A Discrete Büchi Automata Distance for Formal Methods Based Control

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Outline

1. Problem and Motivation
   - Formal Methods Based Control
   - The Product Automaton
   - Execution: Path Finding
2. Our Contribution
   - Büchi Distance and Algorithm
3. Performance on Common Formulas
   - Workspace
   - Reachability While Avoiding Regions
   - Sequencing
   - Coverage
   - Recurrence (Liveness)
4. More Complex Formulas
   - Study of Various Formulas
5. Conclusions

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Linear Temporal Logic

- We will be using **Linear Temporal Logic (LTL)**, defined recursively as $\varphi ::= \top|\alpha|\neg\varphi_1|\varphi_1 \vee \varphi_2|\mathbf{X}\varphi_1|\varphi_1\boldsymbol{\mathcal{U}}\varphi_2$

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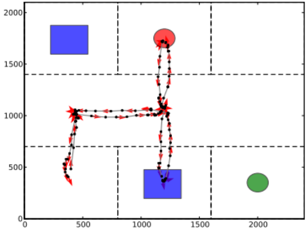
- Why?LTL formulas are versatile; LTL allows us to encode statements about the robot and workspace, and also how events relate to each other in the time domain.
  Ex. from [Guo15a] $\varphi = \diamond(\text{rball} \wedge \diamond\text{basket}) \wedge \diamond\square\text{r1}$
  "Eventually pick up the red ball and put it in one of the baskets. Then go home to r1"



LTL Motion and Task Planning

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- Implementation: Calculate the continuous controllers such that the continuous path will satisfy the discrete path.

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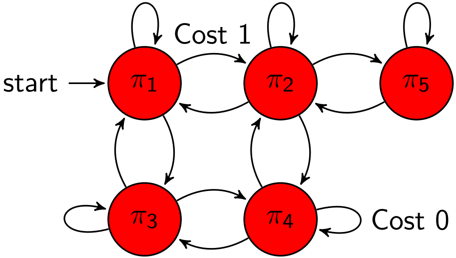
Finite-State Transition System (FTS)

An FTS is a tuple $\mathcal{T} = (\Pi, \rightarrow, \Pi_0, AP, L_D)$ where Π is the set of states, $\rightarrow \subseteq \Pi \times \Pi$ is the transitions, $\Pi_0 \subseteq \Pi$ is the initial state(s), AP is the set of atomic propositions, and $L : \Pi \rightarrow 2^{AP}$ is the labelling function (goes from a state to the set of atomic propositions that are true in that state).



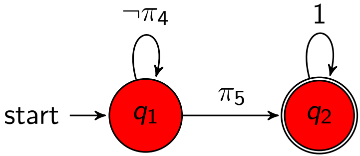
Büchi Automaton

Büchi Automaton

A Büchi automaton is a tuple $\mathcal{A}_\varphi = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$ where $\mathcal{Q}$ is a finite set of states, $\mathcal{Q}_0 \subseteq \mathcal{Q}$ is the set of initial states, 2^{AP} is the alphabet, $\delta : \mathcal{Q} \times 2^{AP} \to 2^{\mathcal{Q}}$ is a transition relation, and $\mathcal{F} \subseteq \mathcal{Q}$ is the set of accepting states.

- A path on a Büchi automaton is accepting if it passes through an accepting state infinitely many times.
- For any LTL formula φ over AP, there exists a Büchi automaton over 2^{AP} corresponding to φ [BKL08]

Reachability while avoiding regions $\varphi = \neg\pi_4 \boldsymbol{\mathcal{U}} \pi_5$

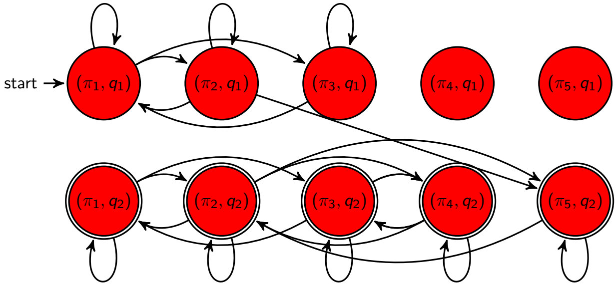


Product Automaton

Product Automaton

$\mathcal{A}_p = \mathcal{T}_w \otimes \mathcal{A}_\varphi = (Q', \delta', Q'_0, \mathcal{F}', W_p)$, where
$Q' = \Pi \times Q = \{\langle \pi, q\rangle \in Q' | \forall \pi \in \Pi, \ \ \forall q \in Q\}$; $\delta' : Q' \to 2^{Q'}$.
$\langle \pi_j, q_n\rangle \in \delta'(\langle \pi_i, q_m\rangle)$ iff $(\pi_i, \pi_j) \in \to_c$ and $q_n \in \delta(q_m, L_d(\pi_j))$;
$Q'_0 = \{\langle \pi, q\rangle | \pi \in \Pi_0, \ \ q_0 \in Q_0\}$, $\mathcal{F}' = \{\langle \pi, q\rangle | \pi \in \Pi, q \in \mathcal{F}\}$

- Also a Büchi automaton



State-Space Explosion Problem

- State-space explosion problem is the is the combinatorial explosion of the number of states in the product automaton.
- Number of states in Büchi automaton can be exponential in the size of the LTL formula [GL02] and $|\mathcal{Q}'| = |\Pi| \cdot |\mathcal{Q}|$
- State-space explosion problem is the bottle neck of formal methods based control synthesis.

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- We prefer a prefix, suffix structure
  $R = \langle R_{pre}, R_{suf} \rangle = q'_0 q'_1 \dots q'_f [q'_f q'_{f+1} \dots q'_n]^\omega$

Here is the algorithm currently used in the literature [Guo15a],[FGKGP09],[KB08],[STBR10]

Procedure 1 OptRun() [Guo15a]

Input: Input $\mathcal{A}_p$

Output: R_{opt}

1: For initial state $q'_0 \in \mathcal{Q}'_0$, find the optimal path to each $q'_f \in \mathcal{F}$.

2: For each accepting state $q'_f \in \mathcal{F}'$, calculate the optimal path back to q'_f.

3: Find the pair of $(q'_{0,opt}, q'_{f,opt})$ that minimizes the total cost

4: Optimal accepting run R_{opt}, prefix: shortest path from q'_{0*} to q_{f*}; suffix: the shortest cycle from q'_{f*} and back to itself.

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Büchi Distance Measure

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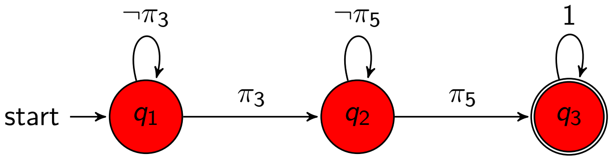
Ex. Sequencing $\diamond(\pi_3 \wedge \diamond\pi_5)$



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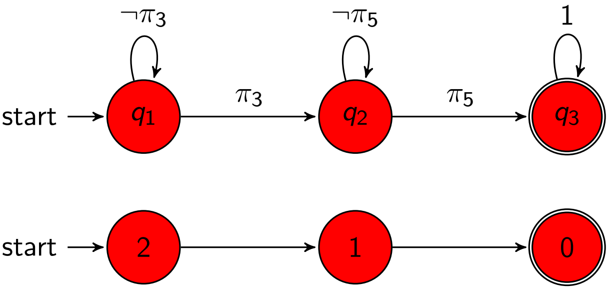

- Denoted $d_p : \mathcal{Q} \to \mathbb{Z}$, e.g. $d_p(q_2) = 1$

Note: Transitions with && are removed from LTL2BA because regions do not overlap

Our Algorithm

- Motivation for our algorithm: $\mathcal{F}' = \{\langle \pi, q\rangle | \pi \in \Pi, q \in \mathcal{F}\}$
- We greedily find the optimal path which reduces the Büchi distance at each step

---

Procedure 2 GreedyRun()

Input: Input $\mathcal{A}_{p,d}$
Output: R_g
1: LEVEL $= d_p(q'_0 \in \mathcal{Q}'_0)$
2: **while** LEVEL > 0 **do**
3: find optimal path down to q'_n s.t. $d_p(q'_n) == LEVEL - 1$
4: Level = Level - 1
5: Find optimal path from q'_n back to itself
6: Accepting run R_g, prefix: the optimal paths calculated in the while loop concatenated together; suffix: optimal path from q'_n back to itself.

Why?

- **We approximate the globally optimal path with a series of locally optimal paths**
- **We sacrifice a degree of optimality for easier computation!**

Code

- The code for the accepted algorithm is from the P_MAS_TG GitHub Repository [Guo15b].
- The code for our algorithm is a modified version of code from P_MAS_TG.
- All computations were done on a 2.5 GHz MacBook Pro and used Python 2.7.5.

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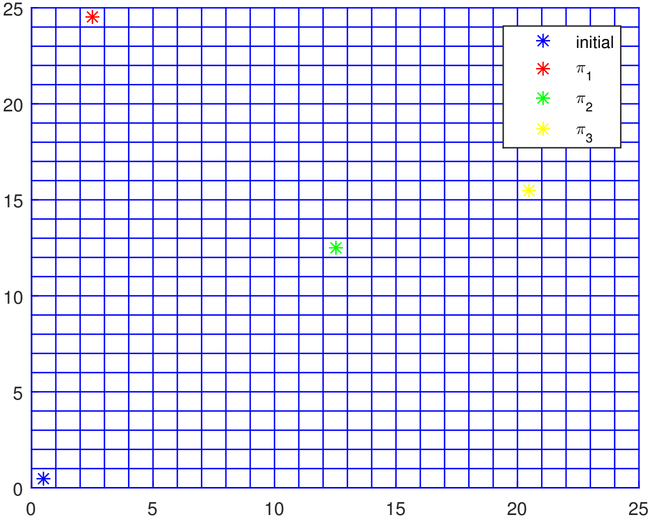
Workspace



Outline

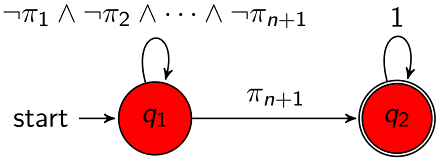


Figure: Büchi automaton corresponding to $\neg(\pi_1 \vee \pi_2 \vee \ldots \pi_n)\mathcal{U}\pi_{n+1}$

$$d_p(q_1) = 1 \text{ and } d_p(q_2) = 0$$

```
Accepted Algorithm
==================
plan done within 0.02s: precost 37.00, sufcost 0.00
...
full construction and synthesis done within 0.11s
```

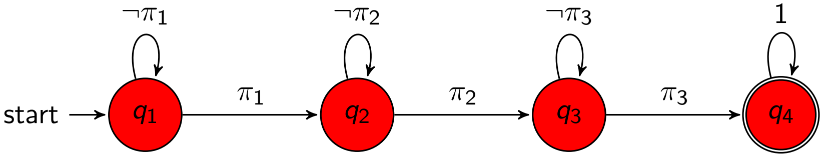
```
Our Algorithm
==================
plan done within 0.01s: precost 37.00, sufcost 0.00
...
full construction and synthesis done within 0.10s
```

Animation

asdf

Outline

- We look at the sequencing formula $\diamond(\pi_1 \wedge \diamond(\pi_2 \wedge \diamond\pi_3))$



- $d_p(q_1) = 3,\ d_p(q_2) = 2,\ d_p(q_3) = 1,\ d_p(q_4) = 0$
- Only one path down $\rightarrow$ Both algorithms calculate the same path!

Simulation

```
Accepted Algorithm
==================
plan done within 0.04s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.19s
```

Our algorithm computed the same path, with an output of

```
Our Algorithm
==================
plan done within 0.02s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.17s
```

Nodes Searched

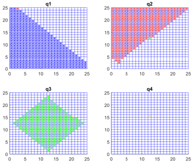


Figure: Nodes searched with our Algorithm

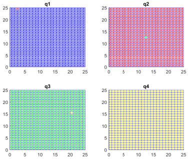


Figure: Nodes searched with the accepted Algorithm

Outline

Coverage

- $\varphi = \Diamond\pi_1 \wedge \Diamond\pi_2 \wedge \cdots \wedge \Diamond\pi_n$.
- A coverage formula represents the statement visit $\pi_1, \pi_2, \ldots, \pi_n$ in any order. Ex. $\varphi = \Diamond\pi_1 \wedge \Diamond\pi_2 \wedge \pi_3$

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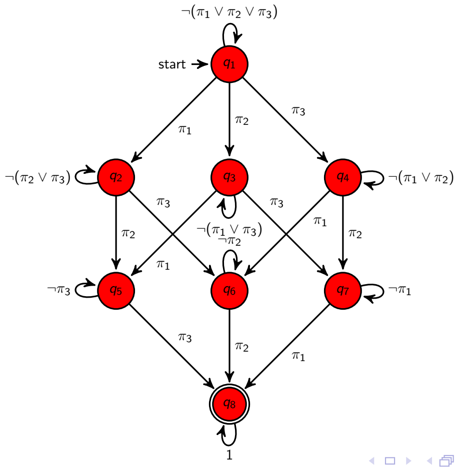
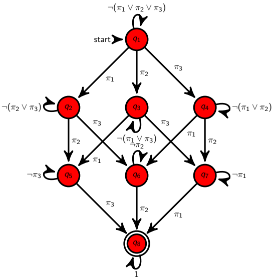


Coverage



- Now we have "choices" that have to be made!
- Choices $\rightarrow$ we lose optimality. The path our algorithm finds will likely cost more than the path calculated by the accepted algorithm. However our path is computed faster.

Coverage

```
Accepted Algorithm
==================
plan done within 0.08s: precost 59.00, sufcost 0.00
...
full construction and synthesis done within 0.43s
```

and our algorithm is

```
Our Algorithm
==================
plan done within 0.02s: precost 62.00, sufcost 0.00
...
full construction and synthesis done within 0.38s
```

Cost Bound

- The travelling salesperson problem: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?"
- It has been shown [RSL74] that for an n-node travelling salesperson problem which satisfies the triangle inequality

$$\text{NEARNEIBR} \leq (\frac{1}{2}\lceil \log(n) \rceil + \frac{1}{2})\text{OPTIMAL}$$

- [LK75] shows how to formulate seeming unrelated problems as travelling salesperson problems by introducing a dummy node.
- We introduce a dummy node * which is $\max_{i,j} c_{i,j}$ where $c_{i,j}$ is the cost from π_i to π_j.
- Then our bound is

$$\text{GREEDY} + 2\max_{i,j} c_{i,j} \leq (\frac{1}{2}\lceil \log(n) \rceil + \frac{1}{2})(\text{ACCEPT} + 2\max_{i,j} c_{i,j})$$

Outline

- Recurrence: "Visit π_1, π_2, ..., π_n infinitely many times."

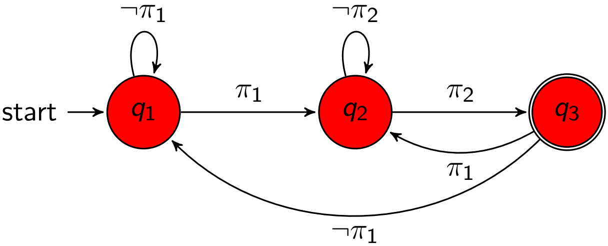


Figure: Büchi Automaton for $\Box(\Diamond\pi_1 \wedge \Diamond\pi_2)$ 1

- Automaton from LTL2BA. Note: Not tight.

Recurrence (Liveness)

- Recurrence: "Visit $\pi_1, \pi_2, \ldots, \pi_n$ infinitely many times."

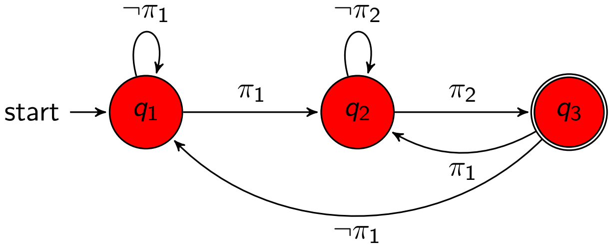


Figure: Büchi Automaton for $\square(\diamond\pi_1 \wedge \diamond\pi_2)$ 1

- Automaton from LTL2BA. Note: Not tight.
- For the first time we have a formula that has a **non-trivial suffix**.

Case Study

- Remember that the accepted algorithm computes the suffix from **every** accepting state. That implies a lot of work for this formula.

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```
Accepted Algorithm
==========================
plan done within 16.17s: precost 62.00, sufcost 60.00
------------------------------------------
...
full construction and synthesis done within 16.35s
```

while our algorithm did it in

```
Our Algorithm
==========================
plan done within 0.04s: precost 62.00, sufcost 60.00
------------------------------------------
...
full construction and synthesis done within 0.21s
```

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Analysis

- More complex formulas prove difficult to analyze because the Büchi automaton becomes very big and cannot be visualized.
- Took formulas from [SB00] to show extensive and non-biased experimentation.

Formula	Accepted Cost prefix, suffix	Accepted Time	Our Cost prefix, suffix	Our Time
'(!r223 U r445) || (!r268 U r435)'	27 ,0	0.04	27,0	0.01
'!r62 U(!r266 U r422)'	38.00 ,0	0.05	38,0	0.02
'[]<> r0 − > []<> r317'	1,0	5.06	1,0	0.00
'[]<> r0 < − > []<> r317'	1,0	10.70	1,0	0.00
'!(<><> r498 < − > r541)'	42.00	0.03	42.00	0.02
'!([]<> r3 − > []<>r591)'	3.00, 0	5.06	3.00, 0	0.00
'!([]<> r3 < − > []<>r591)'	3.00, 0	10.31	39, 0	0.01
'!r532 R (!r432 —— r321)'	0,0	4.97	0,0	0.01
'<> r114 && [](r114 − ><> r12) && ((X r114 U X r12) || !X(r114 U r12))'	24.00	0.08	24.00	0.01
'<> pickrball && [](pickrball − ><> droprball) && ((X pickrball U X droprball) || !X(pickrball U droprball))'	47.00,0	28.87	47.00,0	0.03
' <> r124 && <> !r124'	28.00,0	0.05	28.00,0	0.01

Table: Comparison of Accepted Algorithm with Our Algorithm on Various Examples

Outline

Benefits

- Works very well on reachability while avoiding regions, sequencing, and recurrence (when the automaton is not tight). Guaranteed to get the same path faster!
- Saves a lot of time when the formula does not have a trivial suffix.

Drawbacks

- Hard to analyze the performance on more complex formulas.
- When there is a trivial suffix, the majority of the time is spent on constructing the graph and the search is usually quick. Future work could be to use this algorithm for on-the-fly construction.

References I

Calin Belta, Antonio Bicchi, Magnus Egerstedt, Emilio Frazzoli, Eric Klavins, and George J Pappas, *Symbolic planning and control of robot motion [grand challenges of robotics]*, IEEE Robotics & Automation Magazine **14** (2007), no. 1, 61–70.

Christel Baier, Joost-Pieter Katoen, and Kim Guldstrand Larsen, *Principles of model checking*, MIT press, 2008.

Georgios E Fainekos, Antoine Girard, Hadas Kress-Gazit, and George J Pappas, *Temporal logic motion planning for dynamic robots*, Automatica **45** (2009), no. 2, 343–352.

Dimitra Giannakopoulou and Flavio Lerda, *From states to transitions: Improving translation of ltl formulae to büchi automata*, Formal Techniques for Networked and Distributed SytemsFORTE 2002 (2002), 308–326.

References II

- Meng Guo, *Hybrid control of multi-robot systems under complex temporal tasks*, Ph.D. thesis, KTH Royal Institute of Technology, 2015.
- Meng Guo, *P-mas-tg*, `https://github.com/MengGuo/P_MAS_TG`, 2015.
- Marius Kloetzer and Calin Belta, *A fully automated framework for control of linear systems from temporal logic specifications*, IEEE Transactions on Automatic Control **53** (2008), no. 1, 287–297.
- Jan K Lenstra and AHG Rinnooy Kan, *Some simple applications of the travelling salesman problem*, Journal of the Operational Research Society **26** (1975), no. 4, 717–733.

References III

Daniel J Rosenkrantz, Richard Edwin Stearns, and Philip M Lewis, *Approximate algorithms for the traveling salesperson problem*, Switching and Automata Theory, 1974., IEEE Conference Record of 15th Annual Symposium on, IEEE, 1974, pp. 33–42.

Fabio Somenzi and Roderick Bloem, *Efficient büchi automata from ltl formulae*, International Conference on Computer Aided Verification, Springer, 2000, pp. 248–263.

Stephen L Smith, Jana Tumova, Calin Belta, and Daniela Rus, *Optimal path planning under temporal logic constraints*, Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on, IEEE, 2010, pp. 3288–3293.