

Engine Dynamics

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1 Problem Description and Data

A long time ago, in a galaxy far far away. That is, during my masters degree circa 2013, we did a dynamometer test of a front wheel drive vehicle to get the torque-speed-power curves of the internal combustion engine. This was the final project of a vehicle dynamics course, and fortunately, I was able to dig out the data from the experiment to show my abilities as dynamical systems and control engineer. For this example, I can only show my abilities to model dynamical systems. So, lets begin with a description of the experiment.

1.1 The Vehicle and the Environment

The test subject was a 2007 Ford Fusion. A front wheel drive vehicle equipped with a four stroke V6 engine that displaces a volume of 3 liters. According to the technical specification, the engine of this vehicle should be capable of producing a maximum 221 metric horse-power, i.e., 162.5 [kW] at 6250 [RPM], and a maximum torque of 278 [Nm] at at 4800 [RPM], sea level, when new. All these power gets to the front wheels via a torque converter automatic gearbox.

After 5 years of use, in an altitude of 2600 [m] above sea level (yes, Bogotá is a city up in the clouds), and a power delivery via the automatic gearbox, the measured torque and calculated power will definitely be way lower. Lets see.

1.2 The data

The testing rig is a four wheel drive 2000 [hp] axle-hub dynamometer, with Dyno-max Pro software acquisition software. This combo can provide temperature, atmospheric pressure and relative humidity measurements to make the test compliant with the SAR J1349 Engine Test Power Code. In an ideal world, we could have taken all those factors into consideration. For this test though, those variables were not available.

The axle-hub dynamometer has an Eddy current absorber able to provide a steady state RPM value according to the provided current, varying the absorber load, this is the input to the system. For the outputs, we have the torque measurement at the axle, and the RPM of the engine.

Again, in an ideal world where we would have a measurement of the output torque throughout the whole range of engine RPM. Unfortunately, this is not the case. When testing, we did not have a proper gas extraction system, the test rig had a single hose for gas extraction, while the vehicle had double exhaust pipes (no worries, the garage at the laboratory had a good ventilation system). Then, for security reasons we had a single run of test. In addition, it was not possible to make an automated test setting up a ladder of RPM values throughout the whole range because we did not have the knowledge to set it up. Then, we opted for a manual variation of the coil current to measure the torque and RPM output at 9 different absorber RPM values for within 2000 and 4000 RPM of the engine. Definitely not an ideal situation because we are not even at the RPM value for max torque. Making this exercise much more interesting.

Let see a picture of the experimental data to get a clearer *picture* of the problem at hand. Figure 1 shows the motor RPM, and the torque measurement at the absorber according to the different RPM of the absorber.

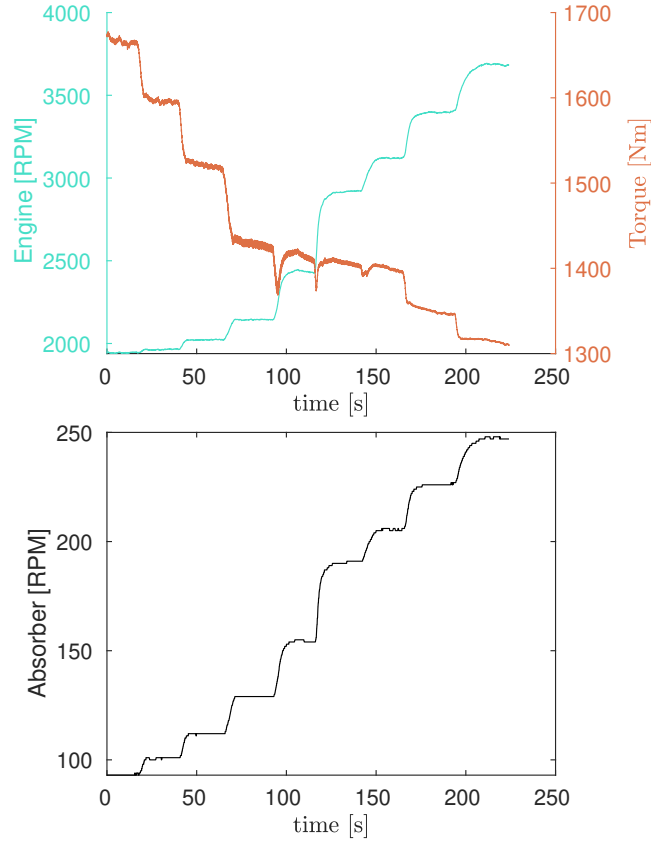


Figure 1: Outputs: Motor RPM and Torque. Input: Absorber RPM

So, one experiment for a not so ideal range of RPM, from a non so systematic

Table 1: Gear ratios

Gear Ratios	five-speed	six-speed
1st	3.31	4.10
3rd	1.23:1	1.56:1
Final Drive	4.39:1	3.46:1
1st×Final	14.53	14.35
3rd×Final	5.39	5.39

test, with some questionable behavior around 2500 RPM of the engine.

Other consideration include: The gear in which the vehicle was during the test is unknown. With the power google, I have the ratios for each gear, the final drive ratio, and the information that these tests are normally in third gear.

Let us see what we can make out of this data.

1.3 Objectives

Test my algorithm: I have a **data-driven** development, notice the emphasis on data-driven, this is not within the traditional *machine learning* (ML) methods or what the mainstream media is calling AI. It is called (drum-roll please); the *extended dynamic mode decomposition* (EDMD) with pq-quasi norm reduction or, as I like to call it: the pqEDMD.

Apply traditional identification methods: On top of EDMD based identification methods, I have extensive knowledge of traditional modeling and identification techniques. Then, what can we do with the information from the data, and the model from the pqEDMD.

2 The pqEDMD

Without getting too technical, the pqEDMD is an algorithm that takes *input/output* data from an arbitrary system and produces an approximation of its dynamics using a family of polynomial functions, and a mathematical structure similar to a linear system identification, that is, we get an A, B, C and maybe D matrices. Although is similar to a linear identification, this method captures the nonlinear behavior of a dynamical system. Where the A matrix is called an operator, because it acts on a set of functions (the polynomial functions) and not directly on the states, like in a linear system.

Figure 2 shows the result of applying the pqEDMD algorithm to the data-set. The testing methodology is equivalent to an initial value problem in dynamical systems. From the same initial condition, the algorithm applies the same sequence of inputs that drive the experimental data. Therefore, there will be some error, not this much error, but still, some deviation from the actual curves.

To be clear about this point, this is not a one step look ahead predictor, the evolution is just from the same initial condition.

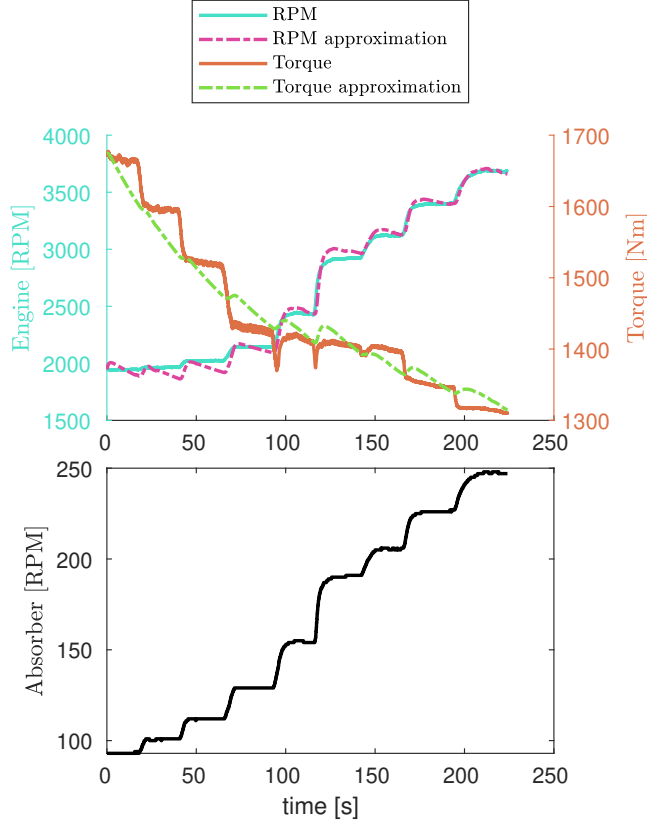


Figure 2: pqEDMD approximation of the Engine Dynamics

Not being able to have a better approximation came as a surprise for me. The algorithm is generally much better at identifying dynamical systems from data. What is happening then?

It would be necessary to go deeper into the analysis and possible refinement of the parameters to make the approximation. What is definitely true, is the fact that the whole system has many interconnected parts. The engine has its dynamics, then we have the torque converter and planetary gearbox dynamics before we get the shaft. The torque measurement at the shaft, also depends on the dynamics of the dynamometer; there is a feedback control loop that maintains a constant RPM at the absorber for a set value of the break current. This is not a strictly open loop system.

On top not having an open loop system, we also have to consider the gearbox dynamics. In a perfect world, we would have just an efficiency factor from the effect of the gearbox. Lets see then the behavior of the absorber RPM vs., the

engine RPM.

2.1 Gearbox analysis

This exercise is about the estimating the engine torque and eventually calculating the power at different RPM. For this, we could say that the engine RPM is proportional to the absorber RPM. Lets check this hypothesis. Figure 3 shows the plots of the absorber RPM vs., engine RPM and the ratio between the two. It is clear, that at low RPM, the ratio is not linear, and therefore it is not just proportional. At high RPM we can confirm that that test was in first gear. So, we have to consider the gearbox dynamics on to of the engine dynamics to do an analysis.

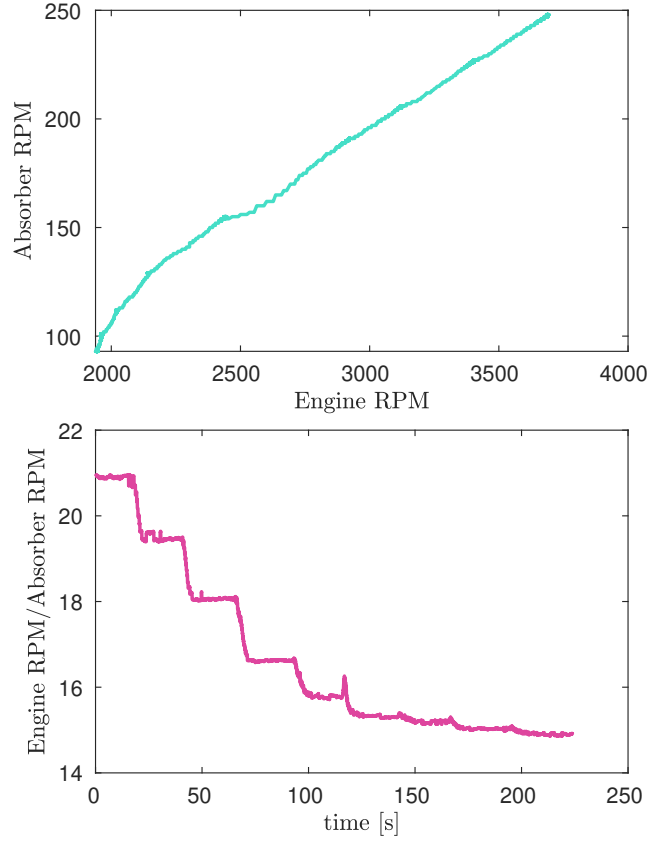


Figure 3: Engine RPM vs., Absorber RPM, and ratio between the two.

Knowing that is necessary to consider all of these dynamics to perform the analysis, my hypothesis is that there are some unobservable dynamics from the output, making an analysis like the pqEDMD not relevant to this problem.

The data driven analysis is not all we have. We still have traditional identification methods.

2.2 Higher order models

The pqEDMD allows for higher order models. The model in Section 2 shows the best approximation that fits the data. After evaluating the performance of other decompositions coming from different parameterizations, we get approximations where the shape suggests that a better data-set could produce more accurate approximations. Consider for example Figure 4, where the overall error is higher than the approximation in Figure 2 but the overall shape suggest that the algorithm has the potential to produce accurate approximations of this *input/output* behavior.

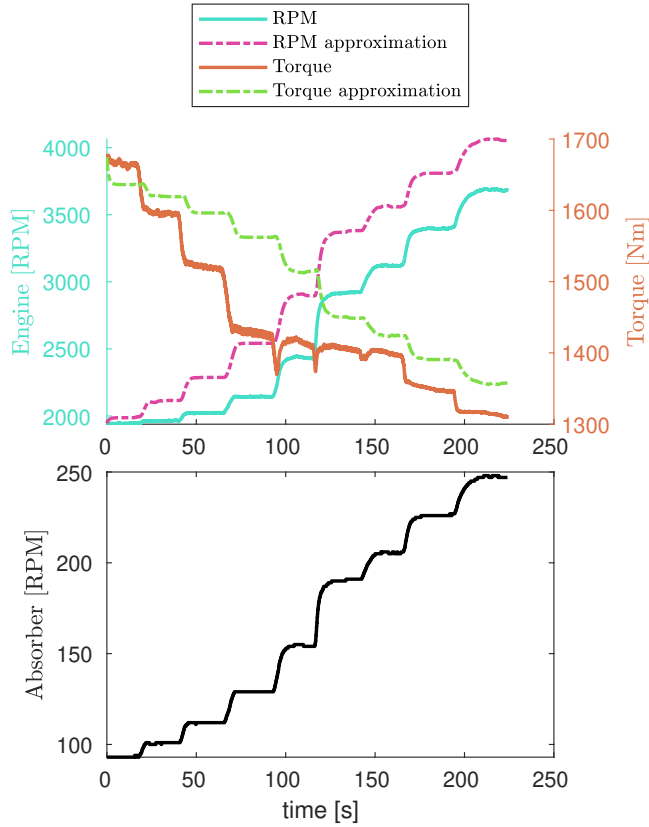


Figure 4: Alternative approximation of the Drivetrain Dynamics using other parameterizations of the pqEDMD.

3 Traditional Identification

At the time, for the development of the project, the exercise was to get the torque-power vs., RPM curve of the engine. To do this, we take the values of the torque at “steady state” for each step, take an average and divide by the gear ratio to get the torque at the engine for the different RPM values. For the power, just multiply by the engine speed in [rad/s] and plot the result. This is a static analysis and pretty simple and boring. Therefore, I will try to produce a dynamical system of the assembly.

Defining:

- k_e : Moment of inertia (and damping effect) of the engine and transmission [kg·m²]
- k_d : Moment of inertia (and damping effect) of the dynamometer and transmission [kg·m²]
- ω_e : Engine angular velocity [rad·s]
- ω_d : Absorber angular velocity [rad·s]
- T_d : Torque measured at the absorber [Nm]

3.1 Linear Approximation

After some attempts at using an adaptation of [1], and trying to include some transmission model, to account for the torque converter/planetary-gearbox assembly. The results were not satisfactory. Instead, consider a more pragmatic approach:

- The engine speed is directly proportional to the absorber speed.

$$\omega_e \approx k_1 \omega_d$$

The constant k_1 is related to the gearbox ratio.

- The absorber torque is inversely proportional to the absorber speed.

$$T_d \approx \frac{k_2}{\omega_d}$$

This comes from the analysis of the power equation $P_d = T_d \omega_d$, at steady state, the conservation of power applies, then, if the angular velocity increases, then the torque must decrease.

Then, we can approximate the dynamics of the system as:

$$k_e \dot{\omega}_e = k_1 \omega_d - \omega_e \tag{1}$$

$$k_d \dot{T}_d = \frac{k_2}{\omega_d} - T_d. \tag{2}$$

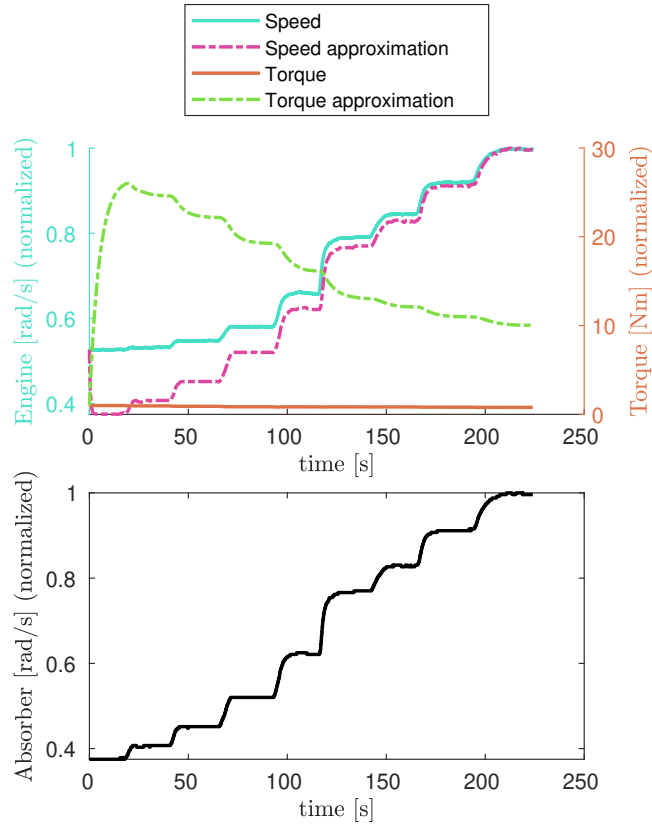


Figure 5: Linear approximation of the engine speed and absorber torque

Then, there are four parameters to estimate. The two “inertias” and the two proportionality constants. Figure 5 shows the validation of the differential equation after identifying the parameters. Even though this is not an accurate model, the overall shape of the response suggests that it is a good starting point to eventually get an approximation of the engine torque.

References

- [1] J. Blumenschein, P. Schrangl, T. E. Passenbrunner, H. Trogmann, and L. del Re, “Easily adaptable model of test benches for internal combustion engines,” in *2013 European Control Conference (ECC)*, pp. 1982–1987, 2013.