EDHEC
Business School
LILLE - NICE

Portfolio Management Final

Professor Philippe Malaise

Garth Mortensen

MSc Finance

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Exercise 1

You have won a mandate to actively manage a $100 million bond portfolio. Your objective is to outperform the Barclays U.S. Treasury 1-3Y Index in the long run. To do so, you have decided to implement a dynamic core-satellite (DCS) strategy allowing for dissymmetric management of tracking error (refresher: this approach helps ensure that the underperformance of the overall portfolio with respect to the benchmark will be limited to a given level, while letting the investor gain access to excess returns that may be generated by the active portfolio, i.e. the satellite portfolio). The risk budget and the max tracking error of the overall portfolio are set at 2% (annual basis).

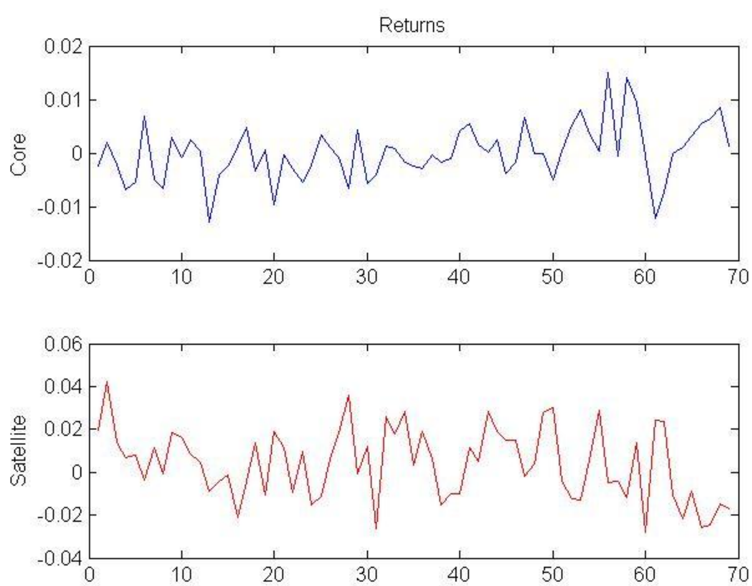
The core portfolio therefore replicates the performance of your benchmark (Barclays U.S. Treasury 1-3Y Index) through an ETF while the satellite portfolio acts as a return enhancer.

1) *As regards the latter, you intend to invest in a fund of L/S equity variable bias funds with a solid track record despite the Lehman Brothers collapse in mid-September 2008. All of them offer weekly liquidity through a managed account platform. Analyze the diversification potential of L/S equity funds (when combined with short-term Treasuries) taking account of the moments and co-moments of their return distribution and then argue the case for an investment in such hedge funds (NB: use the spreadsheet "Exercise 1 in-sample performance data.xls" to answer this question. Fill in the yellow cells).*

We have been handed $100 million of other people's money to invest with the objective to outperform the Barclays U.S. Treasure 1-3Y Index in the long run. To do so, we adopt a passive dynamic core-satellite (DCS) strategy, which allows us to manage our downside risk while enjoying some of the upside active managers have to offer. We assume the bond ETF and L/S equity funds are the best in their classes and proceed from there.

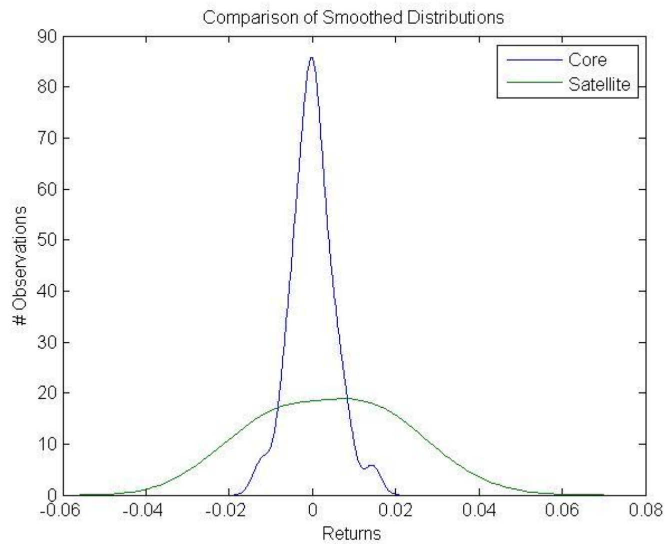
The returns process is shown in **Figure 1**. Eyeballing the returns can give us a good understanding of how differently the prices performed over the sample period. The period included the market downturn (including Lehman's collapse), which we can see at the end of the sample.

Figure 1



To preliminarily check the distribution of these returns, a smoothing kernel density function was used in Matlab (**Figure 2, see Matlab code**). Indeed, these return distributions differ in mean, width, skew and kurtosis. With this understanding, we proceed to the summary statistics.

Figure 2



The overall portfolio's performance (under the default 85% allocation to the core) is shown in **Figure 3 (Excel spreadsheet displayed as Appendix 1).** Combining the core with the satellite offers the benefit of higher return and lower standard deviation.

Figure 3

	Core	Satellite	Overall Portfolio		
Monthly Mean Return	0.003%	0.392%	0.061%		
Monthly Standard Deviation	0.520%	1.659%	0.433%	$\beta_{S,C}$ (2)	-1.016
Monthly Variance	0.003%	0.028%	0.002%		
Covariance(R_S,R_C)	-0.0027%				
Skewness	0.259	0.040	0.014	$\beta_{S,C}$ (3)	-0.536
Coskewness(R_S,R_C)	-0.139				
Excess Kurtosis	0.982	-0.719	0.696		
Max Drawdown	**-6.60%**	**-11.74%**	**-3.63%**		

According to Markowitz, the satellite would be a good contribution if it improves the portfolio's mean and standard deviation. But we have learned that diversification benefits are more complicated when distributions are non-normal, which is a stylized fact of financial returns.

To test whether or not the return series are normal, we can perform a Jarque-Bera test in Matlab **(see Matlab code)**. Funds, like assets, do not return normal distributions. Contrary to expectation, the results of the test applied to our series suggest the distributions are indeed normal. However, the p-value associated to the core and satellite is 0.15 and 0.33, respectively; the statistic is not wholly convincing. Due to the low observation count, these measures are somewhat unreliable. In partial confirmation, normal distributions have zero skewness and excess kurtosis, which are not the case (on an added note, because Excel applies a bias correction to these calculations, results may differ (Alexander, 82). We take note of the results but continue onwards for the purpose of discourse.

When we examine the third and fourth moments, we see that our satellite skewness and kurtosis are worse than the core. We desire a positive skew, to have larger positive returns, indicating a higher probability weight to positive values is larger than negative values. Financial markets typically exhibit negative skewness, as asset price falls tend to be followed by more price falls (Alexander, 83). We should be happy not to be in this category. Regarding kurtosis, it's desirable to have excess kurtosis (that above 3), since it means fewer return swings. Investors like stability. The betas are less than 1 which tells there are diversification benefits for skewness and kurtosis.

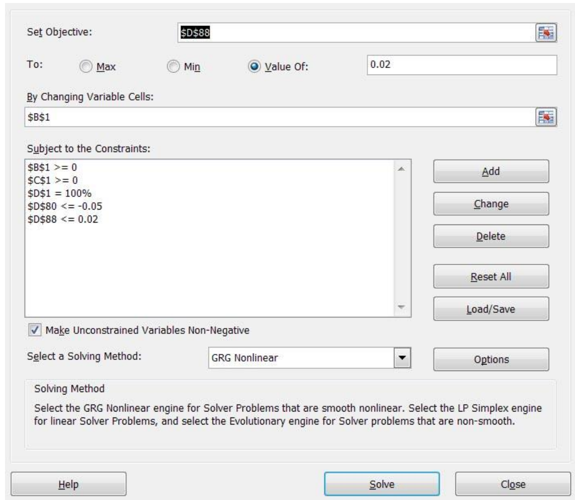
But in the end, we must remember that though our results are thus far positive, these first order approximations are only valid for small error. Furthermore, estimating the higher-order moments and co-moments is a formidable challenge that grows with dimensionality. For only five assets, there are 4,421,275 fourth moment estimations required.

2) *Compute the max drawdown of L/S equity funds (i.e. peak-to-valley drawdown) over the period ranging from April 2003 to December 2008. Assuming that you had implemented a constant-mix strategy on that period (systematic rebalancing at the end of each month - see column D in the spreadsheet "Exercise 1 in-sample performance data.xls" - while complying with a max drawdown constraint of 5%), what would have been the maximum allowable allocation to L/S equity funds?*

The overall portfolio performance is computed as a weighted average of core and satellite monthly returns **(Excel spreadsheet displayed as Appendix 2)**. Fixing tracking error at 2% and adjusting the weights results in a satellite allocation of 31%. This serves as our upper limit for satellite allocation due to the tracking error constraint.

In order to determine our max satellite allocation while complying to our max drawdown constraint, we use Solver to set our portfolio's max drawdown to -5%, allow our weights to change, but constrain each to above 0% and their total to 100% **(Figure 5)**. We're faced with two possible solutions. The first solution is to allocate 7% to the satellite, a number below our 31% limit and resulting in a tracking error of 0.5%. We know from before that we can still move closer to our satellite. A 31% allocation to the satellite results in a tracking error of 2% and a max drawdown of only -3.3%. This satisfies all constraints and offers an even higher satellite allocation. With this max drawdown constraint, we can weather a market storm.

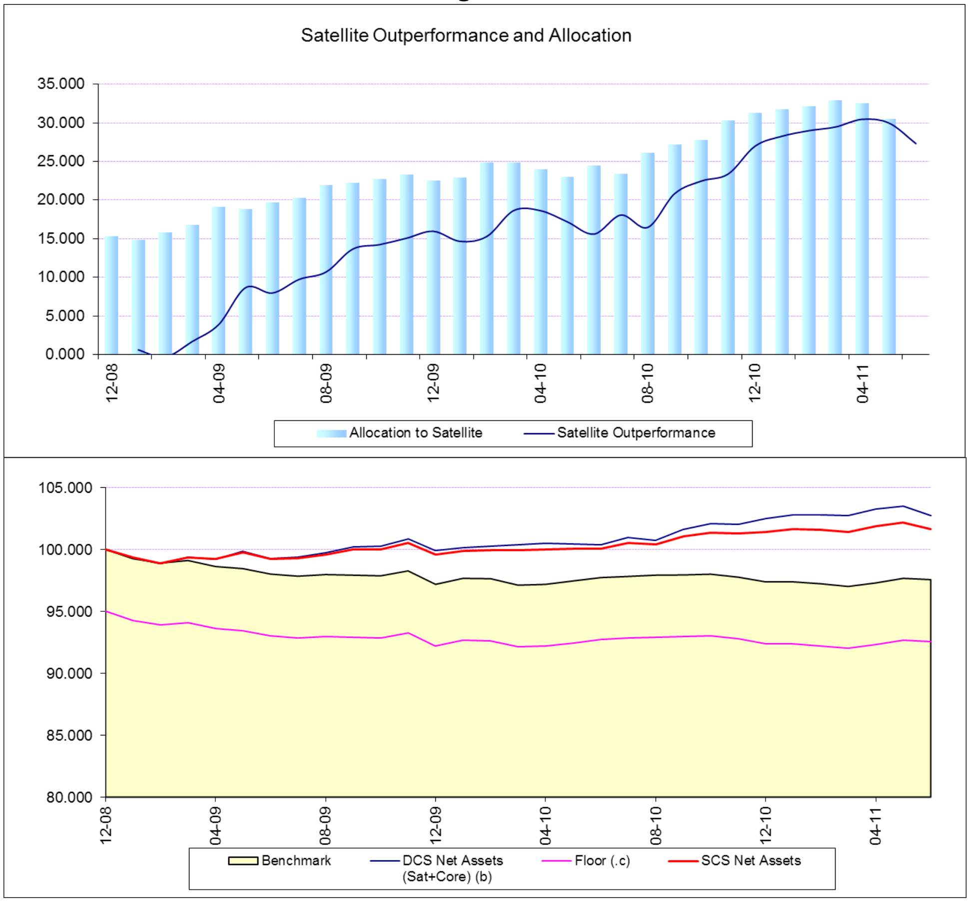
Figure 5



3) *The DCS strategy has been implemented since January 2009 using the following parameters: k = 95%, m = 3 (i.e. initial cushion = \$5 million, initial investment in the satellite portfolio = \$15 million). The floor is indexed on the performance of the benchmark. The guarantee on the relative level of performance is set at: benchmark value – 5 (initial value equal to 100 to simplify matters). Your portfolio has been rebalanced at the end of each month using the above-mentioned parameters over the period ranging from 12/31/2008 to 06/30/2011 (no ceiling rule with regard to the satellite portfolio). Plot the graph of the (re)allocation to the satellite portfolio (after rebalancing) with its cumulative outperformance using the spreadsheet "DCS Case Study LS Equity Funds and Treasuries.xls" (NB: Monthly returns of the L/S equity funds and the benchmark in columns H and I respectively – we assume that the return of the core portfolio is equal to the return of the benchmark - fill in the yellow cells and <u>don't forget to rebalance your portfolio</u>: new allocations to the satellite and core portfolios computed in columns M and N).*

Rebalancing the portfolio on a monthly basis results in a cumulative overall gain of 27.31, revealing how much more strongly the satellite performed relative to the core **(Appendix 3)**. By the end of June 2011, allocation to the satellite stood at 30.46, with the core at 72.25. **Figure 6** shows how these allocations developed over time in response to market conditions, and comparing performances of the DCS and SCS portfolios.

Figure 6



4) *Compute the following risk-return statistics for the DCS portfolio, the benchmark (Barclays U.S. Treasury 1-3Y Index), and a static core-satellite portfolio (i.e. buy-and-hold strategy without rebalancing: initial investment in L/S equity funds = 15% of the assets under management, see column T in the spreadsheet):*
   a. *Cumulative return*
   b. *Annualized return*
   c. *Annualized standard deviation of returns*
   d. *Skewness*
   e. *Excess Kurtosis*
   f. *1-month modified Value-at-Risk (Cornish-Fisher extension with a 99% confidence interval)*

   Compare these results and draw the conclusion.

The satellite and benchmark funds performed very differently over the years, with the former achieving higher returns but also all around higher risk, as seen in **Figure 7**. Notably, the satellite achieves far superior skewness, but far worse kurtosis. An investor wants more excess kurtosis (kurtosis greater than 3) because it erodes the presence of tails, which pose extreme risk. Between the DCS and SCS portfolios, the same risk-return relationship holds true. Quantifying the relationship by dividing return by risk, we see that the satellite's ratio is far better than the benchmark's, and the DCS is slightly better than the SCS portfolio.

Regarding the 99% modified variance at risk, both portfolios outperform the satellite and core in terms of 99% modified VaR. There is a 1% chance of losing more than 1.05% from the DCS portfolio, a larger amount than the 1.01% associated with the SCS. However, as a note, these values do not represent the max potential loss. Hedging can be used to mitigate this risk. Monte Carlo simulation can be used to simulate a large number of drawdowns to further examine these findings. Otherwise, stress testing using a multifactor model could be used.

Figure 7

	Satellite*	Benchmark	DCS Portfolio	SCS Portfolio
Mthly Mean Return	0.76%	-0.08%	0.09%	0.06%
Annualized Return	9.06%	-0.98%	1.08%	0.67%
Monthly SD	1.59%	0.36%	0.44%	0.39%
Annualized SD	5.49%	1.24%	1.52%	1.34%
Skewness	0.39	-0.83	-0.56	-0.75
Excess Kurtosis	-0.14	1.06	-0.08	0.32
99% Modified VaR	-2.34%	-1.13%	-1.05%	-1.01%
Return/Risk	1.65	-0.79	0.71	0.50
Z_c	-2.33	-2.33	-2.33	-2.33
Z	-1.95	-2.92	-2.60	-2.74

5) *Compute the tracking error of your DCS portfolio and compare it with that of the static core-satellite portfolio. Compare the "good" tracking error of your portfolio with the "bad" one. What are your findings?*

Looking at **Figure 8**, we see both portfolios offer more "good" tracking error than "bad" tracking error. Altogether, the DCS portfolio comes with more good and bad tracking error. This is because as the satellite outperforms the benchmark and we give it more weight, tracking error is pulled up alongside. Compared to the SCS, the DCS presents more positive tracking error, in both absolute and relative terms.

The DCS portfolio is superior, but not goes against expectations that it can reduce bad tracking error by allocating less weight to an asset when it underperforms. My interpretation is that it should achieve less bad tracking error than the SCS.

Figure 8

	DCS	SCS	Absolute Difference	Relative Difference
Good TE	0.0025	0.0020	0.0005	0.2702
Bad TE	0.0019	0.0011	0.0007	0.6570
Absolute difference	0.0007	0.0009		

6) *You have to stress-test your portfolio on a monthly basis as from 30 June 2011. The worst case scenario would result in a monthly loss of -10% for the L/S Equity funds but the benchmark would be up 1% on the same period. What would be the resulting allocation to L/S equity funds (i.e. after rebalancing) in that case? Does it make sense to set a cap on the weight of the satellite portfolio given this result? What would be your recommendation?*

If we stress test our portfolio with a 10% monthly loss in our satellite and a 1% gain in our core, the resulting allocation to L/S equity funds after rebalancing comes to 20.566 (and 79.826 to the core out of 100.392 net assets). Of the total, L/S equity funds take 20% allocation **(Figure 9).**

Figure 9

4/30/2011	103.255	92.294	10.961	32.466	70.790	97.294	1.01%	0.29%
5/31/2011	103.495	92.666	10.829	32.854	70.641	97.666	-0.09%	0.38%
6/30/2011	102.716	92.561	10.155	31.785	70.931	97.561	-2.16%	-0.11%
Stress Testing	100.39	93.54	6.86	27.42	72.97	98.537	**-10.00%**	**1.00%**
Rebalance	100.392	93.537	6.855	20.566	79.826			
			Allocation	20%	80%			

What about setting a cap on the weight of the satellite? Doing so would help ensure that the risky satellite asset doesn't too negatively affect our portfolio. If our investor is looking to preserve their wealth, we would want to set a cap to minimize our max drawdown. If we don't want to lose our shirts like we would under the 10% loss stress test, we could set our multiplier to m = 4, and thereby our allocation to risky assets wouldn't surpass 40%.

In the end, the decision lies with our investors' individual utility functions for more wealth. Immediately, one could ask investor age, income level and other factors to approximate their risk tolerance.

Exercise 2

During their recent meeting, the administrators of the pension fund for the European section of the professional footballers' association have decided to select a provider for the management of the equity portion of their portfolio.

The asset management company you work for has been shortlisted to take part in this consultation. Following the recommendations of the consultancy firm AM International Ltd, it has been decided that this specific mandate should be consistent with the latest insights from modern portfolio theory, emphasizing the benefits of benchmark construction. The investment universe is made up of the stocks in the Stoxx Europe 600 index (capitalization-weighted index designed to provide a broad representation of companies across 18 European countries).

The administrators of the pension fund would be inclined to use a strategic benchmark based on a ***minimum variance optimization****, i.e.*

Minimize $var(R_P) = \sum_{i=1}^{i=n}\sum_{j=1}^{n} x_i x_j \operatorname{cov}(R_i, R_j)$

Under the following conditions:

$\Sigma x_i = 1$ *(portfolio constraint)*

$x_i \geq 0$, $i = 1,\ldots, n$ *(positivity constraint, no short sales)*

Where x_i denotes the proportion of asset i held in the portfolio and $cov(R_i, R_j)$ denotes the covariance between asset i and asset j.

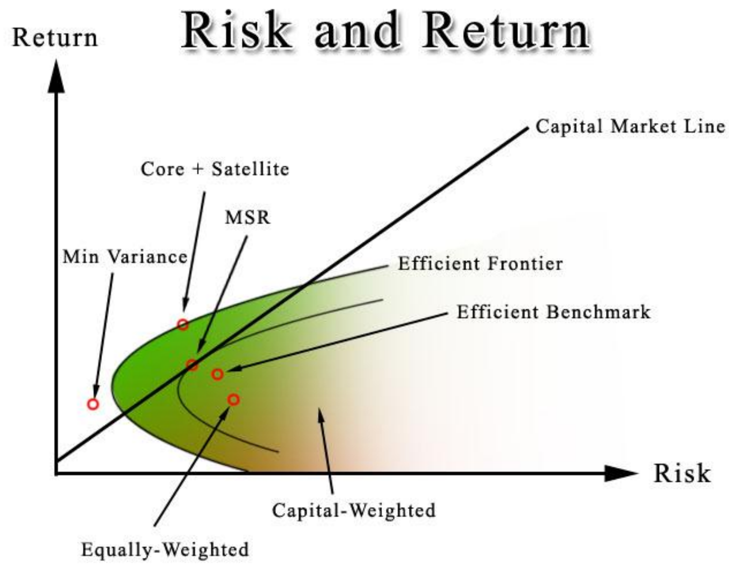
a) Explain to the administrators of the pension fund what are the advantages and disadvantages of the min variance portfolio compared with other benchmarks that are more optimal than the market-cap weighted index?

Capitalization weighted indices (such as the S&P500, CAC40 and FTSE 100) weigh their components by individual asset market capitalization. Modern Portfolio Theory states that the goal of any investor is to hold the so called market portfolio. But what is the market portfolio? It is the cap weighted portfolio of all available assets, which is impossible to construct (there are a number of other unrealistic assumptions, but that is another story). Existing cap-weighted indices stand below and to the right of the efficient frontier line that the perfect portfolio rests on, meaning that they are inefficient representations of the market.

The mechanics of cap weighted indices leads to trend following strategies that provide an inefficient risk-return trade-off. A declining market value of a stock leads to a smaller weight, and vice versa. This underweights the past losers and overweighs the past winners, or that is, it is trend following. There are easy ways to overcome this problem, such as by using an equal-weighting scheme for individual components.

Cap weighting logically leads to high concentration and exposure to idiosyncratic risk. The Herfindhal index can be used to test for concentration. All of these shortcomings urged the development of alternative weighting schemes such as equally-weighted, core satellite, or min variance portfolios **(Figure 10)**. Even small investors can beat cap weighted indices by implementing these alternative weighting schemes and rebalancing quarterly.

Figure 10



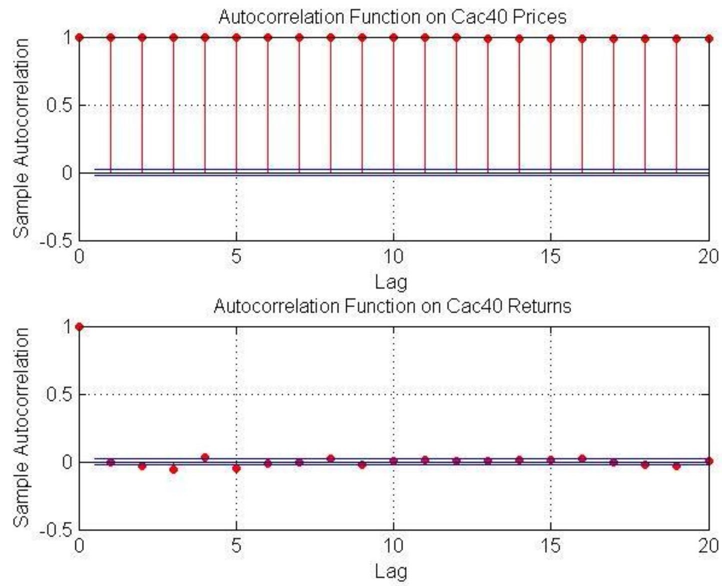
(Adapted from Portfolio Management Lecture 5, Philippe Malaise)

As seen, the disadvantages of cap weighted indices are numerous. The Stoxx Europe 600 is just one such index, but we assume is comprises the entire investment universe. Our next step is to find a superior alternative. The equally-weighted index is one such example, but it is not the best.

The only efficient benchmark in modern portfolio theory is the true tangency portfolio, the one with the highest Sharpe ratio, which is also known as the maximum Sharpe ratio (MSR) portfolio. It is always the most efficient portfolio on the frontier. The MSR is a function of the unknown true parameter values of variances and covariance on one hand and expected returns on the other. The goal when constructing an MSR portfolio is to estimate the parameter values so as to get a decent proxy for the true tangency portfolio so as to approach it. If the initial parameters are correct, we should get something close to the MSR. Of course, performance results heavily depend on the estimation process.

In trying to estimate these parameters we use statistics, which is utterly hopeless in terms of forecasting expected returns, (examining autocorrelation in the price and return series of the CAC40 from start-1990 to end-2011 **(Figure 11)** reveals this dilemma). With this in mind, we should shy away from those who claim they can predict the future path of returns. A notorious lesson of this is the analysts who all gave positive recommendations for Worldcom and Enron right before their fall from grace. The only hope then is to manage risk and keep in mind that there is a risk-return trade-off. That is, the expected return parameters should relate to risk parameters.

Figure 11



The main advantage of min variance is that it does not require such estimation of expected returns, as does the MSR. This frees us of a huge burden. The global min variance (GMV) approach seeks to minimize portfolio variance by optimizing individual asset weights using a Lagrange function **(Figure 12)**. That's all! The weights depend on return variances and covariances, but not the expected return. These parameters are usually estimated by replacing the true covariance matrix with its time series estimator.

Figure 12

$$L = w'\Sigma w - \gamma(w'e - 1)$$
$$L_w = \Sigma w - \gamma e = 0 \Rightarrow w^*_{GMV} = \gamma \Sigma^{-1} e$$
$$L_\gamma = w'e - 1 = 0 \Rightarrow \gamma(\Sigma^{-1}e)e - 1 = 0 \Rightarrow \gamma = (e'\Sigma^{-1}e)^{-1}$$

$$w^*_{GMV} = \frac{\Sigma^{-1}\boldsymbol{e}}{\boldsymbol{e}'\Sigma^{-1}\boldsymbol{e}} \equiv g(\sigma_i, \rho_{ij})$$

From **Figure 10**, we can immediately see that it results in very low risk and achieves a return above the risk free rate. An investor's goal is to push returns up, but do so with equal or lower standard deviation. However, we see the min variance benchmark generates lower Sharpe ratios than the MSR. This should not be seen as a drawback, because risk reduction, which is the main objective of the benchmark, logically results in lower performance. A rational investor tries to get the best risk adjusted performance. Again, to maximize the reward to risk ratio, we need to estimate expected return parameters, which is difficult.

According to Stoyan and Goltz, because min variance portfolios only aim at minimizing variance, they do however tend to overly concentrate in low volatility assets at the expense of exploiting correlation properties. The portfolios therefor may be only best fit to serve those investors looking for “defensive” positions (just what a pension fund should be). Stoyan and Goltz also point to other work that shows min variance portfolios have a difficult time outperforming even simple equal-weighted portfolio Sharpe ratios.

Furthermore, filtering for those assets which together return the minimum variance comes with a drawback. As items are pulled from the portfolio, we are left holding fewer assets and our exposure to specific (idiosyncratic) risk rises. To avoid this potential risk, we can impose minimum weighting constraints to ensure we are invested in the same sample of underlying assets. The true min variance portfolio is always located on the left of the efficient frontier.

And even in the min variance portfolio, besides the risk free asset, there is always the one key risk factor of credit (counter party) risk.

b) To achieve the best possible result with a min variance optimization program, you are required to try and use efficient estimators of the variance-covariance matrix. What solutions can you suggest to the administrators of the pension fund? Argue the pros and cons of each solution

The problem with Markowitz optimization (max Sharpe ratio) is that you need many inputs. As previously stated, estimating parameters grows exponentially more difficult as the number of assets in a portfolio rises. Even a concentrated portfolio of 5 assets requires 4,421,275 estimates. Since it is difficult to estimate all of these input parameters, practitioners will often reduce the dimensionality through rough approximation. A number of these fixes have been proposed.

As early as 1973, Elton & Gruber offered up their constant correlation estimator. In the long run, this method works very well. It offers a drastic reduction in parameters compared with the number of a portfolio with 50 or 100 underlying assets. There is just one estimator, the average of coefficients of correlation (*See CAC40 PCA.xls, sheet ‘CAC40 Underlying Asset Returns’*). The method is easy to implement as well. If you’re working with the CAC40, you calculate 40 time series returns’ coefficients of correlations, and take their average. This works much better than the Markowitz optimization. It is more stable over time. *On a side note, through my thesis work I found the dynamic correlation method, which is a calculation easy enough for Excel, worked well as an upgrade to the overly simplistic constant correlation. We expect correlation to rise in a down market and fall in an up market, indicating that correlation is dynamic.*

A superior approach to the constant correlation method is factor-based estimation. This can be done with a single-factor model, such as the CAPM, but these are not enough to replicate the performance of a portfolio. Superior results come from multi-factor models, either explicit or implicit.

Implicit multi-factor models are based on principal component analysis, which boasts the advantage of each risk factor being uncorrelated with all others. But the problem here is that these factors are not observable, which means you can’t use them to hedge your portfolio. Besides its complex nature, this may be why the industry has not readily adopted this approach, and instead focuses on explicit multi-factor models (such as the ever popular Fama-French multi-factor model).

One drawback of explicit models is that you must test a large number of combinations of factors. You must search through various combinations to find lower correlations between factors, thereby minimizing co-linearity problems and achieving a better model. And with fewer factors, your model will be more healthy and robust.

But the main advantage of this technique is that all the risk factors are potentially investible vehicles to hedge with. Imagine that you (an asset manager) wake up in the morning to find there has been a severe market reversal. If you adopted the implicit multi-factor approach, you would have to open your statistics book and remind yourself what eigenvectors are. If you follow the explicit multi-factor model approach, you can quickly hedge your position and mitigate extreme risk, and 30 angry footballers don't show up at your doorstep the next day. Rather, you get a Christmas bonus. The advantages are very real.

As said, with these multi-factor models, you can reduce the number of input parameters from say 100,000 to perhaps three or four. If we're managing many more funds than this one pension, the multi-factor model can also reduce our oversight costs.

Another solution is to use statistical shrinkage estimators. This is a rough estimator but still superior to Markowitz optimization. The main idea here is to shrink individual return means to a grand sample mean. Unfortunately, this introduces bias which increases when we introduce various asset classes (equities or bonds) that perform differently from each other.

In the end, explicit multi-factor models appear the most attractive choice given their readily understandable, easy to use, robust and can be used to hedge with.

Exercise 3

As the CIO of an endowment fund, you are responsible for managing and monitoring a \$100m core-satellite portfolio that must comply with the following rules:

- *Satellite portfolio fully invested in a fund of high yield bonds*
- *Core portfolio invested in U.S. 5-7Y Treasuries*
- *Floor: \$95m indexed to the performance of the Barclays U.S. 5-7Y Treasury index*
- *Wealth level T_t (from which you will shift from a goal-oriented approach to a risk-management approach) initially set at \$104m*
- *Performance cap G_t initially set at \$106m*
- *Multipliers: $m_1 = 4$, $m_2 = 2$*

a) You have to implement this DCS strategy from January 2009 to December 2009 using the "Exercise 3 File" (1st spreadsheet -> input data: 'Return History' – to simplify matters, we assume that 1) the core portfolio replicates the returns of the Barclays U.S.5-7Y Treasury index and 2) the satellite portfolio replicates the returns of the Barclays Macro EM High Yield index; 2nd spreadsheet -> output data: 'DCS Portfolio' - you have to fill in the blanks with the right formulae).

Draw up the annual report on this portfolio and explain the results obtained over that period.

Compare them with the performance of (and the risk incurred by):

- *a CPPI strategy whose floor is indexed to the performance of U.S. Treasuries,*
- *a constant mix strategy (20% invested in high yield bonds),*
- *a buy-and-hold strategy (20% initially invested in high yield bonds),*
- *U.S. 5-7Y Treasuries.*

Argue the case for the reduced allocation to high yield bonds in Q4 2009.

A DCS strategy investing its core in the Barclays U.S. 5-7Y Treasury index and its satellite in the Barclays Macro EM High Yield index over the course of 2009 resulted in either positive or negative gains relative to the goal-direct strategy depending on your risk controls. The goal-directed strategy gained a strong 4.12%. The ratchet effect of this strategy locks in profits and reinvests the proceeds into risk-free assets. For our purpose, gains from the high yield bonds were channelled into safely into the 5-7Y Treasure index for safe keeping.

The Constant Proportion Portfolio Insurance (CPPI) risk control strategy results in the highest final return of 5.474% by year end. The performance outpaced our goal-direct strategy. This strategy is a trend follower, and since 2009 was a great year for high yield bonds, it paid off handsomely. Speaking of its risk control, the strategy lowers its allocation to those assets when their prices are falling. When prices fall too low, all assets are invested in the risk free asset.

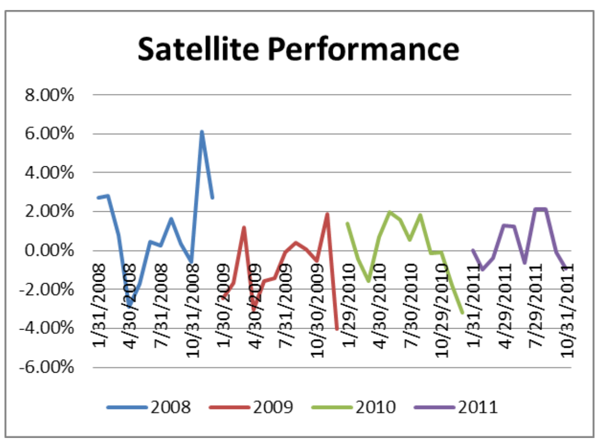
During 2009, the constant mix actually lost 2.84%. But in terms of risk control, this method results in smoother performance during a rollercoaster market thanks to lower market sensitivity. As a contrarian strategy, it didn't benefit as much from the great year for high yield bonds. Finally, the buy and hold strategy goes negative as well (-1.55%), solely due to the poor performance of treasuries.

Another way to look at this decision is to consider what may happen in the future. If high yield bonds are expected to climb, fall, or undergo flat volatility, then our allocation should change, and so perhaps should our risk controls. Each of the investment styles will perform differently during an increasing, decreasing, and flat but volatile market, as illustrated with a random price walk in **Figure 13**. A (difficult) forecast of high yield bonds is therefore crucial in this decision.

Figure 13

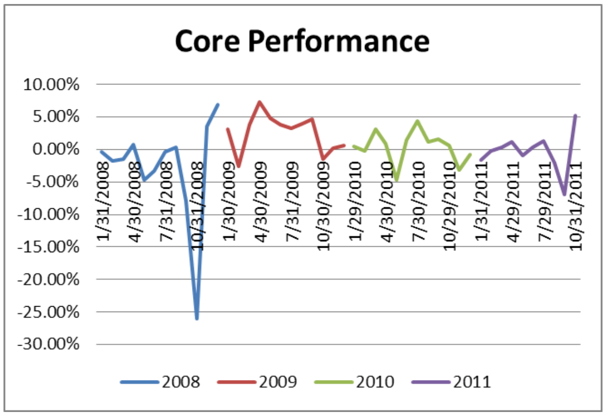
The expectation that CPPI tromps other investment styles in downward market conditions holds true for 2009. Previously in 2008, the satellite (high yield bond) performance was a strong performer. In 2009, the asset jumped from positive to negative several times **(Figure 14)**.

Figure 14



Meanwhile, the core (treasury) performance in 2009 achieved higher returns with less volatility **(Figure 15)**. The discrepancy in historical performance may be used to consider future weighting.

Figure 15



At some level of wealth, investors are more concerned about maintaining it than they are growing it. Diminishing marginal returns dictates that the marginal value of each additional dollar decreases with their amount. It is therefore natural that using a goal-directed strategy, we account for this through wealth and performance levels. Using the goal-directed strategy, we are allowed to consider that at this wealth level, the investor's utility preference demands a change in allocation as we switch between hope- and fear-dominated behaviour. Additionally, when we switch to investing in the risk-free asset, we enjoy lower management costs.

For our purposes, if net assets are below than the wealth level, then the cushion is equal to net assets minus floor. If net assets are below the performance cap, then the cushion is equal to performance cap minus net assets. If our net assets have climbed beyond both the wealth level and the performance cap, then we can set our cushion to 0. Indeed, as of 2009Q4, our wealth has surpassed our performance cap and we can remove the risky high yield bonds from our portfolio.

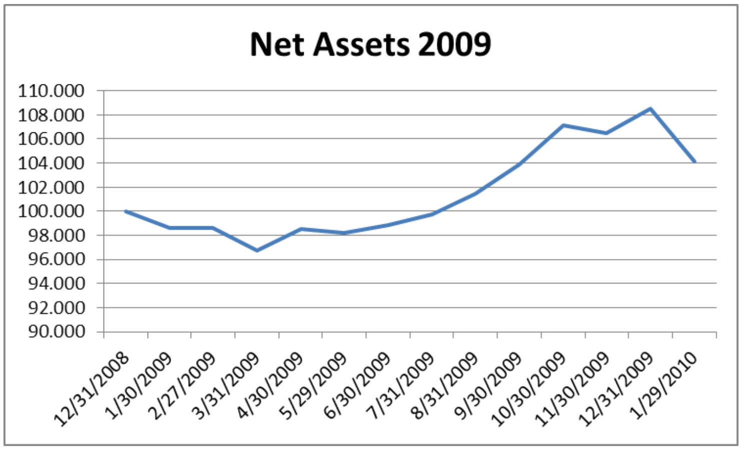
b) The wealth level T_t (for fiscal year 2010) has been cautiously adjusted upwards, from \$104m to \$105.5m as from 31 December 2009, simultaneously with the performance cap G_t (now set at \$107.5m).

What is the YTD return as of 31 December 2010 assuming that the floor has been updated as from the end of December 2009 (new value initialized as of 31 December 2009 = \$95m – see cell AF29 in the DCS portfolio spreadsheet -, then indexed to the performance of the Barclays U.S. 5-7Y Treasury index)?*

To what extent was it justified to simultaneously increase the floor, the wealth level and the performance cap?

Over the course of 2010, the goal-directed strategy managed to earn 1.85% in cumulative returns upon rebalancing. As to increasing all three factors simultaneously, we can see (**Figure 16**) that since 2009Q3, net assets have been climbing. Perhaps we expect them to continue on.

Figure 16



The wealth level is where you begin reducing your risk exposure. The performance cap is where you switch to investing in the risk-free asset. Increasing these amounts means that we are ready to remain invested in higher risk assets for a longer duration in order to earn higher returns. This also effectively increases the size of our cushion.

Beginning at our wealth level 104, the investor's risk tolerance drops until we reach zero, when we reach our desired performance cap of 106. Pushing these two factors higher will push these points to the right. As visually displayed in **Figure 17**, if you push wealth level (T) to the right, you are willing to take on higher risk until you reach a higher net asset value. The same can be said of the performance cap.

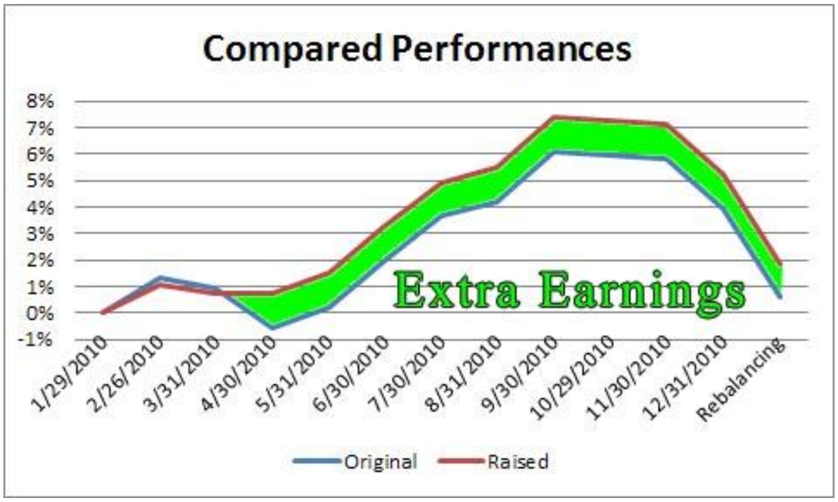
Figure 17

Our floor was also lifted just over 10 points to 95. The floor level determines our allocation to the low risk treasury index. Raising the floor increases our allocation to the core and lowers risk.

By raising all three factors together, we stay invested in riskier assets until we reach a larger NAV worth, but we also invest more in the core to reduce risk. We therefore can't simply say we are either raising or lowering total risk, but we are changing its dynamics.

If we hadn't increased these factors, upon rebalancing at yearend 2010, our cumulative return would have hit only 0.61% as opposed to the realized 1.85%. Retrospectively, we can see in **Figure 18** the choice to raise the factors paid off well.

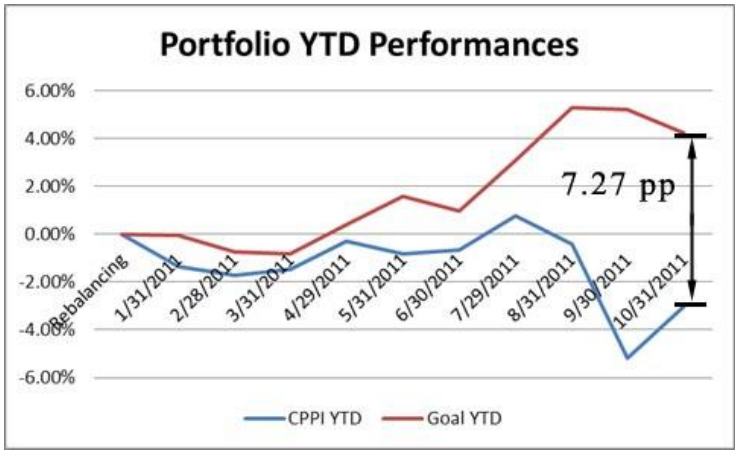
Figure 18



c) After the strong rebound in the high yield bond market (+42.15% in 2009 and 2010, knowing that the cumulated loss - peak-to-valley drawdown - from April 2007 to October 2008 was -42.17%), you should be extra cautious if you decide to revise the wealth level T_t for the next fiscal year (2011). From this perspective, it has eventually been set at \$106m as from 31 December 2010. At the same time, the performance cap G_t has been increased from \$107.5m to \$108m, the floor remaining unchanged (FYI: floor value as at the end of December 2010 = 95.617). What is the YTD return of the DCS portfolio as from 31 October 2011? Compare it with the result obtained through the CPPI portfolio. What conclusion can you draw in light of those results?

Over the course of 2011 until that October, the goal-directed strategy managed to earn 4.24% in cumulative returns upon rebalancing. The CPPI portfolio meanwhile lost 3.03%, standing a tall 7.27 percentage points below the goal-directed strategy. Shortly before October the difference was even larger for a short period. The evolution of these figures is displayed in **Figure 19**.

Figure 19



A look at the equations which contrast the two investment strategies reveal that as government bonds performed well, the DCS method allocated more assets to them and thus achieved higher net assets. Given these results, we can say that raising the three factors even higher would have improved end performance.

Appendix

Matlab Code

```
%Garth Mortensen
%mortensengarth@hotmail.com
```

Jarque-Bera Code

```
%% Prepare matrices
[matrix,~,~]       = xlsread('C:\Exercise 1.xls','Monthly Return History');
core               = matrix(4:72,2:2);
sat                = matrix(4:72,3:3);

%% Jarque-Bera test
[hcore,pcore]      = jbtest(core,.05);
[hsat,psat]        = jbtest(sat,.05);

%% Plotting
figure
subplot(2,1,1)
plot(core,'b')
ylabel('Core')
title('Returns')

subplot(2,1,2)
plot(sat,'r')
ylabel('Satellite')

figure;
ksdensity(core);
hold all;
ksdensity(sat);
legend('Core','Satellite','Location','NorthEast');
ylabel('# Observations')
xlabel('Returns')
title('Comparison of Smoothed Distributions')
```

Cac40 Code

```
%% Autocorrelation in Cac40

cacreturn = price2ret(cac40);

figure
subplot(2,1,1)
autocorr(cac40)
title('Autocorrelation Function on Cac40 Prices')

subplot(2,1,2)
autocorr(cacreturn)
title('Autocorrelation Function on Cac40 Returns')
```

Appendix 1 (fully scalable)

Weights	85%	15%	100%	Core		Satellite		Excess Return		Overall			
Name	US Treasuries 1-3Y	L/S Equity Funds	Overall Portfolio										
Strategy	Core	Satellite	Systematic Rebalancing	Core Monthly Return	Drawdown	Satellite Monthly Return	Drawdown	Satellite Excess Return (Sat. - Core)	Overall Excess Return (Overall - Core)	Overall Monthly Return	Drawdown	Skewness of C Decomposition	Coskewness of S decomposition
4/30/2003	-0.25%	1.93%	0.08%	99.8%	0.00%	101.9%	0.00%	2.18%	0.33%	100.1%	0.00%	-48.3016576%	-11.2690188%	0.0000097%
5/31/2003	0.20%	4.23%	0.80%	100.0%	0.20%	106.2%	4.23%	4.02%	0.60%	100.9%	0.80%	38.0214980%	5.4965182%	0.0000150%
6/30/2003	-0.22%	1.37%	0.02%	99.7%	-0.22%	107.7%	1.37%	1.59%	0.24%	100.9%	0.02%	-42.8179166%	-7.8501254%	0.0000048%
7/31/2003	-0.68%	0.68%	-0.48%	99.1%	-0.90%	108.4%	0.68%	1.36%	0.20%	100.4%	-0.48%	-131.2085970%	-225.8847310%	0.0000134%
8/31/2003	-0.54%	0.80%	-0.34%	98.5%	-1.43%	109.3%	0.80%	1.34%	0.20%	100.1%	-0.81%	-104.3198145%	-113.5273284%	0.0000120%
9/30/2003	0.68%	-0.34%	0.52%	99.2%	-0.76%	108.9%	-0.34%	-1.02%	-0.15%	100.6%	-0.29%	129.7495005%	218.4324130%	-0.0000335%
10/31/2003	-0.50%	1.12%	-0.26%	98.7%	-1.26%	110.1%	0.77%	1.62%	0.24%	100.3%	-0.55%	-96.7816965%	-90.6524804%	0.0000185%
11/30/2003	-0.66%	-0.07%	-0.57%	98.0%	-1.91%	110.1%	-0.07%	0.59%	0.09%	99.8%	-1.11%	-127.0077233%	-204.8756732%	-0.0000200%
12/31/2003	0.29%	1.87%	0.53%	98.3%	-1.62%	112.1%	1.80%	1.58%	0.24%	100.3%	-0.59%	55.5350081%	17.1277580%	0.0000123%
1/31/2004	-0.08%	1.59%	0.17%	98.3%	-1.70%	113.9%	1.59%	1.66%	0.25%	100.5%	-0.42%	-15.5275576%	-0.3743772%	0.0000008%
2/29/2004	0.24%	0.86%	0.34%	98.5%	-1.46%	114.9%	0.86%	0.62%	0.09%	100.8%	-0.09%	46.0790290%	9.7838539%	0.0000027%
3/31/2004	0.03%	0.46%	0.09%	98.5%	-1.43%	115.4%	0.46%	0.43%	0.06%	100.9%	0.01%	4.9866813%	0.0124004%	0.0000000%
4/30/2004	-1.29%	-0.90%	-1.23%	97.3%	-2.70%	114.4%	-0.90%	0.39%	0.06%	99.7%	-1.23%	-248.2681379%	-1530.2520165%	-0.0002151%
5/31/2004	-0.41%	-0.45%	-0.42%	96.9%	-3.10%	113.9%	-1.34%	-0.03%	-0.01%	99.3%	-1.64%	-79.8321594%	-50.8784217%	-0.0000144%
6/30/2004	-0.25%	-0.13%	-0.23%	96.6%	-3.34%	113.7%	-1.47%	0.11%	0.02%	99.0%	-1.87%	-47.9576428%	-11.0299486%	-0.0000033%
7/30/2004	0.12%	-2.11%	-0.22%	96.7%	-3.23%	111.3%	-3.56%	-2.23%	-0.33%	98.8%	-2.08%	22.1856865%	1.0919899%	-0.0000033%
8/31/2004	0.48%	-0.41%	0.35%	97.2%	-2.76%	110.9%	-3.95%	-0.89%	-0.13%	99.2%	-1.74%	92.3357944%	78.7245651%	-0.0000185%
9/30/2004	-0.34%	1.40%	-0.08%	96.9%	-3.09%	112.4%	-2.60%	1.75%	0.26%	99.1%	-1.82%	-66.6659111%	-29.6286222%	0.0000122%
10/29/2004	0.06%	-1.06%	-0.11%	96.9%	-3.03%	111.2%	-3.64%	-1.12%	-0.17%	99.0%	-1.92%	10.7629675%	0.1246798%	-0.0000005%
11/30/2004	-0.97%	1.87%	-0.55%	96.0%	-3.98%	113.3%	-1.83%	2.85%	0.43%	98.4%	-2.46%	-188.0029026%	-664.4979776%	0.0001416%
12/31/2004	-0.03%	1.13%	0.14%	95.9%	-4.01%	114.6%	-0.72%	1.16%	0.17%	98.6%	-2.32%	-6.3361165%	-0.0254372%	0.0000001%
1/31/2005	-0.30%	-0.97%	-0.40%	95.7%	-4.29%	113.5%	-1.69%	-0.68%	-0.10%	98.2%	-2.71%	-57.9655841%	-19.4764881%	-0.0000124%
2/28/2005	-0.54%	0.96%	-0.31%	95.1%	-4.81%	114.6%	-0.75%	1.50%	0.23%	97.9%	-3.02%	-104.1657948%	-113.0252290%	0.0000168%
3/31/2005	-0.23%	-1.55%	-0.43%	94.9%	-5.03%	112.8%	-2.28%	-1.32%	-0.20%	97.4%	-3.43%	-44.9509612%	-9.0827414%	-0.0000106%
4/29/2005	0.34%	-1.09%	0.13%	95.2%	-4.70%	111.5%	-3.35%	-1.44%	-0.22%	97.6%	-3.31%	65.1103135%	27.6025598%	-0.0000170%
5/31/2005	0.10%	0.62%	0.18%	95.3%	-4.61%	112.2%	-2.75%	0.52%	0.08%	97.7%	-3.14%	18.6597641%	0.6497084%	0.0000002%
6/30/2005	-0.08%	1.85%	0.21%	95.3%	-4.69%	114.3%	-0.96%	1.93%	0.29%	97.9%	-2.93%	-15.9949330%	-0.4092110%	0.0000010%
7/29/2005	-0.66%	3.56%	-0.03%	94.6%	-5.32%	118.4%	2.57%	4.22%	0.63%	97.9%	-2.96%	-127.6996988%	-208.2426199%	0.0001397%
8/31/2005	0.42%	-0.05%	0.35%	95.0%	-4.91%	118.3%	-0.05%	-0.48%	-0.07%	98.3%	-2.62%	80.8173297%	52.7853602%	-0.0000079%
9/30/2005	-0.56%	1.22%	-0.29%	94.5%	-5.45%	119.8%	1.16%	1.78%	0.27%	98.0%	-2.91%	-108.7021357%	-128.4441208%	0.0000264%
10/31/2005	-0.41%	-2.64%	-0.75%	94.1%	-5.84%	116.6%	-2.64%	-2.23%	-0.33%	97.2%	-3.63%	-80.1939124%	-51.5732149%	-0.0000528%
11/30/2005	0.12%	2.61%	0.49%	94.2%	-5.73%	119.6%	-0.11%	2.49%	0.37%	97.7%	-3.16%	22.7907559%	1.1837941%	0.0000031%
12/30/2005	0.08%	1.79%	0.34%	94.3%	-5.65%	121.8%	1.68%	1.71%	0.26%	98.1%	-2.83%	14.9744245%	0.3357766%	0.0000008%
1/31/2006	-0.18%	2.84%	0.27%	94.1%	-5.82%	125.2%	2.84%	3.02%	0.45%	98.3%	-2.57%	-35.6186785%	-4.5189070%	0.0000084%
2/28/2006	-0.23%	0.35%	-0.15%	93.9%	-6.04%	125.7%	0.35%	0.59%	0.09%	98.2%	-2.71%	-45.4273065%	-9.3745615%	-0.0000002%
3/31/2006	-0.28%	1.88%	0.04%	93.6%	-6.31%	128.0%	1.88%	2.17%	0.32%	98.2%	-2.67%	-55.2995424%	-16.9108179%	0.0000123%
4/28/2006	-0.03%	0.61%	0.07%	93.6%	-6.34%	128.8%	0.61%	0.64%	0.10%	98.3%	-2.61%	-6.4786372%	-0.0271926%	0.0000000%
5/31/2006	-0.17%	-1.55%	-0.38%	93.5%	-6.50%	126.8%	-1.55%	-1.38%	-0.21%	97.9%	-2.98%	-33.9158803%	-3.9012994%	-0.0000060%
6/30/2006	-0.11%	-1.02%	-0.25%	93.4%	-6.60%	125.5%	-2.55%	-0.91%	-0.14%	97.7%	-3.22%	-22.1952331%	-1.0934002%	-0.0000019%
7/31/2006	0.41%	-1.00%	0.20%	93.7%	-6.22%	124.3%	-3.53%	-1.41%	-0.21%	97.9%	-3.03%	78.0098416%	47.4731651%	-0.0000229%
8/31/2006	0.54%	1.16%	0.63%	94.2%	-5.72%	125.7%	-2.41%	0.62%	0.09%	98.5%	-2.41%	103.1345237%	109.7014082%	0.0000222%
9/29/2006	0.15%	0.50%	0.20%	94.4%	-5.57%	126.3%	-1.92%	0.35%	0.05%	98.7%	-2.21%	28.6003232%	2.3394449%	0.0000002%
10/31/2006	0.02%	2.84%	0.44%	94.4%	-5.55%	129.9%	0.86%	2.82%	0.42%	99.1%	-1.78%	3.2862527%	0.0035490%	0.0000001%
11/30/2006	0.24%	1.89%	0.49%	94.6%	-5.32%	132.4%	1.89%	1.65%	0.25%	99.6%	-1.30%	46.0411074%	9.7597183%	0.0000086%
12/29/2006	-0.37%	1.49%	-0.09%	94.3%	-5.68%	134.4%	1.49%	1.87%	0.28%	99.5%	-1.39%	-72.3346710%	-37.8477034%	0.0000156%
1/31/2007	-0.17%	1.47%	0.07%	94.1%	-5.84%	136.3%	1.47%	1.64%	0.25%	99.6%	-1.32%	-33.6833127%	-3.8215926%	0.0000033%
2/28/2007	0.66%	-0.21%	0.53%	94.7%	-5.22%	136.1%	-0.21%	-0.87%	-0.13%	100.1%	-0.79%	126.1061520%	200.5436070%	-0.0000258%
3/30/2007	0.00%	0.40%	0.06%	94.7%	-5.22%	136.6%	0.19%	0.40%	0.06%	100.2%	-0.74%	-0.6014093%	-0.0000218%	0.0000000%
4/30/2007	-0.01%	2.77%	0.41%	94.7%	-5.23%	140.4%	2.77%	2.78%	0.42%	100.6%	-0.33%	-2.5379989%	-0.0016348%	0.0000000%
5/31/2007	-0.49%	3.02%	0.03%	94.3%	-5.70%	144.6%	3.02%	3.51%	0.53%	100.6%	-0.30%	-95.5038545%	-87.1089340%	0.0000648%
6/29/2007	0.04%	-0.42%	-0.03%	94.3%	-5.66%	144.0%	-0.42%	-0.46%	-0.07%	100.6%	-0.33%	7.1841433%	0.0370787%	-0.0000001%
7/31/2007	0.52%	-1.24%	0.25%	94.8%	-5.17%	142.2%	-1.66%	-1.76%	-0.26%	100.8%	-0.08%	98.6242227%	95.9291907%	-0.0000429%
8/31/2007	0.81%	-1.28%	0.49%	95.5%	-4.41%	140.4%	-2.91%	-2.08%	-0.31%	101.3%	0.42%	154.2478032%	366.9923090%	-0.0001073%
9/28/2007	0.35%	0.56%	0.38%	95.9%	-4.07%	141.2%	-2.37%	0.21%	0.03%	101.7%	0.38%	66.6040687%	29.5462441%	0.0000020%
10/31/2007	0.03%	2.90%	0.46%	95.9%	-4.05%	145.3%	0.46%	2.87%	0.43%	102.2%	0.46%	5.1390029%	0.0135718%	0.0000002%
11/30/2007	1.49%	-0.47%	1.20%	97.3%	-2.61%	144.6%	-0.47%	-1.96%	-0.29%	103.4%	1.20%	286.3335655%	2347.5604484%	-0.0001899%
12/31/2007	-0.07%	-0.44%	-0.12%	97.3%	-2.68%	144.0%	-0.90%	-0.37%	-0.06%	103.3%	-0.12%	-13.7948885%	-0.2625153%	-0.0000004%
1/31/2008	1.40%	-1.17%	1.02%	98.6%	-1.32%	142.3%	-2.06%	-2.57%	-0.39%	104.3%	0.89%	269.1075255%	1948.8460295%	-0.0003058%
2/29/2008	0.94%	1.39%	1.01%	99.6%	-0.39%	144.3%	-0.70%	0.45%	0.07%	105.4%	1.01%	179.8181012%	581.4337296%	0.0000870%
3/31/2008	-0.06%	-2.81%	-0.47%	99.5%	-0.45%	140.2%	-3.50%	-2.75%	-0.41%	104.9%	-0.47%	-11.6576698%	-0.1584292%	-0.0000012%
4/30/2008	-1.22%	2.40%	-0.67%	98.3%	-1.66%	143.6%	-1.18%	3.62%	0.54%	104.2%	-1.14%	-234.7601893%	-1293.8184884%	0.0002997%
5/30/2008	-0.73%	2.39%	-0.26%	97.6%	-2.38%	147.0%	1.19%	3.12%	0.47%	103.9%	-1.40%	-140.5883657%	-277.8741502%	0.0001067%
6/30/2008	0.00%	-1.06%	-0.16%	97.6%	-2.38%	145.4%	-1.06%	-1.06%	-0.16%	103.7%	-1.56%	-0.6014093%	-0.0000218%	0.0000000%
7/31/2008	0.11%	-2.15%	-0.23%	97.7%	-2.27%	142.3%	-3.18%	-2.26%	-0.34%	103.5%	-1.78%	20.0805346%	0.8097031%	-0.0000028%
8/29/2008	0.30%	-0.90%	0.12%	98.0%	-1.98%	141.0%	-4.06%	-1.20%	-0.18%	103.6%	-1.66%	57.6214637%	19.1316690%	-0.0000116%
9/30/2008	0.54%	-2.59%	0.07%	98.5%	-1.45%	137.4%	-6.54%	-3.13%	-0.47%	103.7%	-1.60%	102.3854900%	107.3285444%	-0.0000845%
10/31/2008	0.65%	-2.49%	0.18%	99.1%	-0.81%	134.0%	-8.87%	-3.13%	-0.47%	103.9%	-1.42%	124.1871957%	191.5272004%	-0.0001200%
11/28/2008	0.86%	-1.49%	0.50%	100.0%	0.04%	132.0%	-10.22%	-2.34%	-0.35%	104.4%	-0.92%	164.0940396%	441.8536215%	-0.0001368%
12/31/2008	0.13%	-1.69%	-0.14%	100.1%	0.13%	129.7%	-11.74%	-1.82%	-0.27%	104.3%	-1.06%	25.0857899%	1.5786409%	-0.0000035%

	Core	Satellite	Overall Portfolio		
Monthly Mean Return	0.003%	0.392%	0.061%		
Monthly Standard Deviation	0.520%	1.659%	0.433%	$\beta_{s,c}{}^{(2)}$	-1.01566
Monthly Variance	0.003%	0.028%	0.002%		
Covariance(R_S,R_C)	-0.0027%				
Skewness	0.259	0.040	0.014	$\beta_{s,c}{}^{(3)}$	-0.536136247
Excess Kurtosis	0.982	-0.719	0.696		
Max Drawdown	**-6.60%**	**-11.74%**	**-3.63%**		

See Risk Measures.xls

this is third order moment beta. It tells if the satellite can improve the skewness of the overall portfolio. When beta is less than or equal to 1, then it's an improvement. Don't confuse the c

fourth moment beta less than or equal to 1, with respect to the core. Clearly, the excess kurtosis is better. All the lights turn green. Combining the assets makes sense. Now we check whether these results

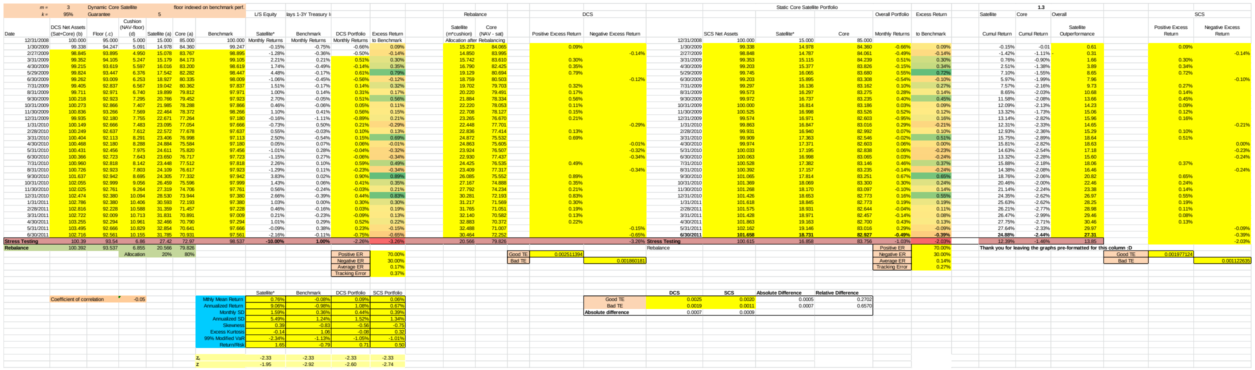
	Core	Satellite	Overall Portfolio	
Annualized Volatility	1.801%	5.747%	1.499%	
Weekly drawdown	-0.013	-0.028	-0.012	
99% Cornish Fisher VaR				
Information Ratio		0.206	0.206	
Monthly Tracking Error		0.019	0.003	the annual tracking error is 2%
Annual Tracking Error		**6.55%**	0.98%	
	weight*TE(Satellite) =	0.0028		the annual risk budget is 2%
	Annualized =	**0.1474**		

Appendix 2

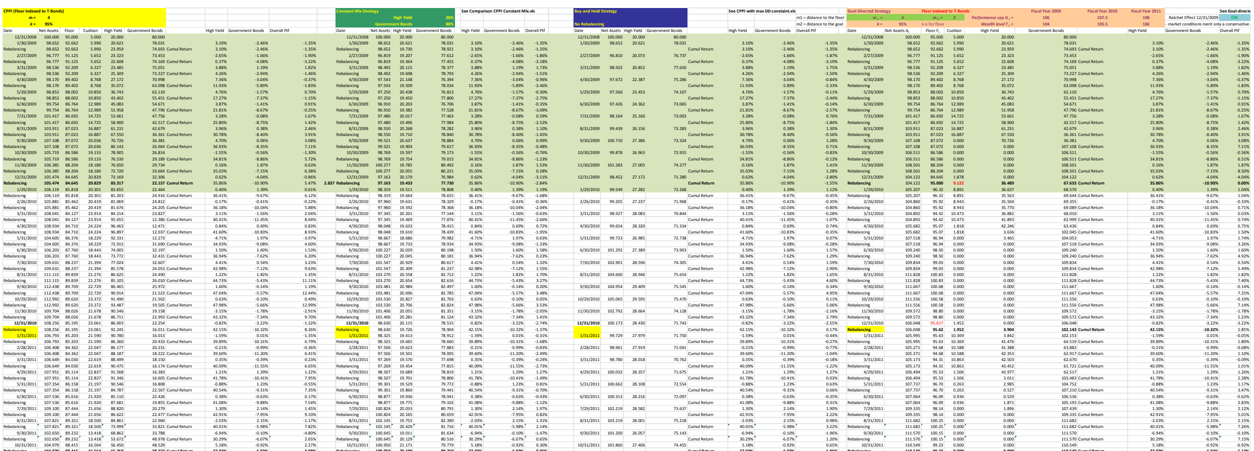
	Core	Satellite	Overall Portfolio		
Monthly Mean Return	0.003%	0.392%	0.122%		
Monthly Standard Deviation	0.520%	1.659%	0.520%	$\beta_{s,c}{}^{(2)}$	-1.01566
Monthly Variance	0.003%	0.028%	0.003%		
Covariance(R_S,R_C)	-0.0027%				
Skewness	0.259	0.040	-0.110	$\beta_{s,c}{}^{(3)}$	-0.536136247
Excess Kurtosis	0.982	-0.719	-0.020		
Max Drawdown	**-6.60%**	**-11.74%**	**-3.30%**		

	Core	Satellite	Overall Portfolio	
Annualized Volatility	1.801%	5.747%	1.802%	
Weekly drawdown	-0.013	-0.028	-0.012	
99% Cornish Fisher VaR				
Information Ratio		0.206	0.206	
Monthly Tracking Error		0.019	0.006	**the annual tracking error is 2%**
Annual Tracking Error		**6.55%**	2.00%	
	weight*TE(Satellite) =	0.0058		**the annual risk budget is 2%**
	Annualized =	**0.3002**		

Appendix 3 (fully scalable)



Appendix 4 (fully scalable)



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