Info

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11.4

Laufzeit $:= x$

a) i) $\dfrac{\log_2(64)}{\log_2(32)} = \dfrac{x}{5s} \Leftrightarrow x = 6s$

ii) $\dfrac{64}{32} = \dfrac{x}{5s} \Leftrightarrow x = 10s$

iii) $\dfrac{64\log_2(64)}{32\log_2(32)} = \dfrac{x}{5s} \Leftrightarrow x = 12s$

iv) $\dfrac{64^2}{32^2} = \dfrac{x}{5s} \Leftrightarrow x = 20s$

v) $\cancel{\dfrac{2^{64}}{2^{32}} = \dfrac{x}{5s} \Leftrightarrow x = 2{,}147 \cdot 10^{10} s}$

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b) Vermutung: for $n \to \infty$

$$1 < \log n < \sqrt{n} < n\log n < n^2 < 2^n < n^n$$
(a) (b) (c) (d) (e) (f)

Beweis:

$\boxed{\text{I} := \text{L'HOPITALS REGEL: } L = \lim\limits_{x \to x_0} \dfrac{f(x_0)}{g(x_0)} \Leftrightarrow L = \lim\limits_{x \to x_0} \dfrac{f'(x_0)}{g'(x_0)}}$

(a) $\lim\limits_{n\to\infty} \dfrac{\log n}{1} = \infty$

(b) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{\log n} \overset{\text{I}}{=} \lim\limits_{n\to\infty} \dfrac{\frac{1}{2}\frac{1}{\sqrt{n}}}{\frac{1}{\ln(10)}\cdot\frac{1}{n}} = \lim\limits_{n\to\infty} \dfrac{1}{2\ln(10)} \cdot \sqrt{n} = \infty$

(c) $\lim\limits_{n\to\infty} \dfrac{n\log n}{\sqrt{n}} \overset{\text{I}}{=} \lim\limits_{n\to\infty} \dfrac{n\frac{1}{\ln(10)n} + \log n}{\frac{1}{2\sqrt{n}}} = \lim\limits_{n\to\infty} 2\sqrt{n}\left(\dfrac{1}{10} + \log n\right) = \infty$

(d) $\lim\limits_{n\to\infty} \dfrac{n^2}{n\log n} = \lim\limits_{n\to\infty} \dfrac{n}{\log n} \overset{\text{I}}{=} \lim\limits_{n\to\infty} \dfrac{1}{\frac{1}{\ln(10)n}} = \lim\limits_{n\to\infty} \ln(10)n = \infty$

(e) $\lim\limits_{n\to\infty} \dfrac{2^n}{n^2} \overset{\text{I}}{=} \lim\limits_{n\to\infty} \dfrac{\ln(2)2^n}{2n} \overset{\text{I}}{=} \lim\limits_{n\to\infty} \dfrac{\ln^2(2)2^n}{2} = \infty$

(f) $\lim\limits_{n\to\infty} \dfrac{n^n}{2^n} = \lim\limits_{n\to\infty} \dfrac{\overbrace{n\cdot n\cdot n\cdot \ldots \cdot n}^{n}}{\underbrace{2\cdot 2\cdot 2\cdot 2 \cdot \ldots \cdot 2}_{n}} = \infty$

$n^n = 2^n$ for $n = 2$ und $n^n > 2^n$ für $n \geq 3$

$2^2 = 2^2$ ✓

$3^3 = 27 > 8 = 2^3$ ✓