

**Graduate Program in Software**  
**CSIS 734-01: Data Mining & Predictive Analytics**  
Assignment #9 (100 points)  
Due Date: May 12<sup>th</sup>, 2018

1) What are the Silhouette coefficients for the following 7 points and their clustering results?

Data	Cluster	Silhouette Coefficient
1	1	
2	1	
5	1	
11	1	
12	2	
20	3	
21	3	

**Definitions**

**Internal cohesion**

Find the distance of each point to all others in the same cluster, then average them.

**Center**

Find the center by averaging their locations

**External cohesion**

Find the distance of each point to the centers of the other clusters,  
*you take minimum to penalize it*

**Silhouette coefficient**

$$Sq = (Eq - lq) / \text{Max}(Eq, lq)$$

*1 is the best, -1 is the worse*

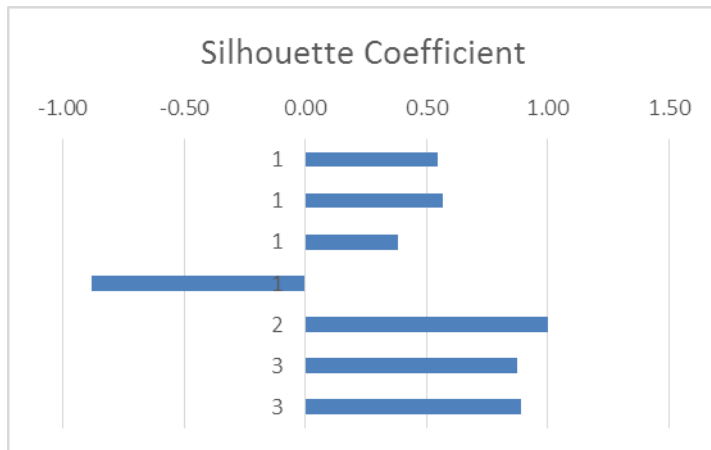
### Equations:

	A	B	C	D	E	F
			<b>Internal</b>		<b>External</b>	<b>Silhouette</b>
1	<b>Data</b>	<b>Clust</b>	<b>Cohesion</b>	<b>Center</b>	<b>Cohesion</b>	<b>Coefficient</b>
2	1	1	$=((A3-A2)+(A4-A2)+(A5-A2))/3$	$=AVERAGE(\$A\$2:\$A\$5)$	$=MIN((D6-A2),(D7-A2))$	$=(E2-C2)/MAX(C2,E2)$
3	2	1	$=((A3-A2)+(A4-A3)+(A5-A3))/3$	$=AVERAGE(\$A\$2:\$A\$5)$	$=MIN((D6-A3),(D7-A3))$	$=(E3-C3)/MAX(C3,E3)$
4	5	1	$=((A4-A2)+(A4-A3)+(A5-A4))/3$	$=AVERAGE(\$A\$2:\$A\$5)$	$=MIN((D6-A4),(D7-A4))$	$=(E4-C4)/MAX(C4,E4)$
5	11	1	$=((A5-A4)+(A5-A3)+(A5-A2))/3$	$=AVERAGE(\$A\$2:\$A\$5)$	$=MIN((D7-A5),(D6-A5))$	$=(E5-C5)/MAX(C5,E5)$
6	12	2	0	$=A6$	$=MIN((D7-A6),(A6-D5))$	$=(E6-C6)/MAX(C6,E6)$
7	20	3	$=(A8-A7)/1$	$=AVERAGE(\$A\$7:\$A\$8)$	$=MIN((A7-D6),(A7-D5))$	$=(E7-C7)/MAX(C7,E7)$
8	21	3	$=(A8-A7)/1$	$=AVERAGE(\$A\$7:\$A\$8)$	$=MIN((A8-D6),(A8-D5))$	$=(E8-C8)/MAX(C8,E8)$

### Answers:

Data	Cluster	Internal Cohesion	Internal Center	External Cohesion	Silhouette Coefficient
1	1	5.00	4.75	11.00	0.55
2	1	4.33	4.75	10.00	0.57
5	1	4.33	4.75	7.00	0.38
11	1	8.33	4.75	1.00	-0.88
12	2	0.00	12.00	7.25	1.00
20	3	1.00	20.50	8.00	0.88
21	3	1.00	20.50	9.00	0.89

### Visual:



2) Use the following distance table to draw two dendrograms: one use single-link method and another one uses complete-link method.

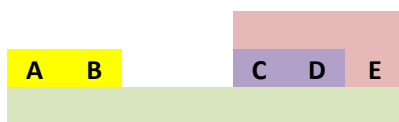
	A	B	C	D	E
A	0	1	4	5	6
B	1	0	3	4	5
C	4	3	0	1	2
D	5	4	1	0	1
E	6	5	2	1	0

Distance table					
	A	B	C	D	E
A	0	1	4	5	6
B	1	0	3	4	5
C	4	3	0	1	2
D	5	4	1	0	1
E	6	5	2	1	0

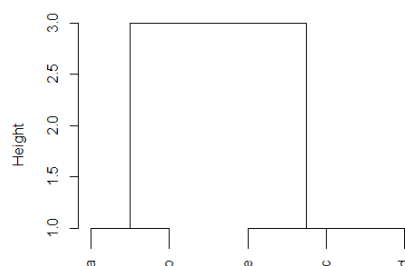
### Single-link

*min of min distances among clusters*

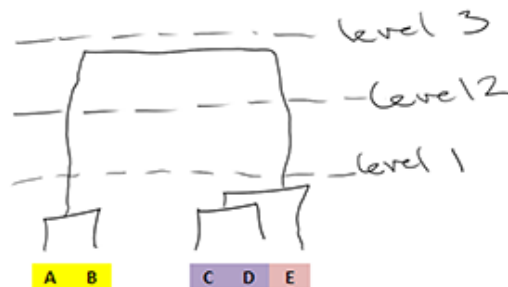
- 1 ((A, B), (C), (D), (E))      A-B has shortest distance 1
- 2 ((A, B), (C, D), (E))      C-D has shortest distance 1      e.g.  $AB \rightarrow C \min(4, 3) = 3$
- 3 (A, B, C, D), (E)       $d(A, B) = 1, d(A, B, C) = 3, d(A, B, D) = 4, d(A, B, E) = 5$   
CD-E has shortest distance 1
- 4 (A, B, C, D, E)      AB-CDE has shortest distance 3



Dendrogram using Single-link method



as.dist(distanceTable)  
hclust(\*, "single")



Distance table					
	A	B	C	D	E
A	0	1	4	5	6
B	1	0	3	4	5
C	4	3	0	1	2
D	5	4	1	0	1
E	6	5	2	1	0

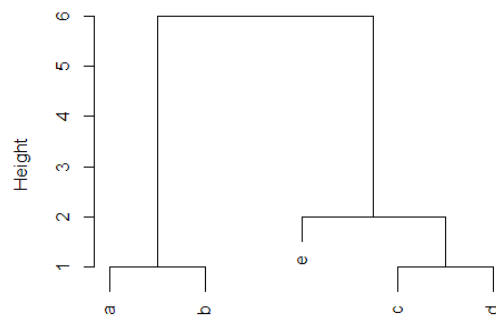
### Complete-link

*min of max distances among clusters*

- 1 ((A, B), (C), (D), (E))      A-B has shortest distance 1
- 2 ((A, B), (C, D), (E))      C-D has shortest distance 1      e.g.  $AB \rightarrow C \max(4, 3) = 4$
- 3 (A, B, C, D), (E)       $d(A, B), (C, D) = 5$ ,  $d(A, B), (E) = 6$ ,  $d(C, D), (E) = 2$   
CD-E has the shortest distance 2
- 4 (A, B), (C, D, E)       $d(A, B), (C, D, E) = 6$       AB-CDE has the shortest distance 6



Dendrogram using Complete-link method



```
as.dist(distanceTable)
hclust("complete")
```

