

To fit the regression model, we choose a model form  $y = f(x; w) - e$ , where  $y$  is the label,  $f$  is the model function,  $x$  are the features,  $w$  are the model weights, and  $e$  is the error. The minus sign ( - ) is chosen for the error term so that a positive error implies the estimate overshoots the actual value, and a negative error indicates an estimate that undercuts the actual value. This allows for a nicely human-interpretable metric.

Constructing an objective function for tuning parameters, consider the error terms by solving for them in the model equation. This gives  $e = f(x; w) - y$ , and using the errors to construct a convex loss function leads to using  $L_2 = \sum_i e_i^2$  or  $L_2 = \sum_i (f(x_i; w) - y_i)^2$ .

In the case of a linear regression, we choose a model where  $f$  is an affine function of the features, and the coefficients are given by the weights. This looks like  $f = b + w_1 x_1 + \dots + w_n x_n$  for a model with  $n$  features, where  $b$  is the bias and the  $w$  are the feature weights. In the notation above, this can be represented by treating  $b$  as a special additional weight in the  $w$  vector, which then gives  $f(x; w) = w_0 + \langle w_*, x \rangle$ .

In this formulation,  $x$  is an  $n$ -dimensional vector of features, and  $w$  is an  $n+1$  dimensional vector of parameters, where  $w_*$  represents the  $n$ -dimensional parameters without the bias term, thus constituting the feature coefficients. The notation  $\langle a, b \rangle$  represents the inner product, which for our purposes means the same as adding the pairwise products of each element in the vector. This is the same as saying  $\langle a, b \rangle = \sum_i a_i b_i$ .