

Thesis

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1 Abstraction of the Workspace

In [1], Belta et al describe robot path planning as consisting of three parts: the specification level, execution level, and the implementation level. The first level, the specification level, involves creating a graph (Büchi automata, which will be defined later) that represents the robot motion. Next is the execution level, which involves finding a path through the graph that satisfies a specification. Lastly, in the implementation level robot controllers are constructed that satisfy the path found in the previous step.

To create a graph that represents the robot motion we need to consider the workspace, denote $W_0 \subset \mathbb{R}^n$, our robot is in and the dynamics of the robot. First we partition our environment, W_0 , into a finite number of equivalence classes. Various partitions can be used i.e. [2] IS THIS TOO SIMILAR TO FAINEKOS2005, however the partition must satisfy the bisimulation property, which will be defined later once more notation has been introduced. We denote the $\Pi = \pi_1, \pi_2, \dots, \pi_w$ to be the set of equivalence classes the workspace has been partitioned into, and thus $\cup_{i=1}^w \pi_i = W_0$ and $\pi_i \cap \pi_j = \emptyset, \forall i, j = 1, 2, \dots, w$ and $i \neq j$.

Definition 1.1. A partition map, $T : W_0 \rightarrow \Pi$ sends each state $x \in W_0$ to the finite set of equivalence classes $\Pi = \pi_i, i = 1, 2, \dots, w$. $T^{-1}(\pi_i)$ is then all the states $x \in W_0$ that are in the equivalence class π_i .

We now introduce atomic propositions, which will be the building blocks for our task specification. Atomic propositions are boolean variables, and will be used to express properties about the state or the robot or the workspace. We define the following set of atomic propositions $AP_r = a_{r,i}, i = 1, 2, \dots, w$ where

$$\alpha_{r,i} = \begin{cases} \top & \text{if the robot is in region } \pi_i \\ \perp & \text{else} \end{cases}$$

which represent the robot's location. Other things we want to be able to express are potential tasks, denote $AP_p = \alpha_{p,i}, i = 1, 2, \dots, m$. These can be ... We now define the set of all propositions $AP = AP_r \cup AP_p$.

Definition 1.2. The labelling function $L_C : W_0 \rightarrow 2^{AP}$ maps a point $x \in W_0$ to the set of atomic propositions satisfied by π_i . From meng

We also include a definition of the discrete counterpart

Definition 1.3. The labelling function $L_D : \Pi \rightarrow 2^{AP}$ maps a region $\pi_i \in \Pi$ to the set of atomic propositions satisfied by π_i . From meng

Note: 2^{AP} is the powerset of AP , i.e. the set of all subsets of AP include the null set and AP .

For example...

Next, we must define transitions to show the relationship between the regions. We define a transition as follows

Definition 1.4. There is a continuous transition, $\rightarrow_C \subset W_0 \times W_0$ from x to x' , denoted $x \rightarrow_C x'$ if it is possible to construct a trajectory $x(t)$ for $0 \leq t \leq T$ with $x(0) = x$ and $x(T) = x'$ and we have $x(t) \in (T^{-1}(T(x)) \cup T^{-1}(T(x')))$ From dynamic robots

Definition 1.5. There is a discrete transition, $\rightarrow_D \subset \Pi \times \Pi$, from π_i to π_j , denoted $\pi_i \rightarrow_D \pi_j$ if there exists x and x' such that $T(x) = \pi_i$, $T(x') = \pi_j$ and $x \rightarrow_C x'$

We can now define bisimulations

Definition 1.6. A partition $T : W_0 \rightarrow \Pi$ is called a bisimulation if the following properties hold for all $x, y \in W_0$

1. (Observation Preserving): If $T(x) = T(y)$, then $L_C(x) = L_C(y)$.
2. (Reachability Preserving): If $T(x) = T(y)$, then if $x \rightarrow_C x'$ then $y \rightarrow_C y'$ for some y' with $T(x') = T(y')$

The Observation Preserving requirement makes sure we do not allow the situation where part of π_i fulfils $\alpha \in AP$ while part of π_i does not, and the Reachability Preserving requirement ensures that for every point in region π_i , there exists a trajectory to some point x' , such that $T(x') = \pi_j$ if $\pi_i \rightarrow_D \pi_j$. These two requirements together guarantee that the discrete path we compute is feasible at the continuous level.

We can now define Finite-State Transition System (FTS), which is how we will represent our workspace.

Definition 1.7. An FTS is a tuple $\mathcal{T}_C = (\Pi, \rightarrow_D, \Pi_0, AP, L_C)$ where Π is the set of states, $\rightarrow_D \subseteq \Pi \times \Pi$ is the transitions relation where $(\pi_i, \pi_j) \in \rightarrow_C$ iff there is a transition from π_i to π_j as defined in definition ???. In adherence to common notation, we will write $\pi_i \rightarrow_C \pi_j$. Note: $\pi_i \rightarrow_C \pi_i$, $\forall 1, 2, \dots w$. $\Pi_0 \subseteq \Pi$ is the initial state(s), $AP = AP_r \cup AP_p$ is the set of atomic propositions, and $L_C : \Pi \rightarrow 2^{AP}$ is the labelling function defined in definition 3.1.

In this thesis, we will also consider the weighted FTS (WFTS)

Definition 1.8. A WTFS is a tuple $\mathcal{T}_C = (\Pi, \rightarrow_C, \Pi_0, AP, L_C, W_C)$ where Π , \rightarrow_C , Π_0 , AP , and L_C are defined as in definition 1.8 and $W_C : \Pi \times \Pi \rightarrow \mathbb{R}^+$ is the weight function i.e. the cost of a transition in \rightarrow_C .

When we use a WFTS, we will estimate the cost of a transition by the Euclidean distance between the centers of the two regions. Note: $W_C(\pi_i, \pi_i) = 0$.

When talking about FTS, it is helpful to use the notation $\text{Pre}(\pi_i) = \{\pi_j \in \Pi | \pi_j \rightarrow_D \pi_i\}$ to define the predecessors of the state π_i and $\text{Post}(\pi_i) = \{\pi_j \in \Pi | \pi_j \rightarrow_D \pi_i\}$ to define the successors of the state π_i . In this thesis, we will only be dealing with infinite paths. An infinite path of \mathcal{T} is an infinite sequence of states $\tau = \pi_1 \pi_2 \dots$ such that $\pi_i \in \Pi_0$ and $\pi_i \in \text{Post}(\pi_{i-1}) \forall i > 0$. The trace of a path is the sequence of sets of atomic propositions that are true in the states along a path i.e. $\text{trace}(\tau) = L_D(\pi_1) L_D(\pi_2) \dots$

2 Linear Temporal Logic (LTL)

To define tasks for our robot we must choose a high level language. Temporal logics are especially suited for defining robot tasks because of their ability to express not only formulas constructed of atomic propositions and standard boolean connectives (conjunction, disjunction, and negation), but also temporal specifications e.g. α is true at some point of time. The particular temporal logic we will be using is known as linear temporal logic (LTL) [3]. LTL formulas are defined over a set of atomic propositions AP according to the following grammar:

$$\varphi ::= \top | \alpha | \neg\varphi_1 | \varphi_1 \vee \varphi_2 | \mathbf{X}\varphi_1 | \varphi_1 \mathcal{U}\varphi_2$$

where \top is the predicate true, $\alpha \in AP$ is an atomic proposition, φ_1 and φ_2 are LTL formulas, \neg and \vee denote the standard Boolean connectives negation and disjunction respectively, \mathbf{X} being the "Next" operator. \mathcal{U} is the temporal operator "Until", with $\varphi_1 \mathcal{U}\varphi_2$ meaning φ_1 is true until φ_2 becomes true. Given these operators, we can define the following additional propositional operators:

$$\begin{aligned} \text{Conjunction: } \varphi_1 \wedge \varphi_2 &= \neg(\neg\varphi_1 \vee \neg\varphi_2) \\ \text{Implication: } \varphi_1 \Rightarrow \varphi_2 &= \neg\varphi_1 \vee \varphi_2 \\ \text{Equivalence: } \varphi_1 \Leftrightarrow \varphi_2 &= (\varphi_1 \Rightarrow \varphi_2) \wedge (\varphi_2 \Rightarrow \varphi_1) \end{aligned}$$

We note quickly that we have the false predicate, $\perp = \neg\top$. We are also able to derive the following additional temporal operators:

$$\begin{aligned} \text{Eventuality: } \diamond\varphi_1 &= \top \mathcal{U}\varphi_1 \\ \text{Always: } \square\varphi_1 &= \neg\diamond\neg\varphi_1 \end{aligned}$$

There is a growing interest in path and mission planning in robots using temporal logic specifications given the easy extension from natural language to temporal logic. We now give examples to illustrate this point. First, the atomic operators generally capture properties of the robot or the environment i.e. "the robot is in region 1", "the ball is in region 2", "the robot is holding a ball". We now give examples of common tasks converted to LTL formulas [5]

1. **Reachability while avoiding regions:** "Go to region π_{n+1} while avoiding regions $\pi_1, \pi_2, \dots, \pi_n$ "

$$\neg(\pi_1 \vee \pi_2 \dots \pi_n) \mathcal{U} \pi_{n+1}$$
2. **Sequencing:** "Visit regions π_1, π_2, π_3 in that order"

$$\diamond(\pi_1 \wedge \diamond(\pi_2 \wedge \diamond\pi_3))$$
3. **Coverage:** "Visit regions $\pi_1, \pi_2, \dots, \pi_n$ in any order"

$$\diamond\pi_1 \wedge \diamond\pi_2 \wedge \dots \wedge \diamond\pi_n$$
4. **Recurrence (Liveness):** "Visit regions π_1, \dots, π_n in any order over and over again"

$$\square(\diamond\pi_1 \wedge \diamond\pi_2 \wedge \dots \wedge \diamond\pi_n)$$

Of course more complicated tasks are also expressible in LTL, and atomic propositions need not only refer to the location of the robot. Here is an example given in [6]: "Pick up the red ball, drop it to one of the baskets and then stay in room one"

$$\diamond(rball \wedge \diamond basket) \wedge \diamond\square r1$$

As it is possible to develop computational interfaces between natural language and temporal logics[7], LTL.

An infinite word over the alphabet 2^{AP} is an infinite sequence $\sigma \in (2^{AP})^\omega$, i.e. $\sigma = S_0 S_1 S_2 \dots$ where $S_k \in 2^{AP}$ for all $k = 1, 2, \dots$ where S_k is the set of atomic propositions that are true at the time stop k .

Definition 2.1. The semantics of LTL are defined as follows:

$$\begin{aligned} (\sigma, k) \models \alpha & \quad \text{if} \quad \alpha \in S_k \\ (\sigma, k) \models \neg \varphi & \quad \text{if} \quad (\sigma, k) \not\models \varphi \\ (\sigma, k) \models \mathbf{X}\varphi & \quad \text{if} \quad (\sigma, k+1) \models \varphi \\ (\sigma, k) \models \varphi_1 \vee \varphi_2 & \quad \text{if} \quad (\sigma, k) \models \varphi_1 \text{ or } (\sigma, k) \models \varphi_2 \\ (\sigma, k) \models \varphi_1 \mathcal{U} \varphi_2 & \quad \text{if} \quad \exists k' \in [k, +\infty], (\sigma, k') \models \varphi_2 \text{ and} \\ & \quad \forall k'' \in (k, k'), (\sigma, k'') \models \varphi_1 \end{aligned}$$

In the interest of the reader, we will denote $(\sigma, 0) \models \varphi$ by $\sigma \models \varphi$, which is what is used for all this thesis. The connection to an infinite path τ of an FTS is that the trace of the path, $\text{trace}(\tau)$, is a word over the alphabet 2^{AP} . Given the LTL semantics, we can now verify if a path satisfies an LTL formula! We will say an infinite path τ satisfies φ if its trace satisfies φ , i.e. $\tau \models \varphi$ if $\text{trace}(\tau) \models \varphi$. A path satisfying φ will be referred to as a plan for φ .

We now describe how to construct a path of an FTS that satisfies an LTL formula.

3 Büchi Automata

Given an LTL formula φ over AP , there exists a Nondeterministic Büchi automaton (NBA) over 2^{AP} corresponding to φ , denoted A_φ according to [1, Theorem 5.37] Lars' paper

Definition 3.1. An NBA \mathcal{A}_φ is defined by a five-tuple:

$$\mathcal{A}_\varphi = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$$

where \mathcal{Q} is a finite set of states, $\mathcal{Q}_0 \subseteq \mathcal{Q}$ is the set of initial states, 2^{AP} is the alphabet, $\delta : \mathcal{Q} \times 2^{AP} \rightarrow 2^{\mathcal{Q}}$ is a transition relation, and $\mathcal{F} \subseteq \mathcal{Q}$ is the set of accepting states

An infinite run of an NBA is an infinite sequence of states that starts from an initial state and follows the transition relation i.e. $r = q_0 q_1 \dots$ where $q_0 \in \mathcal{Q}_0$ and $q_{k+1} \in \delta(q_k, S)$ for some $S \in 2^{AP}$, $k = 0, 1, \dots$. An infinite run r is accepting if $\text{Inf}(r) \cap \mathcal{F} \neq \emptyset$, where $\text{Inf}(r)$ is the set of states that appear infinitely often in r . from meng As with the FTS, we denote the predecessors of $q_m \in \mathcal{Q}$ $\text{Pre}(q_m)$ and the successors $\text{Post}(q_m)$

Definition 3.2. Given an infinite word $\sigma = S_0 S_1 S_2 \dots$ over 2^{AP} , its resulting run in \mathcal{A}_σ is denoted by $r_\sigma = q_0 q_1 q_2 \dots$, which satisfies: (i) $q_0 \in \mathcal{Q}_0$; (ii) $q_{i+1} \in \delta(q_i, S_i)$, $\forall i = 1, 2, \dots$. Similar statement holds for a finite word $\bar{\sigma} = S_0 S_1 S_2 \dots S_{N+1}$.

4 Product Automata

These two structures are then combined to create the product automation. The product automata is also a Büchi automata and is defined as follows:

Definition 4.1. The weighted product Büchi automaton is defined by $\mathcal{A}_p = \mathcal{T} \otimes \mathcal{A}_\varphi = (Q', \delta', Q'_0, \mathcal{F}', W_p)$, where $Q' = \Pi \times Q = \{\langle \pi, q \rangle \in Q' \mid \forall \pi \in \Pi, \forall q \in Q\}$; $\delta: Q' \rightarrow 2^{Q'}$. $\langle \pi_j, q_n \rangle \in \delta'(\langle \pi_i, q_m \rangle)$ iff $(\pi_i, \pi_j) \in \rightarrow_c$ and $q_n \in \delta(q_m, L_c(pi))$; $Q'_0 = \{\langle \pi, q \rangle \mid \pi \in \Pi_0, q_0 \pi Q_0\}$, the set of initial states: $\mathcal{F}' = \{\langle \pi, q \rangle \mid \pi \in \Pi, q \in \mathcal{F}\}$, the set of accepting states; $W_p: Q' \times Q' \rightarrow \mathbb{R}^+$ is the weight function: $W_p(\langle \pi_i, q_m \rangle, \langle \pi_j, q_n \rangle) = W_c(\pi_i, \pi_j)$, where $\langle \pi_j, q_n \rangle \in \delta'(\langle \pi_i, q_m \rangle)$ meng

Given a state $q' = \langle \pi, q \rangle \in Q'$, its projection on Π is denoted by $q'|_\Pi = \pi$ and its projection on Q is denoted by $q'|_Q = q$. Given an infinite run $R = q'_0 q'_1 a'_2 \dots$ of \mathcal{A}_p , its projection on Π is denoted by $R|_\Pi = q'_0|_\Pi q'_1|_\Pi q'_2|_\Pi \dots$ and its projection on Q is denoted by $R|_Q = q'_0|_Q q'_1|_Q q'_2|_Q \dots$ meng

Given that \mathcal{A}_p is a Büchi automaton, the requirements of an accepting run is the same as before i.e. $\text{Inf } \mathcal{A}_p \cap \mathcal{F} \neq \emptyset$

Lemma 4.1. *If there exists an infinite path τ of \mathcal{T}_c such that $\tau \models \varphi$, then at least one accepting run of \mathcal{A}_p exists.*

Proof. see the proof of Theorem from [11] meng

Lemma 4.2. *If R is an accepting run of \mathcal{A}_p , then $R|_\Pi \models \varphi$*

Proof. see proof in meng

Given 4.2, our problem is now to find an accepting run of \mathcal{A}_p . Given that an accepting run is a infinite sequence of states, and there are infinitely many possibilities, the process of finding one, nonetheless finding one that has measure of optimality is non-trivial, both theoretically and practically meng "both in theory and software implementation". Therefore we restrict our view of accepting runs to runs that satisfy the prefix-suffix structure i.e.

$$\tau = \langle \tau_{pre}, \tau_{suf} \rangle = \tau_{pre} [\tau_{suf}]^\omega$$

The prefix τ_{pre} is traversed only once and the suffix τ_{suf} is repeated infinitely often meng (which is the meaning of the ω superscript). Plans of this form are much easier to deal with because, while they are still infinite plans, they have a finite representation that we can exploit.

5 Cost of a Run

We are focusing on the accepting runs of \mathcal{A}_p with the prefix-suffix structure

$$\begin{aligned} R &= \langle R_{pre}, R_{suf} \rangle = q_0 q_1 \dots q_f [q_f q_{f+1} \dots q_n]^\omega \\ &= \langle \pi_0, q_0 \rangle \dots \langle \pi_{f-1}, q_{f-1} \rangle [\langle \pi_f, q_f \rangle \langle \pi_f, q_f \rangle \dots \langle \pi_n, q_n \rangle]^\omega \end{aligned}$$

from meng where $q_0 = \langle \pi_0, q_0 \rangle \in \mathcal{Q}_0$ and $q_f = \langle \pi_f, q_f \rangle \in \mathcal{F}$. There is a finite set of transitions in our infinite path i.e.

$$\text{Edge}(R) = \{(q_i, q_{i+1}), i = 0, 1, \dots, (n-1)\} \cup \{(q_n, q_f)\}.$$

Each of these transitions has a cost, given by $W_p(q_i, q_{i+1})$ allowing us to define the total cost of R as

$$\begin{aligned} \text{Cost}(R, \mathcal{A}_p) &= \sum_{i=0}^{f-1} W_p(q_i, q_{i+1}) + \gamma \sum_{i=f}^{n-1} W_p(q_i, q_{i+1}) \\ &= \sum_{i=0}^{f-1} W_c(\pi_i, \pi_{i+1}) + \gamma \sum_{i=f}^{n-1} W_c(\pi_i, \pi_{i+1}) \end{aligned}$$

where $\gamma \geq 0$ is the relative weighting of the transient response (prefix) cost and steady response (suffix) cost. We then look for the accepting run with prefix-suffix structure that minimizes the total cost.

We will denote this accepting run as R_{opt} , with the corresponding plan $\tau_{opt} = R_{opt}|_{\Pi}$. We note however that this plan may not actually be the true optimal plan with prefix-suffix structure. In [8] we see that simplifications in the translation from LTL formulas to NBA can result in a loss of optimality. This will be important when we analyse the paths our algorithm generates.

6 Search Algorithm

The search algorithm used in many recent works on the specific type of control planning synthesis comes from this prefix-suffix structure. The basic idea is to find a path from an initial node, q_0 to an accepting node, q_f , and then find a path from the q_f back to itself. The first part from q_0 to q_f is the prefix and the second part q_f back to q_f is the suffix. Then the resulting path, τ , will be the prefix, followed by the suffix repeated infinitely many times. This path is thus accepting because the suffix finds the path from an initial state back to itself, and thus contains the initial state, and is repeated infinitely many times $q_f \in \text{Inf} \tau \Rightarrow \text{Inf} \tau \cap \mathcal{F} \neq \emptyset$.

Algorithm 1, from [6], gives pseudocode of how to compute R_{opt} .

Procedure 1 OptRun()

Input: Input $\mathcal{A}_p, S' = \mathcal{Q}'_0$ by default

Output: R_{opt}

- 1: If \mathcal{Q}'_0 or \mathcal{F}' is empty, construct \mathcal{Q}'_0 or \mathcal{F}' first.
 - 2: For each initial state $q'_0 \in S'$, call $\text{DijksTargets}(\mathcal{A}_p, q'_0, \mathcal{F}')$.
 - 3: For each accepting state $q'_f \in \mathcal{F}'$, call $\text{DijksCycle}(\mathcal{A}_p, q'_f)$.
 - 4: Find the pair of $(q'_{0,opt}, q'_{f,opt})$ that minimizes the total cost
 - 5: Optimal accepting run R_{opt} , prefix: shortest path from $q'_{0,*}$ to $q'_{f,*}$; suffix: the shortest cycle from $q'_{f,*}$ and back to itself.
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The functions used in algorithm 1 are defined as follows. $\text{DijksTargets}(\mathcal{A}_p, q'_s, \mathcal{F})$ computes the shortest paths in \mathcal{A}_p from initial states $q'_s \in \mathcal{Q}_0$ to every accepting node in \mathcal{F} using Dijkstra's algorithm [4]

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