

1.

a)

$$a \equiv b \pmod{n} \Rightarrow n|a - b \Rightarrow n|-1(a - b) \Rightarrow n|b - a \Rightarrow b \equiv a \pmod{n}$$

b)

$$a \equiv b \pmod{n} \text{ and } b \equiv c \pmod{n} \Rightarrow$$

$$n|a - b \text{ and } n|b - c \Rightarrow n|(a - b) + (b - c) \Rightarrow n|a - c \Rightarrow a \equiv c \pmod{n}$$

2.

a) -1082

b) -571

c) 550

3.

a) $x^3 + 1 = (x + 1)(x^2 - x + 1)$ reducible

b) irreducible

c) irreducible

4.

a) 1 b) $x + 1$

5.

$$H(K|C) = H(K) + H(P) - H(C)$$

$$H(X) = - \sum_{i=1}^n p_i \log_2 p_i$$

$$H(K) = H(X)|X = \frac{1}{2}, \frac{1}{4}, \frac{1}{4} = 1.5$$

$$H(K) = H(X)|X = \frac{1}{3}, \frac{1}{6}, \frac{1}{2} = 1.459$$

$$P_C(C_i) = \sum_{\forall e_{k_i}(p_j)=C_i} P(k_i) * P(p_j)$$

$$H(C) = H(X)|X = \frac{7}{24}, \frac{5}{12}, \frac{1}{8}, \frac{1}{6} = 1.851$$

$$H(K) + H(P) - H(C) = 1.108$$