Practical Session 1: Path planning with A*

1 Introduction

Path planning is concerned with the problem of moving an entity from an initial configuration to a goal configuration. The resulting route may include intermediate tasks and assignments that must be completed before the entity reaches the goal configuration. Path planning algorithms can be classified as either global or local. Global path planning takes into account all the information in the environment when finding a route from the initial position to the final goal configuration. Local planning algorithms are designed to avoid obstacles within a close vicinity of the entity; therefore, only information about nearby obstacles is used. This project will refer to global path planning algorithms where the entire path is generated from start to finish before the entity makes its first move.

Typically, planning operations require searching through (a subset of) all the possible routes to find the most optimal or efficient route among all possibilities. For instance, in a robotics application that must move a robot between five locations in the shortest amount of time, the path planner must perform a search to determine in what order to complete these tasks. Conventional AI search algorithms in conjunction with graph theory are used to accomplish this task. The graph nodes are placed at choice points and edges connect each node in the graph. A choice point is anywhere in the graph where a decision must be made as to where to go next. The search begins at the choice point placed at the initial location, or node, and ends when all choice points have been reached. Exhaustive, simplistic search algorithms such as breadth-first and depth-first suffer from memory issues or suboptimal results. In practice, more advanced approaches, incorporating heuristic assumptions tend to find optimal solutions within a reasonable amount of time and better memory requirements.

A popular example of this is the A* algorithm. This algorithm keeps track of the current cheapest path from the start node to an arbitrary node n by $g(n)$. Furthermore, it uses a heuristic function $h(n)$ to estimate the distance from a node n to the goal node. As a consequence, the estimated cost of the cheapest solution through node n will be given by:

$$f(n) = g(n) + h(n). \tag{1}$$

Along any path from the start node, f will always be non-decreasing provided that the heuristic function is *consistent* (i.e. its estimate is always less than or equal to the estimated distance from any neighboring vertex to the goal, plus the cost of reaching that neighbor). Most of the time, we only pay attention that the heuristic is *admissible* (i.e. it never overestimates the actual cost to get to the goal node), even though this is a weaker claim – all consistent heuristics are admissible, while the converse is not generally true. However, most "natural" heuristics that are admissible are also consistent. Assuming consistency, it can be proven that the first solution found must be

the optimal one, because all subsequent nodes will have a higher f-cost. Furthermore, because it makes the most efficient use of the heuristic function, no search that uses the same heuristic function $h(n)$ and finds optimal paths will expand fewer nodes than A*. An easy heuristic for $h(n)$ is the straight-line (Euclidean) distance between n and the goal. This distance is always an underestimation of the real path between n and the goal.

2 Assignment

In this practical session, we will apply the A* algorithm on a real-world New York traffic network data set. You can download this data and additional Python source files via Ufora under "Content/B. Practicals/Practicals 1/a-star.zip". All the code is in one file, a_star.ipynb. Downloading the data you will be working on is already implemented in this notebook, but we have also included these files in the zip. The easiest option is to upload and run your notebook file in Google Colab: https://colab.research.google.com.

2.1 Loading the data

Load the provided distance graph and coordinate data sets (in files USA-road-d.NY.gr and USA-road-d.NY.co, respectively) into a graph object and an array, using the readGraph and readCoordinates functions from graph.py.

Question 1. Verify the number of vertices (or nodes) and edges are contained within the graph using the Graph and Vertex class functions. You can check your outcome at the top of the USA-road-d.NY.gr text file. Find out which unit of measurement is used in each file.

2.2 A* implementation

Implement the A* algorithm in the a_star_search function, which will take a graph object, a start and goal node as input, and return an optimal path (i.e. a path with minimal travel distance) from the start to the goal node. The pseudocode for this algorithm is as follows:

```
function cameFrom = A*(start,goal)
    closed = {}
    open = {start}
    cameFrom = {}

    g = map with default value of ∞
    g(start) = 0
    f = map with default value of ∞
    f(start) = g(start) + h(start, goal)

    while open is not empty
        current = the node in open having the lowest f value
        if current = goal
            return cameFrom

        open.remove(current)
```

```
closed.add(current)
for each neighbor of current
    if neighbor in closed
        continue
    tentative_g_score = g(current) + dist(current,neighbor)
    if neighbor not in open
        open.add(neighbor)
    else if tentative_g_score >= g(neighbor)
        continue

    cameFrom(neighbor) = current
    g(neighbor) = tentative_g_score
    f(neighbor) = g(neighbor) + h(neighbor, goal)
```

Use the Euclidean distance as a heuristic function:

$$d_E(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}. \tag{2}$$

Take into account the use of different units of measurement. Method `angles2centimeters` might be useful. Additionally, the provided priority queue class definition can be useful in the A* implementation. Generate random start and goal nodes as follows:

```
import random
random.seed(gn)
start = random.randint(1, N+1)
goal = random.randint(1, N+1)
```

where `gn` is your groupnumber and `N` is the number of vertices in the graph. Give the shortest route (including the total distance) from `start` to `goal`.

Question 2.   Explain in your own words (in detail) how the A* algorithm works. Give relevant definitions, such as of admissible and consistent heuristic functions. Explain how the choice of heuristic function influences the performance of A*.

Question 3.   Implement the A* algorithm as described above. Did you get the shortest-distance path? You can verify your results in the `distances.txt` file.

2.3   Different heuristics

In order for the A* algorithm to find the optimal solution, the heuristic it uses has to be consistent (therefore also admissible). Each heuristic has its own advantages and disadvantages. In this section, we will implement different heuristic functions. For each, think about whether it returns the **optimal solution**, and take note of the **running time** of your algorithm and explain.

2.3.1 No heuristics

Question 4. What happens if you leave out the heuristic function ($h(\cdot) = 0$)? Is the found solution optimal?

2.3.2 Manhattan distance heuristics

An alternative heuristic function is the Manhattan distance:

$$d_M(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} |x_i - y_i|. \tag{3}$$

This distance function is characterized by the fact that it measures distances along horizontal and vertical lines.

Question 5. Compare your results of both the Euclidean and Manhattan distance for the previously mentioned start and stop node. What are your conclusions and what are the benefits/disadvantages of both?

2.3.3 Non-consistent admissible heuristics

As we have mentioned before, all consistent heuristics are admissible, but not all admissible heuristics are consistent.

Question 6. Give an example of an admissible heuristics that is not consistent. Implement your A* search for the shortest path using this heuristics. Did you get an optimal solution?

2.4 Shortest time path

In most GPS applications, the user prefers the shortest path with the respect to time, rather than distance. The file `USA-road-t.NY.gr` contains the same graph structure as the one we have been working on, however, the weights $w_{i,j}$ are now stating the required transportation time to get from node i to node j.

Question 7. Compute the minimal time path for the same start and goal nodes as in the previous section, using different heuristics (Eucledian, Manhattan, no heuristics). What are your conclusions?

Question 8. Implement and describe what you think is the best heuristic function for calculating the shortest-time path.

2.5 Submission

The project is done in groups of two students. Each group should write a report (2-3 pages), answering the questions. The source file `a_star.ipynb` should print out a list of subsequent nodes corresponding to the requested shortest path (according to the Euclidean heuristics and with respect to minimal distance). Please do not use any additional files.

The report and your .ipynb file should be submitted as a zip-file in your group on Ufora. The file should be named after both authors and specify the project subject (e.g. Frodo Baggins & Samwise Gamgee: a-star-fbaggins-sgamgee.zip). Deadline for the submission is **Friday, November 1, 2019 (23:59)**.