Garry Huang

Professor Stacey Suver

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Usages of the Satisfiability Reduction to Show NP-Completeness in Modern Problems

The satisfiability problem (SAT), describes a problem in which a boolean expression can or cannot be satisfied. SAT is the first problem proven to be NP-complete. NP-complete refers to the set of problems in which no algorithm exists that may solve the given problem in polynomial time. One way of proving NP-completeness in other problems is to reduce the SAT problem into the specified problem. This also leads into the problem of $p = np$, which states that there is a way to solve all non-polynomial algorithms in polynomial time. If such a proof existed, technology as a whole would be severely impacted. However, the goal of this proposal is not to prove $p = np$, but to find other modern problems that are NP-complete using the reduction from SAT. By showing that a problem is NP-complete, we can use those reductions on even more problems and in doing so, simplify reduction proofs for computer scientists working on algorithmic problems.

In the research article, "The NP-Completeness of Edge-Coloring," by Ian Holyer, describes the problem and the reduction proof from 3SAT, a type of SAT problem, into the 3-edge coloring problem. Holyer defines the problem where a graph is presented and the goal is to find out how many colors are required in order to color all the edges of the graph such that no two adjacent edges may have the same color. Holyer's proof of reduction showed that edge-coloring is in fact NP-complete. Holyer's proof would later be used by many other researchers in order to show NP-completeness. The next step in researching reductions from SAT to modern problems would be to prove more reductions. As a result, the research being proposed is in a sense, a continuation or extension of Holyer's work since all NP-complete problems may be derived using SAT.

In the research article, "Sudoku as a SAT Problem" by Inês Lynce and Joël Ouaknine, describe how to reduce SAT into a sudoku puzzle. The researchers then took a SAT problem and translated the problem such that for every literal inside the given SAT problem represents one tile on the Sudoku board and every clause represents one of the rules of sudoku. Through the usage of many clauses and many literals, the researchers were able to prove that sudoku is a SAT problem. Using their research, we can provide a basis for our reduction. This basis will involve using many literals and clauses to represent our problem and prove the reduction of our problem.

The first step in proving a reduction from SAT to a given problem, is first developing a well-thought problem. Inspiration for this problem can be derived from the previous research articles mentioned previously. For this research proposal, the problem will be working to prove whether or not the sock-sorting problem is NP-complete. This problem is described as, given *n* amount of socks, where each sock has only one other matching sock, put all of the matching socks together. The reduction from SAT to the sock-sorting problem is also very similar to the proof as described by "Sudoku as a SAT problem." In order to start the proof, we must find a way to translate a SAT problem such that it becomes the sock-sorting problem. It is likely that for every literal in the SAT problem, will relate to a pair of socks and whether or not those socks are matching. Every clause in this problem would relate to the rules regarding matching the socks. Although the process of which proving NP-completeness using a reduction from SAT is very similar to previous proofs, the goal is not to redefine how we prove NP-completeness but to further extend the problems by which we can reduce from. Another important note is that this problem follows the property of $p = np$, which states if SAT were proven to be polynomial, then the sock-sorting problem could also be solved in polynomial time. This problem is very flexible and can be easily changed to fit the constraints of other problems that researchers may want to prove NP-completeness in the future.

Although this research will not solve the million dollar question, of $p = np$, this proof of this problem will extend the list of problems that would be applicable to $p = np$. Not only that, but the proof of NP-completeness in the sock-sorting problem will also lead to more reductions in the future. This research will hopefully impact how we prove NP-completeness of problems and reveal different viewpoints in proving reductions from SAT.

Works Cited

Holyer, Ian. "SIAM J. Comput." *SIAM Journal on Computing*. Society for Industrial and Applied Mathetmatics, 13 July 2006. Web. 8 April 2017.

Lynce, Inês Lynce, and Joël Ouaknine. "Sudoku as a SAT Problem." (2006): n. pag. *Google Scholar*. Web. 8 April 2017.