting state. That implies a lot of work for this formula.

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```
Accepted Algorithm
==========================
plan done within 16.17s: precost 62.00, sufcost 60.00
----------------------------------------

...
full construction and synthesis done within 16.35s
```

while our algorithm did it in

```
Greedy Algorithm
==========================
plan done within 0.04s: precost 62.00, sufcost 60.00
----------------------------------------

...
full construction and synthesis done within 0.21s
```

Outline

- More complex formulas prove difficult to analyze because the Büchi automaton becomes very big and cannot be visualized.

Formula	Accepted Cost	Accepted Time	Greedy Cost	Greedy Time
'(!r223 U r445) || (!r268 U r435)'	27	0.04	27	0.01
'!r62 U(!r266 U r422)'	38	0.05	38	0.02
'[]<> r0 − > []<> r317'	1	5.06	1	0.00
'[]<> r0 < − > []<> r317'	1	10.70	1	0.00
'!(<><> r498 < − > r541)'	42	0.03	42	0.02
'!([]<> r3 − > []<>r591)'	3	5.06	3	0.00
'!([]<> r3 < − > []<>r591)'	3	10.31	39	0.01
'!r532 R (!r432 || r321)'	0	4.97	0	0.01
'<> r114 && [](r114 − ><> r12) && ((X r114 U X r12) || !X(r114 U r12))'	24	0.08	24	0.01
'<> pickrball && [](pickrball − ><> droprball) && ((X pickrball U X droprball) || !X(pickrball U droprball))'	47	28.87	47	0.03
' <> r124 && <> !r124'	28	0.05	28	0.01

Table: Comparison of Accepted Algorithm with Greedy Algorithm

Outline

Benefits

- Works very well on reachability while avoiding regions, sequencing, and recurrence (when the automaton is not tight). Guaranteed to get the same path faster!
- Saves a lot of time when the formula does not have a trivial suffix.

Considerations

- Hard to analyze the performance on more complex formulas.
- When there is a trivial suffix, the majority of the time is spent on constructing the graph and the search is usually quick. Future work could be to use this algorithm for on-the-fly construction.

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THANK YOU!

Thanks for listening, any questions? :)