$(2,2)\,(1,1)$

$(2^n, 2^n)$

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Q1

$X =$ All multiples of 5

$R = \{(a,b) \mid a|b \ \forall\, a,b \in x\}$

$(5, 10, 15, 20, 25 \ldots)$

R is Reflexive $\forall\, a \in x,\ a|a$

R is antisymmetric $\rightarrow (a,b) \in R \Rightarrow (b,a) \notin R$ if $a \neq b$ $\forall\, a,b \in x$

$5|10$ but $10 \nmid 5$

$5|-5$ and $-5|5$ $\quad -5 \neq 5$

either symmetric nor antisymmetric

R is transitive.

Q2

$X = 2^n,\ n = 1, 2 \ldots \infty$

$R = \{(a,b) \mid a|b,\ \forall\, a,b \in x\}$

$x = \{2, 4, 8, 16 \ldots \infty\}$

R is Reflexive $\forall\, a \in x,\ a|a$

antisymmetric $\rightarrow (a,b) \in R \Rightarrow (b,a) \notin R$ if $a \neq b$

$\forall,\ a,b \in x,\ a|b \Rightarrow b \nmid a\ \forall\ a \neq b$

For transitive $\rightarrow (a,b) \in R \Rightarrow (b,c) \in R$

$(a,c) \in R$

(5,10) (10,15)

$a^2 = b$, $b^2 = c$ $\quad b = a^3$ $\quad c = b^3$

$c = (a^2)^2$ $\quad c = (a^3)^3 = a^9$

Q3 $\quad X = 2^n, \; n = 1, 2, \dots \infty$

$R = \{(a, b) \mid b = 2^r, \; r \in \text{even} \; \forall a, b \in X\}$

$b = 4, 16, 64, 256 \dots$

- every element is not a even power of set.

$\boxed{b = a^r} \rightarrow$ r is odd, $\forall a, b \in X$

$r = 1, 3, 5, 7, \dots n$

Q4 Reflexive, antisymmetric and transitive.

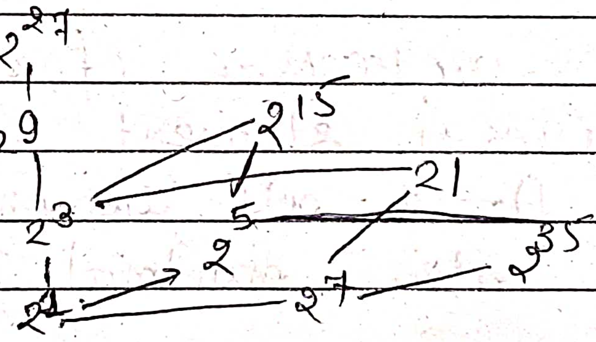
$\boxed{\text{POSET}}$

$b = a^3 \quad c = b^3$

$c = (a^3)^3 = a^9$

$a^1 \Rightarrow$ minimal power

maximal power.



Q. $X = \{\text{Natural Numbers}\}$

$R = \{(a, b) \mid b = a^2 \quad \forall a, b \in X$

- R is not reflexive

antisymmetric

$R \neq b = a^2$

$a = b^2$

$b = a^2$

$b = a^2 \quad c = b^2 \Rightarrow c = (a^2)^2 = a^4$

Not transitive.

b c $i^2 = -1$

$-i^2 \quad i^2$

$(i, -i, 1, -1)$

Q: $X = \{ \text{complex numbers} \}$

$R = \{ (a,b) \mid a|b \ \forall \ a, b \in X \}$

- R is Reflexive $\quad a|a$

$i|-i \qquad -i|i \qquad -i \neq i$

- R is symmetric. $\quad 1/i = \frac{-i^2}{i} = -i \qquad \frac{-1}{i} = \frac{i^2}{i}$

- R is transitive. $\quad i|-i \qquad -i|1$

$1 \qquad -i$

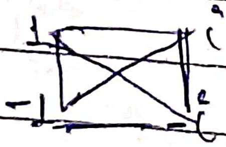
$(a,b) \in R \ \& \ (b,c) \in R \Rightarrow (a,c) \in R$

$(a|b)$ or $(b,c) \Rightarrow (c|a)$

equivalence Relation

\# 1 is predecessor of all element so 1 is minimal element.
$(i, -i, 1) \longrightarrow$ all element are predecessor so -1 is also minimal element.

\# When minimal or maximal element both are same so we formed circuit.



$\longrightarrow$ all are direct relationship

\# This is case when all the element are comparable but not a chain.

$\frac{i^2}{-i} = -i$ (a,b) $(b,a) \in R$

$b = a^2$ $a^2 = b$ b/a^2

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Q4. $X = \{i, -i, 1, -1\}$

$R = \{(a,b) \mid b = a^r,\ r = \text{even}\ \forall\ a,b \in X$

$b = a^2, a^4, a^6, \ldots\ldots a^\infty$

- R is not reflexive. $\{$ bcz all are not power of a^2
- R is not symmetric
  it is anti symmetric $\frac{i^2}{-i} = -i$ $\frac{i^2}{-i} = \frac{+1}{+i} = \frac{1}{i}$
- R is not transitive. $\frac{(-i)}{i^2} \Rightarrow \frac{+i}{+1} \Rightarrow \frac{i}{1}$

$\frac{+i^2}{+i} = i$ $\frac{-i^2}{i^2}$

$b = a^2$ $c = b^2$

$c = (a^2)^2$

$\boxed{c = a^4}$

- $R = \{(a,b) \mid b = a^r,\ r = \text{odd}\ \forall\ a,b \in X$
- R is not Reflexive
- R is neither symmetric nor antisymmetric

$(i, i^3) \in R$ $(i, i^1) \in R$

$(i, -i) \in R$ $(-i, (-i)^3) = (-i, i) \in R$

$(-i)^3 = (-1)^3 \times i^3 = i$

- R is transitive.

(i, i^3), $(i^3, (i^3)^3) \Rightarrow (i, i^9) \Rightarrow (i, i)$

$(i, -i)$, $(-i, i)$ (i, i)

This

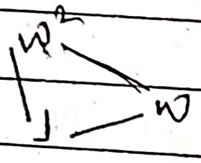
Que $x = \{1, \omega, \omega^2, \omega^3\}$ ($\omega^3 \to 1$)

$R = \{(a,b) \mid a|b \quad \forall\ a,b \in x\}$

- R is reflexive $\qquad a|a \quad \forall\ a,b \in x$.
- $(1/\omega), (\omega/1)$ $\qquad (a/b)\ (b/a) \in R \Rightarrow \forall a \in x$ and $a \neq b$

R is symmetric

- $(1/\omega^2), (\omega/\omega^2) \to (1/\omega^2)$ $\qquad (a/b)\ (b/c) \Rightarrow (a/c)$ $\forall\ a,b,c \in x$



every pair is compar

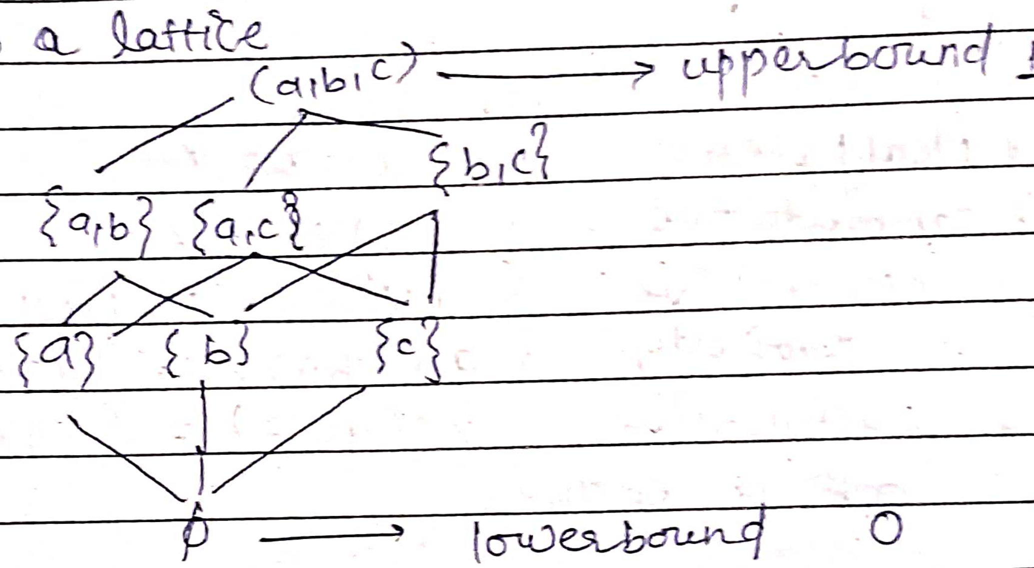
Meet $\wedge$ $\cap$ $a \wedge b = lub(a,b)$

Join $\vee$ $\cup$ $a \vee b = glb(a,b)$

$X = \{a,b,c\}$ $P(X)$ is a lattice

$R = \{(x,y) \mid \forall\ x,y \in P(X),\ x \subseteq y\}$

$(R, \cap, \cup)$ is a lattice



\# ϕ is the least count

$0 \wedge x = 0\ \forall x \in L$

$1 \vee x = x \vee 1 = 1\ \forall x \in L$

y is complement of x if

$x \wedge y = 0$, $x, y \in L$

$x \vee y = 1$

$\{a\} \wedge \{b,c\} = \phi = 0$

$\{a\} \vee \{b,c\} = \{a,b,c\} = 1$

$\{b\} \cap \{a,c\} = \phi = 0$

$\{b\} \cup \{a,c\} = \{a,b,c\} = 1$

$\{c\} \wedge \{a,b\} = \phi = 0$

$\{c\} \vee \{a,b\} = \{a,b,c\} = 1$

Complemented Lattice :-

A lattice $\{L, \wedge, \vee\}$ is said to be complemented if $\exists$ a complemented of every element $x \in L$

- identpatent $\quad x \wedge x = x$
- Commutative $\quad x \wedge y = y \wedge x \qquad x \cup y = y \cup x$
- Associative property $\begin{cases} a \vee (b \vee c) = (a \cup b) \cup c \\ a \cap (b \cap c) = (a \cap b) \cap c \end{cases}$
- Distributive $\quad x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$

a lattice

$\phi \cap (\{a\} \vee \{b\}) = (\phi \cap \{a\}) \vee (\phi \cap b)$

$(\{a,b\}) = \phi \vee \phi$

$\{a\} \wedge (\{b\} \vee \{a,c\}) = \qquad = (\{a\} \cap \{b\}) \vee (\{a\} \cap$

$\{a\} \wedge \{a,b,c\} \qquad = \phi \vee \{a\}$

$a = a$

$\{a\} \vee (\{b\} \wedge \{c\}) = (\{a\} \vee \{b\}) \wedge (\{a\} \vee \{c\})$

$\{a\} \vee \phi = \{a,b\} \wedge \{a,c\}$

$\{a\} = \{a\}$

$\phi \wedge (\{a\} \vee \{a,b\}) = (\phi \wedge \{a\}) \vee (\phi \wedge \{a,b\})$

$\phi \wedge \{a,b,c\} = \phi$
$\phi \vee \{a,b,c\} = \{a,b,c\}$
$0 \wedge 1 = 0$
$1 \vee 0 = 1$

$\phi \wedge \{a,b\} = \phi \vee \phi$
$\phi = \phi$

take $\quad x, y, z \in L$
$x \wedge y = x \wedge z$
$x \vee y = x \vee z$

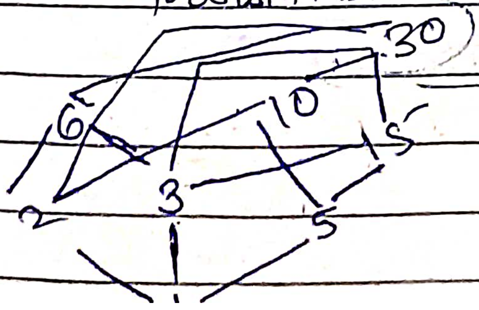
- $\{a\} \wedge (\{b\} \vee \{c\}) = (\{a\} \wedge \{b\}) \vee (\{a\} \wedge \{c\})$
  $\{a\} \wedge \{b,c\} = \phi \wedge \phi$

- $\{a\} \vee (\{b\} \wedge \{c\}) = (\{a\} \vee \{b\}) \wedge (\{a\} \vee \{c\})$
  $\{a\} \vee \phi = \{a,b\} \wedge \{a,c\}$
  $a = a$.

Boolean algebra:- A bounded, complimented distributive lattice is called boolean algebra.

Q $\quad S = \{1, 2, 3, 5, 6, 10, 15, 30\}$
$\{S, a/b\}$
$R = \{(a,b) \mid \forall\ a, b \in S,\ a/b\}$ using poset

Sol^n
- Reflexive
- antisymmetric
- transitive.



a(1,2)
$\frac{1}{2}, \frac{2}{1} \quad 1/2$
$\frac{1}{2}, 2$
$(a,b)\ (b,c) \Rightarrow (a,c)$
$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) \Rightarrow \frac{1}{3}$

$\phi = \{ \}$

$\{ \} = \{ \}$

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Group → Let X is any non-empty set. Let ~~start~~ $*$ denotes the arithmetic operation. Then mapping $X * X \longrightarrow X$ forms a group G with respect to operation $*$ if it satisfies following properties.

$\forall\ a, b \in X, \quad a * b \in X$ — Closure

$\forall\ a, b, c \in X \quad a*(b*c) = (a*b)*c$ — Associative

$\forall\ a \in X, \ \exists\ e \in X$ such that $a * e = e * a = a$

e is the entity element.

$\forall\ a \in X, \ \exists\ a^{-1} \in X$, such that $a * a^{-1} = a^{-1} * a = e$

a^{-1} is universe of a.

$(X, *)$ is a group

Q Let Y is a set of non-negative integers, $(Y, +)$

$0+1 \quad 1+2, \quad 2+3$

Sol^n

$1+2+0 = 2+0+1$ ✓

$\forall\ a, b \in Y, \quad a+b \in Y$

$a, b, c \in Y \quad (a+b)+c = a+(b+c)$ — assoc

$\forall\ a \in Y, \ \exists\ 0 \in Y$ such that $a+0 = 0+a = a$

0 is the idendity ~~nothing~~ element

$\forall\ a \in Y\ \exists\ -a \in Y$ such that $a+(-a) = (-a)+a = 0$

$-a$ is inverse of a

$(Y, +)$ is a group.

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Q. Let I is a set of integers $(I, +)$

$\forall\ a, b \in I,\quad a+b \in I$

$\forall\ a, b, c \in I,\quad (a+b)+c = a+(b+c)$ Associative

$\forall\ a \in I,\ \exists\ 0 \in I,$ such that $a+0 = 0+a = a$

0 is the identity element

$\forall\ a \in I\ \exists\ -a \in I$ such that $a+(-a) = (-a)+a = 0$

$-a$ is inverse of a.

$(X, *)$ is a group.

$(R, +)$ is a group

$(I, +)$ is a group.

Que. Let I is a set of integers $(I, \cdot)$

$\forall\ a, b \in I \Rightarrow a \cdot b \in I$

$\forall\ a, b, c \in I\quad (a \cdot b) \cdot c = a \cdot (b \cdot c)$ Associative

~~$\forall\ a \in I\ \exists\ a \in I$ such that $a+0 = 0+a = a$~~

~~$\forall\ a \in I\ \exists,\ -a \in R\quad a+(-a)$~~

$\forall\ a \in I\ \exists,\ e \in R$ such that $a \cdot e = e \cdot a = a\qquad e = 1$

1 is the identity element.

$\forall\ a \in I\ \nexists\ \frac{1}{a} \in I$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

$-a$ is inverse of a

$(I, \cdot)$ is not a group.

a^{-1}

Q Let R is a set of real $(R, \cdot)$

$\forall\ a, b \in R \Rightarrow a \cdot b \in R$ closure.

$\forall\ a, b, c \in R \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$ Associat

$\forall\ a \in R\ \exists\ e \in R$ such that $a \cdot e = e \cdot a = a$ $e=1$ is the identity elem

$\forall\ a \in R\ \nexists\ \frac{1}{a} \in R$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

$\frac{1}{a}$ is the inverse of a.

$(R, \cdot)$ is not a group

Q $(R - \{0\}, \cdot)$

$\forall\ a, b \in R - \{0\},\ a \cdot b \in R$ closure.

$\forall\ a, b, c \in R - \{0\} \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$

$\forall\ a \in R - \{0\}\ \exists\ e \in R - \{0\}$ such that $a \cdot e = e \cdot a = a$

$e = 1$ is the idendity element.

$\forall\ a \in R - \{0\}\ \nexists\ \frac{1}{a} \in R - \{0\}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

$\frac{1}{a}$ is the inverse of a.

$(R - \{0\}, \cdot)$ is a group.

Que $X = \{i, -i, 1, -1\}$

$(X, \cdot) \longrightarrow$ is group or not

$\forall\ a, b \in X,\ a \cdot b \in X$

$\forall\ a, b, c \in X \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$

$\forall\ a \in X\ \exists\ e \in X$ such that $a \cdot e = e \cdot a = a$

$e = 1$ is the identity ele...

in plus case

$0 \rightarrow$ identity elem

and multiply case 1 is $\rightarrow$ identity element

R → Real.

I → integer

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$\forall\, a \in X \quad \exists\, \frac{1}{a} \in X$ such $\frac{1}{a} \cdot a = \frac{1}{a} \cdot a = 1$

$\frac{1}{a}$ is inverse of X

$(X, \cdot) \longrightarrow$ is a group

Quasi group (Groupoid)

Binary operation Let G be non-empty set. Then operation $*$ is a binary operation if it associate a pair of elements in G such that

$*: G \times G \longrightarrow G$ ie. $*$ is a function.

Algebraic structure:-

Let G be a non-empty set, and $*$ is a binary operation in G, then $(G, *)$ is an algebraic structure if it satisfies following property.

$\forall\, a, b \in G, \quad a * b \in G$. closure.

Semi group:-

Let G be a non-empty set and $*$ is a binary operation in G. Then $(G, *)$ is ~~an algebraic~~ a semi-group.

1) $\forall\, a, b \in G, \ a * b \in G$. Closure

2) $\forall\, a, b, c \in G;\ (a * b) * c = a * (b * c)$ Associative.

Monoid Let G be a non-empty set and $*$ is a binary operation in G. Then $(G, *)$ is a monoid.

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1) $\forall$, $a, b \in G$, $a * b \in G$. closure,
2) $\forall a, b, c \in G$, $(a*b)*c = a*(b*c)$ ' Associative
3) $\forall a \in G \ \exists \ e \in G$ such that $a * e = e * a = a$ ide

Group:-
Let G be a non-empty set and $*$ is a binary oper in G. Then $(G, *)$ is a group.

1) $*$ $a, b \in G$, $a * b \in G$. closure
2) $\forall$ $a, b, c \in$ $(a*b)*c = a*(b*c)$
3) $\forall$ $a \in G$ $\exists$, $e \in G$ such that $a * e = e * a = a$ identi
4) $\forall$ $a \in G$ $\exists$ $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$
Inverse.

Abelian Group:-
all 4 properties then
(5) $\forall$ $a, b \in G$ $a * b = b * a$ commutative.

Que $a * b = \frac{ab}{2}$ where $a, b \in I$

i) $\forall$ $a, b \in R$, where $a * b = \frac{ab}{2} \in R$

ii) $\forall$ $a, b, c \in R$ $(a * b) * c = a * (b * c)$

$$\frac{ab}{2} * c = a * \frac{bc}{2}$$

$$\frac{\frac{ab}{2} \cdot c}{2} = \frac{a \cdot \frac{bc}{2}}{2} = \frac{abc}{4}$$

$(4)\frac{a^{-1}}{2} = 0$

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(iii) $\forall a \in G \quad \exists e \in G$ such that $a * e = e * a = a$

$$\frac{ae}{2} = \frac{ea}{2} \neq a \qquad e = 2$$

(iv) $\forall a \in G \quad \exists a^{-1} \in G$ such $a * a^{-1} = e, \ \frac{aa^{-1}}{2} = e$

$$\frac{aa^{-1}}{2} = 2$$

$$\Rightarrow a^{-1} = \frac{4}{a} \in R$$

its not group

Que $a * b = a+b+ab$ where $a, b \in (R - \{1\})$

if $a+b+ab \neq 1$
$a+b+ab = 1$
$a+b(1+a) = 1$

1) $\forall a, b \in R, \ a * b = a+b+ab \in R$

2) $\forall a, b, c \in R \quad (a*b) * c = a * (b*c)$

$$(a+b+ab) * c = a * (b+c+bc)$$

$$(a+b+ab) + c + (a+b+ab).c = a + (b+c+bc) + a.(b+c+bc)$$

$$\Rightarrow a+b+ab+c+ac+bc+abc = a+b+c+bc+ab+ac+abc$$

3) $\forall a \in G \quad \exists e \in G$ such that $a * e = e * a = a$

$$a+e+ae = e+a+ea = a$$

$$\Rightarrow e + ea = 0$$

$$e = -ea$$

$$1 = -a$$

$$a = -1$$

$\cancel{e(1+a) = 0}$

$\cancel{e = 0}$

$e(1+a) = 0$

$$b(1+a) = 1 - a$$

$$b = \frac{1-a}{1+a} \in R$$

(ii) $\forall a \in G$, $a^{-1} \in G$ such that $a * a^{-1} = e$

$$a + a^{-1} + aa^{-1} = 0$$

$$a + a^{-1}(1+a) = 0$$

$$a^{-1}(1+a) = -a$$

$$\boxed{a^{-1} = \frac{-a}{1+a}}$$

It is ~~not~~ a group.

Que $G = \{a^n \mid a, n \in I\}$

$\{G, \cdot\}$ is a group?

~~$\forall a, b \in I \Rightarrow a \cdot b \in R$~~

i) ~~$\forall a^m, b^n \in G \Rightarrow a^m \cdot b^n \in G$~~ $\forall a, b \in G \Rightarrow a \cdot b \in G$

~~2) $(\forall a^n, b^n, c^n \in G \Rightarrow (a^n \cdot b^m) \cdot c^n = (a^n) \cdot (b^n \cdot c^n)$~~

$$a^m \cdot a^n \in G \Rightarrow a^m \cdot a^n = a^{m+n} \in G, \quad m+n \in I$$

ii) $a^r, a^m, a^n \in G = a^r \cdot (a^m \cdot a^n) = a^{r+m+n} (a^r \cdot a^m) \cdot a^n$

$$\Rightarrow a^r (a^{m+n}) = (a^{r+m}) \cdot a^n$$

$$\Rightarrow a^{r+m+n} = a^{r+m+n} \quad \in I$$

(iii) $a^n \in G$, $a^{-n} \in G \Rightarrow a^n \cdot a^{-n} = a^0 \in G \Rightarrow a^0 = 1$

$a^n \cdot e = a^n$ $\boxed{e = 1}$

(iv) $a^n \in G$, $a^0 \in G \Rightarrow a^n \cdot a^0 = a^{n+0} = a^n$

$a^n \times \boxed{\frac{?}{a^n}} = 1$

$\boxed{a^{-n}}$

It is a group.

$a \times \frac{1}{a}$ a^{-1}

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Q

$X = \{a^m \mid m \in I$, and a is a fixed real number$\}$ it is abelian group or not.
$\{x, \cdot\}$

i) ~~$\forall a, b \in G \Rightarrow a \cdot b \in G$~~

$\cancel{a^m}$ a^n, $a^m \in x$

$a^r \cdot a^m = a^{(r+m)} \in x$ as $r+m \in I$

~~$r, m \in I$~~

ii) $a^k, a^j, a^m \in x$, $k, j, m \in I$,

$a^k \cdot (a^j \cdot a^m) = (a^k \cdot a^j) \cdot a^m \Rightarrow a^k \cdot a^{j+m} = a^{k+j} \cdot a^m$

$a^{k+j+m} = a^{k+j+m}$

iii) Inverse $\forall$ $m, -m \in I$, $a^m, a^{-m} \in x$ such that

$a^m \cdot a^{-m} = a^{-m} \cdot a^m = a^0 = 1$

(iii) $a^0 \in x$, $a^m \in x$ $a^m \cdot a^0 = a^0 \cdot a^m = a^{m+0} = a^m$

Identity.

(v) $\forall a^m, a^n \in x$ $a^m \cdot a^n = a^n \cdot a^m$

$\Rightarrow a^{m+n} = a^{m+n}$

$\{x, \cdot\}$ is an abelian group.

ORDER OF GROUP

The group $\{G, *\}$ will be finite or infinite to underlying set G is finite or infinite

If $(G, *)$ is finite then order of the group will equal to the number of elements in underlying

$$Y = \{i, 1, -1, -i\} \qquad (Y, \cdot)$$
$$O(Y) = 4$$

$$X = \{1, \omega, \omega^2\} \Rightarrow \{X, \cdot\}$$
$$O(X) = 3$$

Congruence modulo n

let $a, b \in I$ if $a-b$ is divisible by n, where $n \in I$ then a is congruence modulo n

$$a \equiv b \,(\text{modulo } n)$$

$\Rightarrow$ $a-b$ is divisible by n

$a-b = kn$ where k is an integer.

$a = b + kn$

Congruence modulo n class

The set of all elements a congruence modulo n given $a, n \in I$ denated by $[a]$,

$$[a] = \{x \mid x \in I,\ x \equiv a \,(\text{modulo } n)\}$$
$$[a] = \{x \mid x \in I,\ (x-a) \text{ is divisible by } n\}$$
$$[a] = \{x \mid x \in I,\ x-a = kn,\ k \in I\} =$$
$$\{x \mid x \in I,\ x = a + kn,\ k \in I\}$$

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$[a] = \{x \mid x \in I, \ x \equiv a \ (\text{modulo } 5)\}$

$23 \equiv 3$ modulo 5 $\Rightarrow$

$15 \equiv 0$ modulo 5

$31 \equiv 1$ modulo 5, $64 \equiv 4$ modulo 5

$42 \equiv 2$ modulo 5 $\quad 53 \equiv 3$ modulo 5

$[a] = \{x \mid x \in I, \ x \equiv a \ (\text{modulo } 5)\}$

$[0] = \{x \mid x \in I \quad x \equiv 0 \ (\text{modulo } 5)\} = \{\ldots -10, -5, 0, 5, 10, \ldots\}$

$[1] = \{x \mid x \in I, \ x \equiv 1 \ (\text{modulo } 5)\} = \{\ldots -9, -4, 1, 6, 11, 16 \ldots\}$

$[2] = \{x \mid x \in I, \ x \equiv 2 \ (\text{modulo } 5)\} = \{\ldots -8, -3, 2, 7, 12, 17 \ldots\}$

$[3] = \{x \mid x \in I, \ x \equiv 3 \ (\text{modulo } 5)\} = \{\ldots -7, -2, 3, 8, 13 \ldots\}$

$[4] = \{x \mid x \in I, \ x \equiv 4 \ (\text{modulo } 5)\} = \{\ldots -6, -1, 4, 9, 14 \ldots\}$

$[5] = \{x \mid x \in I, \ x \equiv 5 \ (\text{modulo } 5)\} = 0$

$I_5 = \{[0], [1], [2], [3], [4]\}$

The set of all elements a congruence modulo n, gives $a, n \in G$

is called equivalence class,

properties

$I_n = \{[0], [1], [2] \ldots \ldots [n-1]\}$

(i) $[n] = [0]$, $\quad [n+1] = [1]$

(ii) union of all residue classes of I_n will be I

$[0] \cup [1] \cup [2] \cup [3] \cup [4] = I$

(iii) Intersection of ~~any~~ two residue classes is null set ϕ

Addition.

$$[a] +_n [b] = \begin{cases} [a+b] & \text{if } a+b < n \\ [r] & \text{if } a+b \geq n \end{cases}$$

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* $[1] +_5 [2] \Rightarrow [1+2] = [3]$
* $[3] + [4] = [7] > 5$
$= [2]$
$7 = 2$ modulo 5

Multiplication,

$$[a] \times_n [b] = \begin{cases} [a.b] & \text{if } ab < n \\ [r] & \text{if } a.b \geq n \end{cases}$$

$[2].[1] \Rightarrow [2] < [5]$
$[3][4] \Rightarrow [12] > [5] \Rightarrow$ 2 modulo of 5
$= [2]$

Properties of a group

- Let $(G, *)$ be a group. Then the identity element is unique.

proof :- Let e_1 & e_2 are two identity elements in G $e_1 \neq e_2$

$e_2 * e_1 = e_1 * e_2 = e_1$ — ① e_2 is identity element
$e_1 * e_2$
$e_2 * e_1 = e_1 * e_2 = e_2$ — ② e_1 is identity element

From (i) and (ii), $e_1 = e_2$

Hence proved

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2. Let $(G, *)$ be a group. Then the inverse element is unique.

proof- Let a^{-1} and $(a')^{-1}$ are two inverses of $a^{-1} \neq (a')^{-1}$

$\forall a \in G \; \exists \; a^{-1} \in G$: $a * a^{-1} = a^{-1} * a = e$

$\forall a \in G \; \exists \; (a')^{-1} \in G$: $a * (a')^{-1} = (a')^{-1} * a = e$

$\forall (a')^{-1} \in G \; \exists \; e \in G \Rightarrow (a^{-1})^{-1} = (a')^{-1} * e = (a')^{-1} * (a * a^{-1}) = ((a')^{-1} * a) * a^{-1}$

$\Rightarrow e * a^{-1} = a^{-1}$

$\Rightarrow (a')^{-1} = a^{-1}$

3. Let $(G, *)$ be a group. Then left and right cancellation holds for the elements of G.

(i) $a * b = a * c \Rightarrow b = c$

(ii) $b * a = c * a \Rightarrow b = c$

Proof:- Let $a * b = a * c$

$\forall a \in G \; \exists \; a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

$a^{-1} * (a * b) = a^{-1} * (a * c)$

$(a^{-1} * a) * b = (a^{-1} * a) * c$

$e * b = e * c$

$b = c$

4. (P) Let $(G, *)$ be a group given $ax = b$ and $ay = b$ $\forall a, b, x, y \in G$ Then $x = y$

proof

Given $ax = b$

$ay = b$

$\forall a \in G \; \exists a^{-1} \in G$

$\Rightarrow a^{-1} * a * x = a^{-1} * b \Rightarrow e * x = a^{-1} * b$

$\Rightarrow x = a^{-1} b$ —— (iii)

$\Rightarrow a^{-1} * ay = a^{-1} * b \Rightarrow e * y = a^{-1} * b \Rightarrow y = a^{-1} b$ —

$\therefore$ from (iii) and (iv) we get $x = y$.

ORDER OF ELEMENT OF A GROUP

Let $(G, *)$ be a group and $a \in G$. Then if $\exists$ a positive integer m such that $a^m = e$ then m is said to be the order of element a in G.

for additive group, $ma = e$

if we cannot find such m then the order of the element a will be said to be infinite or zero

$G = \{1, w, w^2\}$

$(w)^3 = w^6 = w^9 = \dots = 1$

$O(w) = 3$

$O(w^2) = 3$

$O(1) = 1$

$(1)^1 = (1)^2 = (1)^3 = \dots = 1$

$(1)^m = 1$

$(w^2)^3 = (w^2)^6 = (w^2)^9 = \dots = 1$

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let $a \in G$, $e \in G$
$a \neq e$ and a is of order 1
The order of identity element is always 1.

$$a^1 = e \Rightarrow a = e$$

Let I is a set of integers. $(I, +)$ is a group.
$ma = 0$ $\quad O(0) = 1$
$0 = 1.0 = 2.0 = 3.0 = \ldots$

$m.a = 0$ $\quad m.1 \neq 0$
$m.2 \neq 0$

The order $O(1) = O(2) = O(3) \ldots = 0$ or infinite

~~order of element~~

Que $G = \{1, i, -1, -i\}$ $(G, \cdot)$ is a group.
$O(-1) = 2$
$O(1) = 1$ $\quad (1)^m = 1$
$(1)^2 = (1)^2 = (1)^3 = \ldots 1$

$(i)^4 = i^8 = i^{12} = \ldots = 1$
$O(i) = 4$
$O(-i) = 4$

Sub-semi group
Let $(X, *)$ is a semi group. Then non-empty subset Y of X. Then $(Y, *)$ will be sub-semi group of $(X, *)$ if $(Y, *)$ forms a semi-group.

Sub-Monoid →

Let $(X, *)$ is a monoid. Let Y is non-empty subset of X. Then $(Y, *)$ will be submonoid of $(X, *)$ if $(Y, *)$ forms a monoid.

Sub-Group

Let $(G, *)$ is a group. Let H is non-empty subset of G. Then $(H, *)$ will be subgroup of $(G, *)$ if $(H, *)$ forms a group.

$(I, +)$ is a group.

$H = 3I = \{0, \pm 3, \pm 6, \pm 9, \pm 12, ---\}$

$H = 3I \subset I$

set of complex number C

$G = C - \{0\}$, $(G, \cdot)$ forms a group.

$H = \{1, i, -1, -i\}$ $\quad H \subset G$.

$(H, \cdot)$ is a group.

$\Rightarrow H$ is a subgroup of G.

Let $(G, *)$ be group and a non-empty set $H \subseteq G$. Then $(H, *)$ is a subgroup of G iff it satisfies following properties.

(i) $\forall\, a, b \in H$, $a * b \in H$

and (ii) $\forall\, a \in H$ $\exists\, a^{-1} \in H$ such that $a * a^{-1} = e$

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proof- Given $H \subset G$, and $(G, *)$ is a group
Let H is subgroup of $G \Rightarrow$ H is a group in itself then $\forall a, b \in H$ $a * b \in H$
$\forall a \in H$ $\exists a^{-1} \in H$ such that $a * a^{-1} = e$

Converse $\Rightarrow$ H is subset of G and ~~satisfication~~ satisfies following properties.

(i) $\forall a, b \in H$, $a * b \in H$ Closure (given)

(ii) $\forall a \in H$, $\exists a^{-1} \in H$ such that $a * a^{-1} = e$
Inverse exist (given)

(iii) $\forall a, b, c \in H \Rightarrow a, b, c \in G$, since all the elements of G satisfy associative property.
$a * (b * c) = (a * b) * c$.

(iv) $\forall a, b \in H$, $a * b \in H$
Let $\Rightarrow$ $b = a^{-1}$ $a, a^{-1} \in H$ $a * a^{-1} \in H$ by closure property
$\Rightarrow e \in H$ identity element exist in H.

Theorem:-
Let $(G, *)$ be a group and a non-empty set $H \subset G$. forms a Subgroups $(H, *)$ iff
$\forall a, b \in H$, $a * b^{-1} \in H$

proof
Given $(H, *)$ is a subgroup of G.
(i) H satisfies closure property
$\forall a, b \in H$ $a * b \in H$

(iii) Inverse exists in H

$\forall a \in H, \exists a^{-1} \in H$ such that $a * a^{-1} = e$

from (i) & (iii) that

$\forall a, b \in H, \exists b^{-1} \in H, a * b^{-1} \in H$ by closure property.

<u>Converse</u> $\Rightarrow$ H is a non-empty subset of G and $(G, *)$ is a group and H satisfies following proper

$\forall a, b \in H, \quad a * b^{-1} \in H$

i) Let $c = b^{-1} \Rightarrow \forall a, c \in H \quad a * c \in H$

closure

ii) $\forall a, b, c \in H, \ a, b, c \in G$

$a * (b * c) = (a * b) * c$, since G is associative $\Rightarrow$ its subset will also be associative

iii) $\forall a, b \in H, \ a * b^{-1} \in H$

let $b = a \Rightarrow a * a^{-1} \in H = e \in H$

(iv) $\forall e, b \in H \quad e * b^{-1} \in H \Rightarrow b^{-1} \in H$

thus H is a subgroup of G.

Cyclic group:-

Let $(G, *)$ be a group. Then $(G, *)$ is said to be cyclic if all the elements of G can be expressed as some integral power of $a \in G$.

~~Hae~~ Here is said to generator of group and denoted (a)

$G = \{a^n, n \in I\}$

$G = \{1, i, -1, -i\}$

$\Rightarrow \{i^4, i, i^2, i^3\}$

$(G, \cdot)$ is cyclic group and i is the generator of this group.

$$\{(-i)^4, (-i)^3, (-i)^2, (-i)\}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$1 \quad i \quad -1 \quad -i$$

Que show that $(Z, +)$ is a group is $(Z, \cdot)$ group or not?

Sol^n $a, b \in \cancel{Z}$ $\quad a + b \in G$ closure

$a, b, c \in Z$ $\quad a + (b+c) = (a+b) + (c) \in G$

$(Z, +)$ is group

$\forall a, b \in Z$ $\quad a \cdot b \in Z$ Closure.

$\forall a, b, c \in Z$ $\quad a \cdot (b \cdot c) = (a \cdot b) \cdot (c) \in Z$ associative.

$\forall a \in Z, \nexists \frac{1}{a} \in Z$ $\quad \frac{1}{a} \cdot a = a \cdot \frac{1}{a} = 1$

$\forall a \in Z \Rightarrow \cancel{e \in R}$ such that $a \cdot e = e \cdot a = a$

$e = 1$

$(Z, \cdot)$ is not a group.

Que $(Q, +)$ is group, $(Q, \cdot)$ is not group

$\forall a, b \in Q$ $\quad a + b \in Q$

$\forall a, b, c \in Q$ $\quad a + (b+c) = (a+b) + c$ associative.

$\forall a \in Q$

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Que $Q^* = Q - \{0\}$
Show that $(Q^*, \cdot)$ is a group
→ it is group

Que $(R, +)$, $(R, \cdot)$
✓ ✗

Que Show that $U(n)$ is a group under multiplication operation & $n \geq 2$

$U(n) = \{x \in N \mid 1 \leq x \leq n$ & $gcd(x, n) = 1\}$

$U(8) = \{1, 3, 5, 7\}$

$x = 1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}$
$gcd(x, 8) = 1$

$U(6) = \{1, 5\}$

$U(10) = \{1, 3, 7, 9\}$

$U(8) = \{1, 3, 5, 7\}$

(i) Closure: $\forall a, b \in G$ s.t $a \circ b \in G$
$ax = 1 \pmod n$
$5 \times 7 = 1 \pmod 8$
$35 = 3 \pmod 8$

$ax = 1 \pmod 8$
$3 \times 3 = 1 \pmod 8$
$5 \times 5 = 1 \pmod 8$ identity.
$7 \times 7 = 1 \pmod 8$

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Que $Z_n = \{0, 1, 2, 3, \ldots, n-1\}$ is a group under addition modulo.

$Z_5 = \{0, 1, 2, 3, 4\}$ $\quad \forall a \in Z_n$

$\exists\, n-a \in Z_n$

s.t $a + (n-a) = (n-a) + a$

$= 0$

$Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$ $\quad \forall a \in Z_n$

inverse of
$0 = 0$
$4 = 3$
$2 = 5$
$1 = 6$

order of Group

order of $U(5) = \{1, 2, 3, 4\}$

$\boxed{O = 4}$

order of elements of group.

$G = \{1, -1\}, \quad \{\cdot\}$

$a \in G$

$O(a) = n$ $\quad \boxed{\begin{array}{l} a^n = e \\ n \cdot a = e \end{array}}$ $\quad n =$ least +ve integer.

$U(5) = \{1, 2, 3, 4\}$

$O(1) = 1$

$O(2) = 4$ $\quad 2^4 = 16 \pmod 5$

$O(3) = 4$

$O(4) = 2$ $\quad 81 = 3^4 \rightarrow 1 \pmod 5$

$4^2 = 16 \quad 1 \pmod 5$

$8\overline{)125}(1$

$o(2) =$

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Que $U(8) = \{1, 3, 5, 7\}$

$o(1) = 1$

$o(3) = 2$ $\qquad (3)^2 = 9 \pmod 8$

$o(5) = 2$ $\qquad (5)^2 = 25 \pmod 8$

$o(7) = 2$

$(7)^2 = 49 \pmod 8$

$1 \bmod 8$

Que $Z_3 = \{0, 1, 2\}$ $\Rightarrow$ $n.a \pmod 3 = e = 0$

$n.0 \pmod 3 = 0$

$1.0 \pmod 3 = 0$

$o(0) = 1$

$o(1) = 3$

$o(2) = 3$

Que $Z_5 = \{0, 1, 2, 3, 4\}$

$o(0) = 1$

$o(1) = 5$ $\qquad n.a \pmod 5 = e = 0$

$o(2) = 5$ $\qquad n.0 \pmod 5 = 0$

$o(3) = 5$ $\qquad 1.0 \pmod 5 = 0$

$o(4) = 5$ $\qquad 1 \pmod 5$ $\quad 16 = 1 \pmod 5$

Que $U(n)$ is a group under multiplication modulo. Z_n is a group under addition modulo.

$U(n) \rightarrow e = 1$

$Z_n \rightarrow e = 0$

$U(n) = \{x \in N \mid 1 \leq x \leq n, \gcd(x, n) = 1\}$

$Z_n = \{0, 1, 2, 3 \ldots, n-1\}$

Que theorem:-

$$o(a) = o(a^{-1})$$

proof let $o(a) = n$ ie. $a^n = e$

$o(a^{-1}) = m$ ie. $(a^{-1})^m = e$

i) $a^n = e$

$(a^n)^{-1} = e^{-1} = e$

$(a^{-1})^n = e \Rightarrow m/n$

ii) $(a^{-1})^m = e$

$(a^m)^{-1} = e$

$a^m = e^{-1} = e$

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Cyclic group :- $G = \{-1, 1\}$ with multiplication.

$G = \{1, -1, i, -i\}$ with multiplication.

(i^4) (i^2) $(-i^3)$ (i^3)

1 -1 i $-i$

Que

~~$G = \{-1, 1\}$ with multiplication.~~

Show that $z = \{0, \pm 1, \pm 2 \ldots\}$ is cyclic.
with multiplication

$n.a \Rightarrow a.a.a \ldots$

$\langle a \rangle = \{na \mid n \in z\}$

Q Show that $z_5 = \{0, 1, 2, 3, 4\}$ is cyclic.

$\langle a \rangle = \{na \mid n \in z$

if $a = 0$

$n \; a \pmod 5$

$$\left.\begin{aligned} 1.0 &= 0 \\ 2.0 &= 0 \\ 3.0 &= 0 \end{aligned}\right\} \text{zero its not generator}$$

$4.0 = 0$

$$\left.\begin{aligned} 1.1 &= 1 \\ 2.1 &= 2 \\ 3.1 &= 3 \\ 4.1 &= 4 \end{aligned}\right\} 1 \text{ is generator}$$

$0.2 = 0$
$1.2 = 2$
$2.2 = 4$
$3.2 = 6 \pmod 5$
1
$4.2 = 8 \pmod 5$
$= 3$

2 is generator

3.2

$0.3 = 0$
$1.3 = 3$
$2.3 = 6 \pmod 5$
$= 1$
$3.3 = 9 \pmod 5$
$= 4$
$3.4 = 12 \pmod 5$
$= 2$

3 is generator.

$0.4 = 0$
$1.4 = 4$
$2.4 = 8 \pmod 5$
$= 3$
$3.4 = 12 \pmod 5$
$= 2$
$4.4 = 16 \pmod 5$
$= 1$

4 is generator.

Qu. Show that $Z_4 = \{0, 1, 2, 3\}$ is cyclic

$0.1 = 0$
$0.2 = 0$
$0.3 = 0$

0 is not generator

$1.0 = 0$
$1.1 = 1$
$1.2 = 2$
$1.3 = 3$

1 is generator

$2.0 = 0$
$2.1 = 0$
$2.2 = 4 \pmod 4$
$= 0$
$2.3 = 6 \pmod 4$
$= 2$
$2.4 = 8 \pmod 4$

2 is not generator

$3.0 = 0$
$3.1 = 3$
$3.2 = 6 \pmod 4$
$= 2$
$3.3 = 9 \pmod 4$
$= 1$

3 is generator

* a cyclic group is always abelian group but not every abelian group is a cyclic group.
Rational numbers under addition is not cyclic but is abelian.

Que $Z_6 = \{0, 1, 2, 3, 4, 5\}$

$0.0 = 0$
$0.1 = 0$
$0.2 = 0$
$0.3 = 0$
$0.4 = 0$
$0.5 = 0$

0 is not generator.
1 is generator.

$~~0.2 = 0~~$
$2.1 = 2$
$2.2 = 4$
$2.3 = 6$
$2.4 = 8 \pmod 6$
$= 2$

2 is not generator

3 is not generator

5 is gen

Cyclic group :- A groups $(G, *)$ is said to be cyclic if every element can be represented as integral power of its generator. $a \in G$.

Q. $G = \{1, \omega, \omega^2\}$
$1 = \omega^3$
$(\omega^2)^3 = \omega^6 = (\omega^3)^2 = 1$
$(\omega^3)^3 = \omega^9$

Q. $G = \{1, -i, i, -1\}$
$(-i)^3 = -1$ $(i)^2 = -1$ $(i)^4 = 1$

$(i^3) = -i$
is a generator.

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$(G, +)$ $x \in G$ as $x = n \cdot a$ where a is generator of the group then G is cyclic.

$(Z, +)$ $Z = \{ ---- -3, -2, -1, 0, 1, 2 ----- \}$

$0 \cdot 1 = 0$, $1 \cdot 1 = 1$ $2 \cdot 1 = 2$, $3 \cdot 1 = 3$, $-1 \cdot 1 =$

$\therefore$ 1 is the generator of the group.

$(Z, +)$ is cyclic w.r.t addition.

$x = a^m$

$x = (a^{-1})^{-m}$.

$\forall x \in G$

$x = a^m$

$x = (b)^m = (a^{-1})^m = a^{-m}$

Let Group G is of order n.

and $a^m = e$

if $m < n$ then order of group will be m.

$\Rightarrow m \leq n$

$a_1^m = e$.

$a_2^n = e$ and $m < n$

if GCD of $(m, n) = 1$ then order of group will be m.

Klein-4 group

$G = (e, a, b, c)$ $(G, *)$ $a^2 = b^2 = c^2 = e$

and $a*b = b*a = c$, $a*c = c*a = b$

$b*c = c*b = a$

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

$a*(b*c) = (a*b)*c$

$a*(a) = c*c$

$a^2 = c^2 = e$

There is no generator of this group.

$G = \{a, a^2, a^3, a^4, a^5\}$

a, a^2

$GCD(1, 2) = 1$

$GCD(2, 3) = 1$

$GCD(3, 4) = 1$

$GCD(4, 5) = 1$

$GCD(5, 1) = 1$

$GCD(5, 2) = 1$

$GCD(5, 3) = 1$

$GCD(5, 4) = 1$

if $GCD(m, n) = 1$

$\forall \; m < n$

then order of group $\boxed{o(G) = n}$

$G = \{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8\}$

$GCD(8,1) = 1$ $\quad$ $GCD(7,1)$

$GCD(8,2) = 2$ $\quad$ $GCD(7,2) = 1$

$GCD(8,3) = 1$ $\quad$ $GCD(7,3) = 1$

$GCD(8,4) = 4$ $\quad$ $GCD(7,4) = 1$

$GCD(8,5) = 1$ $\quad$ $GCD(7,5) = 1$

$GCD(8,6) = 2$ $\quad$ $GCD(7,6) = 1$

$GCD(8,7) = 1$ $\quad$ ~~GCD~~ (7.

~~7 is a generator of this group~~

Permutation

let $(x_1, x_2, x_3 \ldots x_n)$. The bijective mapping

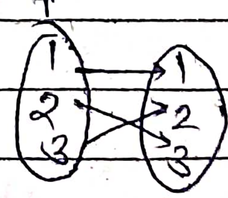
$$\begin{pmatrix} x_1, x_2 \ldots x_n \\ x_{i_1}, x_{i_2} \ldots x_{i_n} \end{pmatrix} = \begin{pmatrix} 1, 2, 3 \ldots n \\ i_1 \; i_2 \ldots i_n \end{pmatrix}$$

giving reassangement of elements S wrt operation given by is called a permutation.

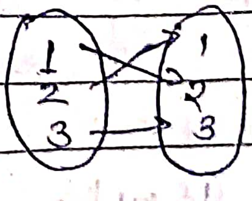
$S = (1, 2, 3)$

$$f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

f



$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$



$$fog = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

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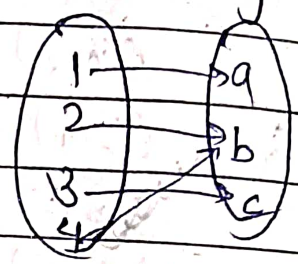
Mapping :-

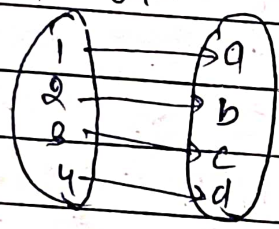
$f: x \to y$

(i) into
(2) onto
(3) one to one.

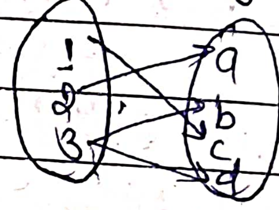
X onto Y



X one to one Y



X into Y



into $\rightarrow$ $\forall x \in X \quad \exists y \in Y, \quad Y = f(x)$

onto $\rightarrow$ $\forall y \in Y \quad \exists x \in X, \quad x = f^{-1}(y)$

one to one $\rightarrow$ $x_1, x_2 \in X, \quad \exists y_1, y_2 \in Y$

such that $y = f(x)$

if $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2$

Mapping of groups

Homomorphism :- Let $(G, \oplus)$ and $(G', \odot)$ are any two groups. Then $f: G \to G'$ is a homomorphism if

$$\boxed{f(x \oplus y) = f(x) \odot f(y)}$$

$Z_3 \oplus = (0, 1, 2)$ $\qquad$ $Z_3 \oplus \overset{x}{=} (a+b) \bmod 3$

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$(G, \oplus_3)$ $\qquad$ $G' = (1, \omega, \omega^2)$

	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

$f(0) = 1$

$f(1) = \omega$

$f(2) = \omega^2$

Que $(Z, +), (R, +)$

$f: Z \rightarrow R$ such that

$$f(x) = x^2$$

$$f(x+y) = (x+y)^2 = x^2 + y^2 + 2xy$$

$$f(x) + f(y) = x^2 + y^2$$

$$f(x+y) \neq f(x) + f(y)$$

Que $(Z, \cdot), (R, \cdot)$

$f: Z \rightarrow R$ such that

$$f(x) = 1/x$$

$$f(x \cdot y) = 1/xy$$

$$f(x) \cdot f(y) = \frac{1}{x} \cdot \frac{1}{y}$$

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Que $(z,+),\ (z,+)$
$z \to z$ such that

$f(x) = 3x \quad \forall x \in z$

$f(x+y) = 3(x+y)$

$f(x)+f(y) = 3x+3y$

$f(x+y) = f(x)+f(y)$

Homomorphic

Que $(R,+) \to (c^{*}, \times)$

$f(x) = \cos x + i\sin x \qquad \because f(x+y) = f(x)\times f($

$f(x+y) = \cos(x+y) + i\sin(x+y)$

$= \cos x\cos y - \sin x\sin y + i(\sin x\cos y + \cos x\sin y)$

$\Rightarrow (\cos x + i\sin x)\cos y + \sin y(-\sin x + i\cos x)$

$\Rightarrow (\cos x + i\sin x)\cos y + \sin y(i^2\sin x + i\cos x)$

$\Rightarrow (\cos x + i\sin x)\cos y + i\sin y(\cos x + i\sin x)$

$\Rightarrow (\cos x + i\sin x)(\cos y + i\sin y)$

$\Rightarrow f(x)+f(y)$

is a homomorphism.

Let G is a group of 2x2 matrices wrt addition
let $p, q, r, s \in R$

f.l.e.g $f: (R,+) \to G$

$$f(x) = \begin{bmatrix} px & qx \\ rx & sx \end{bmatrix}$$

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$$f(x+y) = \begin{bmatrix} p(x+y) & q(x+y) \\ r(x+y) & s(x+y) \end{bmatrix} = \begin{bmatrix} px & qx \\ rx & sx \end{bmatrix} + \begin{bmatrix} py & qy \\ ry & sy \end{bmatrix}$$

$$= f(x) + f(y)$$

is a homomorphism.

Homomorphism:-

$f: (R, \times) \longrightarrow GL_2(R)$ $GL_2(R) \rightarrow$ A set of all 2×2 matrices

$$f(x) = \begin{pmatrix} 3x & x \\ x & x \end{pmatrix}$$

$$f(x \cdot y) = \begin{pmatrix} 3xy & xy \\ xy & xy \end{pmatrix}$$

$$f(x) \cdot f(y) = \begin{pmatrix} 3x & x \\ x & x \end{pmatrix}\begin{pmatrix} 3y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 10xy & 4xy \\ 4xy & 2xy \end{pmatrix} \neq f(x \cdot y)$$

Not a homomorphism.

Isomorphism:-

~~If $f: G_1 \rightarrow G_2$ A one to one and onto and the called~~

A one to one and onto homomorphism is called isomorphism.

$f: (\mathbb{Z}, +) \longrightarrow (\mathbb{Z}, +)$

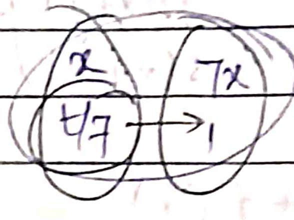
$$f(x) = 7x$$

$$f(x+y) = 7(x+y) = 7x + 7y = f(x) + f(y)$$

if $f(x) = f(y) \Rightarrow y = x$

$7x = 7y \Rightarrow y = x$ one to one.

Let $y = 1 \in \mathbb{Z}$



We cannot find $x \in \mathbb{Z}$ such that $f(x) = 1$

i.e. $7x = 1$

$x = 1/7 \in \mathbb{Z}$

$f(x) = 7x \longrightarrow$ is not an Homorphism.

*

let $GL_n(R)$ is a group of $n \times n$ non-singular matrices

$$f : GL_n(R) \longrightarrow R$$

$$f(A) = |A|$$

$$f(AB) = |AB| = |A||B|$$

for one to one $\longrightarrow$

$$f(A) = f(B) \Rightarrow A = B$$

onto $\quad C = \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} \quad C \in GL_n$

~~$\forall P \in R$~~

$|C| = P$

$\forall P \in R, \ \exists C \in GL_n$

Homomorphism

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 6 \qquad |B| = 6$

$f(A) = f(B) = |A| = |B|$

But $A \neq B$.

Not an isomorphism.

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$f : (R, +) \longrightarrow (R, +)$

$f(x) = 7x$

$f(x+y) = 7(x+y) = 7x + 7y = f(x) + f(y)$

$f(x) = 1 \in R$

$7x = 1 \Rightarrow x = 1/7 \in R$ $\Rightarrow$ onto mapping

Let G_1 is a group of real numbers w.r.t operation and G_2 is a group of complex numbers w.r.t. unit modulus multiplication.

$f : G_1 \longrightarrow G_2$ $f(x) = \cos x + i \sin x$

$f(x+y) = \cos(x+y) + i \sin(x+y) = (\cos x + i \sin x)(\cos y + i \sin y)$

$\Rightarrow$ homomorphism.

one to one.

$f(x) = f(y) \Rightarrow x = y$

$x = 0,$ $y = 2\pi$ $f(x) = 1$ $f(y) = 1$

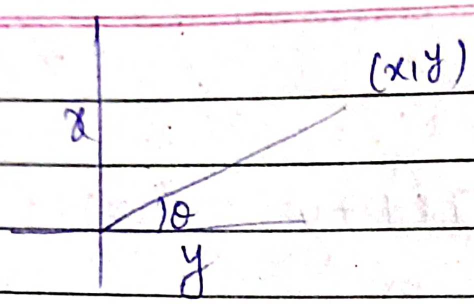
$f(x) = f(y)$ but $x \neq y$ Not one to one.

$0 \leq \theta \leq 2\pi$

$\theta \in R,$ $f(\theta) = \cos\theta + i \sin\theta$

$\forall f(\theta) \in G_2$

$\exists \theta \in R$



$z = x + iy$

Congruence modulo H

Let G be a group and H is a subgroup of G $\forall\, a,b \in G$, $\exists$ $a \equiv b$ modulo H

$ab^{-1} \in H$

Theorem:- $a \equiv b \bmod H$ forms an equivalence relation.

proof:-

Since H is a subgroup $\exists\, e \in H$

$a \equiv a \bmod H$ ~~if~~ $aa^{-1} \in H$ Here $aa^{-1} = e \in H$

It is reflexive.

symmetry:-

$a \equiv b \bmod H$ if $ab^{-1} \in H$ since H is subgroup then

$b \equiv a \bmod H$ if $ba^{-1} \in H$ $\forall\, a,b \in H \;\exists\; a^{-1}, b^{-1} \in H$

and by closure property

$ab^{-1} \in H$ & $ba^{-1} \in H$

Transitive:-

$a \equiv b \bmod H$ $b \equiv c \bmod H \Rightarrow a \equiv c \bmod H$

Since H is a subgroup

$\forall\, a,b,c \in H$, $\exists\; ab^{-1} \in H,\; bc^{-1} \in H$ $\xrightarrow{\text{by closure}}$ $(ab^{-1})(bc^{-1})$

$\Rightarrow (ab^{-1})(bc^{-1}) = ab^{-1}bc^{-1} = ac^{-1} \in H$

$\Rightarrow a \equiv c \bmod H$ Transitive.

It is reflexive.

No of elements in coset = No of elements in subgroup.
or order of coset = order of group.

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p abelian group Here Left coset = Right coset.

COSET :- Let G be a group and H is a subgroup of G. Then $\forall$ $a \in G$, we have aH known as left coset of H in G.

(Ha " " right coset of H)

Let $G = \{1, -1, i, -i\}$
Let $H = \{1, -1\}$
Let

$1 \cdot H = 1 \cdot \{1, -1\} = H$
$-1 \cdot H = -1 \cdot \{1, -1\} = \{-1, 1\} = H$
$i \cdot H = i(1, -1\} = \{i, -i\}$
$-i \cdot H = -i\{1, -1\} = \{-i, i\}$

union of cosets = G
H, iH
$\{1, -1\} \cap \{i, -i\} = \phi$

Group $(Z_4, +_4) = \{[0], [1], [2], [3]\}$
$[0] = \{0, 4, 8, 12, 16 \ldots\}$
$[1] = \{1, 5, 9, 13, 17 \ldots\}$
$[2] = \{2, 6, 10, 14, 18 \ldots\}$
$[3] = \{3, 7, 11, 15, 19 \ldots\}$

$H = \{[0], [2]\}$

$a \in G$
$a + H =$
$a = [0]$

$[0] + [0] = [0]$
$[0] + [2] = [2]$
$[2] + [0] = [2]$
$[2] + [2] = [0]$

$[0] + H = H$
$[2] + H = \{[2], [0]\} = H$
$[1] + H = \{[1], [3]\}$
$[3] + H = \{[3], [1]\}$

total no of distinct ... no operation ...

Or index of subgroup $[G:H] = \frac{O(G)}{O(H)}$

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$[1]+[0]=[1]$

$[1]+[2]=[3]$

caset $[3]+[0]=[3]$

$[3]+[2]=[1]$

Discrete Numeric functions - operations

Let

$$a_r = \begin{cases} 0 & 0 \le r \le 5 \\ 1 & 5 < r \le 8 \\ 2^r & r > 8 \end{cases}$$

$$b_r = \begin{cases} 5 & 0 \le r \le 2 \\ 3 & 3 \le r \le 7 \\ 3^r & r > 7 \end{cases}$$

Addition :-

$$E_r = a_r + b_r$$

$$E_r = \begin{cases} 0+5=5 & 0 \le r \le 2 \\ 0+3=3 & 3 \le r \le 5 \\ 1+3=4 & 6 \le r \le 7 \\ 1+3^r & r=8 \\ 2^r+3^r & r>8 \end{cases}$$

Multiplication :-

$$E_r = a_r * b_r$$

$$E_r = \begin{cases} 0 \times 5 = 0 & 0 \le r \le 2 \\ 0 \times 3 = 0 & 3 \le r \le 5 \\ 1 \times 3 = 3 & 6 \le r \le 7 \end{cases}$$

$$A(z) \times B(z) \Rightarrow \left(\frac{1}{1-2z}\right) \times \left(\frac{1}{1-3z}\right)$$

$$= \frac{\alpha_1}{1-2z} + \frac{\alpha_2}{1-3z}$$

$$\Rightarrow \frac{\alpha_1(1-3z) + \alpha_2(1-2z)}{(1-2z)(1-3z)}$$

$\Rightarrow$ $\alpha_1 + \alpha_2 = 1$ $\boxed{\alpha_1 = -2 \\ \alpha_2 = 3}$

$\alpha_2 = 1 - \alpha_1$

$+3\alpha_1 + 2\alpha_2 = 0$

$\cancel{3\alpha_1 + (1-\alpha_1) \times 2 =} \Rightarrow 3\alpha_1 + 2 - 2\alpha_1 \Rightarrow \boxed{\alpha_1 = -2}$

$-2 \times \{A(z) = -2 - 4z - 8z^2 - 16z^3 + \ldots\}$

$3 \times \{B(z) = 3 + 9z + 27z^2 + 81z^3 + \ldots\}$

$1 + 5z + 19z^2 + 65z^3 + \ldots$

Q

$a_r = 2^r \qquad r \geq 0$

$b_r = (-1)^r 3^r \qquad r \geq 0$

$C_r = a_r \circledast b_r$

$A(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \ldots$

$= 1 + 2z + 4z^2 + 8z^3 + \ldots$

$B(z) = b_0 + b_1 z + b_2 z^2 + b_3 z^3 + \ldots$

$= 1^r(-3)^0 + (-3)^r z + (-9)^r z^2 + (-27)^r z^3 + \ldots$

$\Rightarrow 1 - 3z + 3^2 z^2 - 3^3 z^3 + 3^4 z^4 + \ldots$

$$A(z) = \frac{1}{1-2z}, \quad B(z) = \frac{1}{1+3z}$$

$$\Downarrow$$

$$(1-(1-3z))$$

$$A(z) \times B(z) = \frac{1}{1-2z} \times \frac{1}{1+3z} \Rightarrow \frac{\alpha_1}{1-2z} + \frac{\alpha_2}{1+3z}$$

$$\Rightarrow \frac{\alpha_1(1+3z)+\alpha_2(1-2z)}{(1-2z)(1+3z)} \Rightarrow \frac{(\alpha_1+\alpha_2)+(3\alpha_1-2\alpha_2)z}{(1-2z)(1+3z)}$$

$$\Rightarrow 3\alpha_1 - 2\alpha_2 = 0 \qquad \alpha_2 = 3/2\,\alpha_1$$

$$\alpha_1 + \alpha_2 = 1 \qquad \alpha_1 + 3/2\,\alpha_1 = 1 \qquad \alpha_2 = 3/5$$

$$\frac{5\alpha_1}{2} = 1 \qquad \alpha_1 = 2/5$$

$$c_r = \alpha_1 a_r + \alpha_2 b_r$$

$$c_0 = \frac{2}{5} a_r + \frac{3}{5} b_r \Rightarrow c_1 = \frac{2}{5} \times 2 - \frac{3}{5} \times 3 = -\frac{5}{5} = -1$$

$$c_1 = a_0 b_1 + a_1 b_0 = 1 \times (-3) + 2 \times 1 = -1$$

Q3

$$A(z) = \frac{1}{(1-z)^2} \Rightarrow (1)^r 2 \qquad \frac{1}{(1-z)^2} \times (1+z)$$

$$\frac{1}{(1-z)^2} \times \frac{(1+z)^2}{(1+z)^2} \Rightarrow (1+z)^2$$

$$\boxed{(1-z)^{-a} = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{n!}}$$

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$(a)_n = a(a+1)(a+2)\cdots(a+n-1)$

$(1)_n = 1 \cdot 2 \cdot 3 \cdots n = n!$

$(2)_n = 2 \cdot 3 \cdot 4 \cdots n \cdot (n+1) = (n+1)!$

$(3)_n = 3 \cdot 4 \cdot 5 \cdots \frac{(n+2) \times 2}{2} = \frac{(n+2)!}{2}$

$(4)_n = 4 \cdot 5 \cdot 6 \cdots \frac{(n+3)}{6} \times 6 = \frac{(n+3)!}{3!}$

Q ~~$A(z) = \frac{1}{(1-z)^2(1+z)^2}$~~

$(r)_n = \frac{r(r+1)\cdots(r+n-1)}{(r-1)!} \times (r-1)! = \frac{(n+r-1)!}{(r-1)!}$

$$\Rightarrow \boxed{\sum_{n=0}^{\infty} \frac{(a+n-1)!}{(a-1)!\,n!} z^n \Leftarrow \sum_{n=0}^{\infty} \frac{(a)_n z^n}{n!}}$$

$(1-z)^{-3} = \sum_{n=0}^{\infty} \frac{(n+2)!\,z^n}{2!\,n!} = \sum_{n=0}^{\infty} \frac{(n+2)(n+1)\cancel{n!}}{2!\,\cancel{n!}} z^n$

$\Rightarrow 1+3z+6z^2+10z^3+\cdots$

$(1-z)^{-2} = \sum_{n=0}^{\infty} \frac{(n+1)!}{1!\,n!} z^n = \sum \frac{(n+1)\cancel{n!}}{\cancel{n!}} z^n = 1+2z+3z^2+4z^3+\cdots$

(1) $(1-z)^{-a} \Rightarrow b_n = \frac{(a+n-1)!}{(a-1)!\,n!}$

Q $A(z) = \dfrac{1}{(1-x)^2(1+x)^2}$

$a_r = \dfrac{\alpha_1}{(1-x)^2} + \dfrac{\alpha_2}{(1+x)^2}$

$= \dfrac{1}{(1-x^2)^2} = \dfrac{1}{(1-z)^2}$

$(1-z)^{-2} \Rightarrow \sum_{n=0}^{\infty} = (n+1)z^n = 1+2z+3z^2+4z^3+ \ldots$

$\Rightarrow 1+2x^2+3x^4+4x^6 \ldots$

$a_0 = 1$
$a_1 = 0$
$a_2 = 2$
$a_3 = 0$
$a_4 = 3$

$a_r = (\frac{r}{2}+1)$ for $r =$ even
$= 0$ for $r =$ odd
$a_0 = 1$
$a_1 = 0$
$a_2 = 2$

Q $A(z) = \dfrac{1}{(3+z)(7-4z)}$

$A(z) = \dfrac{1}{(3+z)^{-1}}$ $\quad A(z) = \dfrac{1}{(1+z/3)(1-4z/7)}$

$\Rightarrow \dfrac{1}{(1-4z/3)(1+z/3)} \Rightarrow (1+z/3)^{-1}(1-4z/7)^{-1}$

$$a_r = \frac{\alpha_1}{3(1+z/3)} + \frac{\alpha_2}{7(1-4z/7)}$$

$$\Rightarrow \cancel{7(\alpha_1)(1+z/3)(+3\alpha_2(1-}$$

$$\frac{7(1-4z/7)\alpha_1 + \alpha_2 . 3(1+z/3)}{3(1+z/3)\, 7(1-4z/7)}$$

$$\Rightarrow \frac{7\alpha_1 - 4z\alpha_1 + 3\alpha_2 + z\alpha_2}{(3+3z)(7-4z)}$$

$$\Rightarrow \alpha_1 + \alpha_2 = 1 \qquad \alpha_2 = 1 - \alpha_1$$

$$\frac{4}{7}\alpha_1 + \frac{1}{3}\alpha_2 = 0$$

$$\frac{4}{7}\alpha_1 + \frac{1}{3}(1-\alpha_1) = 0$$

$$\frac{4}{7}\alpha_1 + \frac{1}{3} - \frac{1}{3}\alpha_1 = 0 \Rightarrow \frac{12\alpha_1 + 7 - 7\alpha_1}{21} = 0$$

$$5\alpha_1 = -7$$

$$\boxed{\alpha_1 = -7/5} \qquad \boxed{\alpha_2 = 12/5}$$

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Q $A(z) = \dfrac{1}{1+2z^2+z^4} \Rightarrow \dfrac{1}{(1+z^2)^2}$

$\Rightarrow \dfrac{1}{(1-(-z^2))^2} \Rightarrow \sum \dfrac{(n+1)!}{1!\,n!} (-z^2)^n$

$\Rightarrow \sum_{n=0} \dfrac{(-1)^n (n+1)\, n!\, z^{2n}}{n!}$

$a_r = (-1)^{r/2} (r/2+1)$ for $r =$ even
$= 0$ for $r =$ odd.

$\Rightarrow 1 - 2z^2 + 3z^4 - 4z^6 + 5z^8$

Que

$A(z) = \dfrac{7-4z}{3+z}$

$A(z) = \dfrac{7-4z}{3(1+z/3)} \Rightarrow \dfrac{7(1-4z/7)}{3(1+z/3)} \Rightarrow$

$\dfrac{1}{3}(7-4z)\left[1 + \dfrac{z}{3} + \dfrac{z^2}{9} + \dfrac{z^3}{27} + \cdots\right]$

$\Rightarrow \dfrac{7}{3}\left[1 + \dfrac{z}{3} + \left(\dfrac{z}{3}\right)^2 + \left(\dfrac{z}{3}\right)^3 + \cdots\right] - 4\left[z + \dfrac{z^2}{3} + \dfrac{z^3}{9} + \dfrac{z^4}{3^3}\right]$

$\dfrac{7}{3} + \cancel{\dfrac{7z}{9}} - \left(\dfrac{7}{9}+4\right)z + \left(\dfrac{7}{3^3}+\dfrac{4}{3}\right)z^2 - \left(\dfrac{7}{3^4}+\dfrac{4}{3^2}\right)z^3$

$\frac{7z}{9} \; 42$

$\frac{7}{9}z - 42 \quad \frac{367}{43} \quad 27 \quad 3$

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$$\Rightarrow \frac{7}{3}\left\{\cancel{1 + \frac{z}{3} + \frac{z^2}{9} + \frac{z^3}{27} + \cdots}\right\} - \left\{\cancel{4z + \frac{4z^2}{3} + \frac{4z^3}{9}} + \frac{4z^4}{27} + \cdots\right.$$

$$\cancel{\frac{7}{3} + \frac{29}{9}z + \frac{29z^2}{27} - \frac{29}{81}z^3}$$

$$\boxed{\frac{7}{3} - \frac{43}{9}z + \frac{43}{27}z^2 - \frac{43}{81}z^3 + \cdots}$$

$a_r = \alpha_r + \beta_r$

$\rightarrow \alpha_r = \frac{7}{3} + \left(\frac{-1}{3}\right)^r \quad r \geq 0$

$\rightarrow \beta_r = (-1)^r \frac{4}{3^{r-1}} \quad r \geq 2$

$\beta_0 = 0$

$\beta_1 = -4$

$\beta_0 = 0$

$\beta_1 = -4$

$\beta_2 = +4/3$

$\beta_3 = -4/3^2$

$\beta_4 = +4/3^3$

$a_r = \alpha_r + \beta_r$

$a_0 = 7/3$

$a_1 = \frac{-7}{3^2} - 4$

$\alpha_r = \frac{7}{3}\left(-\frac{1}{3}\right)^r + (-1)^r \frac{\times 4}{3^{r-1}} \quad r \geq 2$

Recurrence Relation

$a_r = 2^r \quad r \geq 0$

$a_r = 2 \cdot 2^{r-1} = 2a_{r-1}$

$\boxed{a_r - 2a_{r-1} = 0}$

$a_n = A\alpha^n$

$$\frac{A\alpha^n - 2A\alpha^{n-1} = 0}{\alpha^{n-1}}$$

$\alpha - 2 = 0$

$\alpha = 2$

$a_n = A \cdot \alpha^n$

initial condition $a_0 = 1$

$a_0 = A\alpha^0$

$1 = A \cdot 1$

$A = 1$

$f_{n+1} = f_n + f_{n-1} \quad n \geq 1$

$f_0 = 1 \quad f_1 = 1$

fibonacci series

$f_{n+1} - f_n - f_{n-1} = 0$

$f_n = A\alpha^n$

$$\frac{A\alpha^{n+1} - A\alpha^n - A\alpha^{n-1}}{A\alpha^{n-1}} = 0$$

$\alpha^2 - \alpha - 1 = 0$

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$$\alpha = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \times (-1)}}{2 \cdot 1}$$

$$= \frac{1 \pm \sqrt{1+4}}{2} \Rightarrow \frac{1 \pm \sqrt{5}}{2}$$

$$\alpha_1 = \frac{1+\sqrt{5}}{2} \qquad \alpha_2 = \frac{1-\sqrt{5}}{2}$$

$$f_n = A_1 \alpha_1^n + A_2 \alpha_2^n$$

$$f_n = A_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + A_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$1 = A_1 (\quad)^0 + A_2 \Rightarrow A_1 + A_2 = 1$$

$$\Rightarrow A_1 \left(\frac{1+\sqrt{5}}{2}\right)^1 + A_2 \left(\frac{1-\sqrt{5}}{2}\right)^1$$

$$A_2 = 1 - A_1$$

$$A_1 \left(\frac{1+\sqrt{5}}{2}\right) + (1 - A_1)\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$A_1 \left(\frac{\cancel{1}+\sqrt{5}}{2} - \left(\frac{\cancel{1}-\sqrt{5}}{2}\right)\right) = 1 - \left(\frac{1-\sqrt{5}}{2}\right)$$

$$A_1 \left(\frac{\cancel{2}\sqrt{5}}{\cancel{2}}\right) = 1 - \left(\frac{1-\sqrt{5}}{2}\right)$$

$$A_1 \sqrt{5} = \frac{1+\sqrt{5}}{2}$$

$$A_1 = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)$$

$$A_2 = 1 - A_1 \Rightarrow 1 - \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)$$

$$A_2 \Rightarrow \frac{2\sqrt{5} - 1(1+\sqrt{5})}{2\sqrt{5}} \Rightarrow \frac{\sqrt{5}-1}{2\sqrt{5}}$$

$\Rightarrow A_2 = \frac{\sqrt{5}-1}{2\sqrt{5}} = \frac{-(1-\sqrt{5})}{2\sqrt{5}}$

$$\boxed{f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}}$$

Homogeneous Recurrence Relation

$f_n + c_1 f_{n-1} + c_2 f_{n-2} + c_3 f_{n-3} + \cdots c_k f_{n-k} = 0$

order $\Rightarrow k$

$$\boxed{f_n + c_1 f_{n-1} + c_2 f_{n-2} + c_3 f_{n-3} + \cdots c_k f_{n-k} = g(n)}$$

$\downarrow$ Non-homogeneous Relation

Q $f_n - 2f_{n-1} = 5$

$f_n - 2f_{n-1} = 0$ (for Homogeneous)

$f_n = A\alpha^n$

$\frac{A\alpha^n - 2A\alpha^{n-1} = 0}{A\alpha^{n-1}}$

$\alpha - 2 = 0$

$\alpha = 2$

$f_n = A\,2^n$

particular sol^n

$f_n = c$

$c - 2c = 5$

$-c = 5$

$\boxed{c = -5}$

$f_n = c_1 + c_2 n$

$(c_1 + c_2 n) - 2(c_1 + c_2(n-1)) = 5n+1$

$-c_1 + c_2(-n+2) = 5n+1$

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$$(2c_2 - c_1) + c_2(-n) = 5n+1$$

$$2c_2 - c_1 = 1$$

$$c_2(-n) = 5n \quad \rightarrow \quad -10 - c_1 = 1$$

$$\boxed{c_2 = -5} \qquad \boxed{c_1 = -11}$$

$$f_n = A2^n - 11 - 5n$$

particular sol^n

$$f_n = c.3^n$$

$$\frac{c.3^n - 2c\,3^{n-1}}{3^{n-1}} = 3^n$$

$$c(3-2) = 3$$

$$\boxed{c = 3}$$

$$f_n - 2f_{n-1} = 3^n$$

$$f_n = A2^n$$

when roots are repeated then $\quad f_n = cn2^n$

$$\frac{c[n2^n - 2(n-1)2^{n-1}]}{2^{n-1}} = 2^n$$

$$c(2n - 2(n-1)) = 2$$

$$\boxed{c = 2}$$

Que Given a sequence 1, 1, 1, 1, 1, 1 form a generating function. for a sequence a_0, a_1, a_2, a_3 ---

$A(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots = \sum_{k=0}^{\infty} a_k z^k$

$G(z) = 1 + z + z^2 + z^3 + z^4 + z^5 + (z^6 + z^7 + \cdots) - (z^6 + z^7 + \cdots)$

$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$

$\Rightarrow \frac{1}{1-z} - z^6(1 + z + z^2 + \cdots) = \frac{1}{1-z} - \frac{z^6}{1-z}$

$\Rightarrow \frac{1-z^6}{1-z}$

Q. let $a_k = c(n,k)$ $\quad a_0 = c(n,0) \quad a_1 = c(n,1)$

$k = 0(1)n$

$G(z) = c(n,0) + c(n,1)z + c(n,2)z^2 + \cdots + c(n,n)z^n$

$= (1+z)^n$

Given $a_k = \alpha^k$

$G(z) = \alpha^0 + \alpha^1 z + \alpha^2 z^2 + \alpha^3 z^3 + \cdots$

$= \frac{1}{1-\alpha z}$

Fibonacci series

$f_n = f_{n-1} + f_{n-2}$, $n \geq 2$, $f_0 = 0$, $f_1 = 1$

$G(z) = f_0 + f_1 z + f_2 z^2 + f_3 z^3 + \cdots = \sum_{n=0}^{\infty} f_n z^n$

$f_2 = f_1 + f_0$

$f_3 = f_2 + f_1$

$f_4 = f_3 + f_2$

$G(z) = f_0 + f_1 z + (f_1 + f_0) z^2 + (f_2 + f_1) z^3 + (f_3 + f_2) z^4 + \cdots$

$\cdots = \sum_{n=2}^{\infty}$

$= 0 + z + z^2 + (f_2 + 1) z^3 + (f_3 + f_2) z^4 + \cdots$

$f_n z^n = f_{n-1} z^n + f_{n-2} z^n \quad n \geq 2$

$\sum_{n=2}^{\infty} f_n z^n = \sum_{n=2}^{\infty} f_{n-1} z^n + \sum_{n=2}^{\infty} f_{n-2} z^n$

$G(z) - f_0 - f_1 z = z(G(z) - f_0) + z^2 G(z)$

$\cancel{G(z) - z G(z) - z^2 G(z) = 1}$

$G(z) - 0 - z = z(G(z) - 0) + z^2 G(z)$

$G(z) - z G(z) - z^2 G z = z$

$G(z)(1 - z - z^2) = z$

$$\boxed{G(z) = \frac{z}{1 - z - z^2}}$$

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Que $y_n + 2y_{n-1} - 15y_{n-2} = 0$, $n \geq 2$, $y_0 = y_1 = 1$

$$G(z) = y_0 + y_1 z + y_2 z^2 + y_3 z^3 + \cdots = \sum_{n=0}^{\infty} y_n z^n$$

$$\sum_{n=2}^{\infty} y_n z^n + 2\sum_{n=2}^{\infty} y_{n-1} z^n - 15\sum_{n=2}^{\infty} y_{n-2} z^n = 0$$

$$(G(z) - y_0 - y_1 z) + 2(zG(z) - y_0 z] - 15[z^2 G(z)] = 0$$

$$(G(z) - 1 - z) + 2(zG(z) - z] - 15[z^2 G(z)] = 0$$

$$G(z) - 1 - z + 2zG(z) - 2z - 15z^2 G(z) = 0$$

$$G(z)[1 + 2z - 15z^2] = 1 + 3z$$

$$G(z) = \frac{1+3z}{1+2z-15z^2} \Rightarrow \frac{1+3z}{(1+5z)(1-3z)}$$

$$\frac{A}{1+5z} + \frac{B}{1-3z} = \frac{1+3z}{(1+5z)(1-3z)}$$

$$\Rightarrow \frac{A(1-3z) + B(1+5z)}{(1+5z)(1-3z)} \Rightarrow \begin{cases} 3A + 3B = 1 \\ -3A + 5B = 3 \end{cases}$$

$8B = 6$

$B = 3/4$

$A = 1/4$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{1+5z} + \frac{3}{1-3z}\right]$$

$$y_k = a_k = \frac{1}{4}\left[(-5)^k + 3(+3)^k\right] \qquad k \geq 0$$

Que $y_n = 3y_{n-1}$, $n \geq 1$, $y_0 = 2$

$$G(z) = y_0 + y_1 z + y_2 z^2 + \cdots = \sum_{n=0}^{\infty} y_n z^n$$

$$\sum_{n=1}^{\infty} y_n z^n = 3\sum_{n=1}^{\infty} y_{n-1} z^n$$

$$\Rightarrow (G(z) - y_0) = 3(zG(z))$$

$$\Rightarrow G(z) - y_0 - 3zG(z)$$

$$\Rightarrow G(z)(1-3z) = y_0$$

$$\Rightarrow G(z) \Rightarrow \frac{2}{(1-3z)}$$

$$y_k = 2 \cdot 3^k \qquad k \geq 0$$

Q $y_n = 3y_{n-1} + 2$, $n \geq 1$, $y_0 = 1$

$$\sum_{n=1}^{\infty} y_n z^n = 3\sum_{n=1}^{\infty} y_{n-1} z^n + 2\sum_{n=1}^{\infty} z^n$$

$$(G(z) - y_0) = 3(zG(z)) + 2$$

$$G(z)(1-3z) = y_0 + \left(\frac{2z}{1-z}\right)$$

$$G(z) = \frac{8}{1-3z}$$

$$\frac{1 + \left(\frac{2z}{1-z}\right)}{1-3z} \Rightarrow \frac{1-z+2z}{(1-z)(1-3z)} \Rightarrow \frac{1+z}{(1-z)(1-3z)}$$

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$$

$$\sum_{n=1}^{\infty} z^n = \frac{1}{1-z} - 1$$

$$\Rightarrow \frac{z}{1-z}$$

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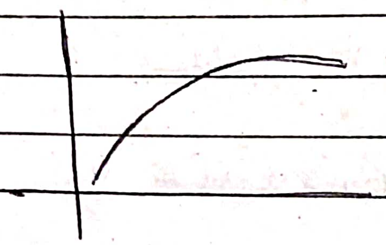
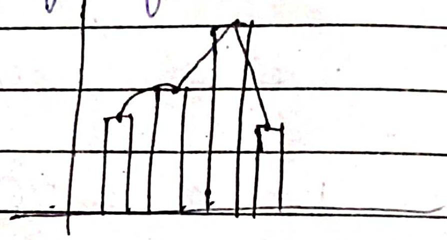
Measures of central Tendency

Mean = salary of employee $x_1, x_2, x_3 - - x_n$

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Mode = Highest frequency of occurence. variable





BAR CHART

Probability :-

Toss of a coin occurence of Head of a toss

No of outcomes = 2
Head, Tail

$$P(\text{Head}) = \frac{\text{frequency of event}}{\text{No of outcomes}}$$

* frequency of event is subset of No of outcomes.

Sample space

Let S is a set of all outcomes and P in the associated probability of each outcomes.

$S = \{H, T\}$

$P(H) = P(T) = 1/2$ (equally likely)

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$\therefore$ Then sample space is a pair (S, P) given by

$P : S \to R$

- ~~Frequency of event is subset of no of outcomes~~

where R is a set of non-negative Real no's

and $\sum_{s \in S} P(s) = 1$

$P(s) \geq 0$

fuzzy set $0 \leq \mu(x) \leq 1$

$0 \leq P(x) \leq 1$

Samarandche Neutrosophic set

Toss of a dice

$S = \{1, 2, 3, 4, 5, 6\}$

all outcomes are equally likely

$P(S) = 1/6$

$(S, P(s))$

Toss of a pair of dice

$S = \{(1,1), (1,2), (1,3) \ldots\ldots \ldots (6,6)\} = 36$

all outcomes are equally likely

$P(s) = 1/36$

Event:- An occurence of a specific / particular outcome A from set S.

$A \subset S$.

$$P(A) = \frac{\text{frequency of } A}{\text{total no of outcomes}}$$

Que

Sum of the outcomes of two dice is 5.

(1,4) (4,1) (2,3) (3,2)

$$P(A) = \frac{4}{36} = \frac{1}{9}$$

Q Toss of 5 coins

{HHHHH, HHHHT, HHHTH ---------- TTTTT}

$2\times2\times2\times2\times2 = 2^5 = 32$

A = only one Head

$$P(A) = \frac{5}{32}$$

Q Toss of 10 dice

total no of outcomes $= 6^{10}$

1 is not come when

(2, 3, 4, 5, 6)

$5\times5\times5\times - - - - = 5^{10}$

$$P(A) = \frac{5^{10}}{6^{10}}$$

Combination of events

sample space (S, P)

event $A \subset S$

$P(A)$

event $B \subset S$

$A \cup B$

$A \cap B$

A Not B $\quad A - B$

B Not A $\quad B - A$

$A' \longrightarrow S - A$

Que on throw of a dice

A = { even numbers }.

B = { prime number }

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\} = \qquad P(A) = 3/6 = 1/2$

$B = \{2, 3, 5\} = \qquad P(B) = 3/6 = 1/2$

$P(A) + P(B) = 1$

$A \cap B = \{2\}$.

$P(A \cap B) = 1/6$

$A \cup B = \{2, 3, 4, 5, 6\} \qquad P(A \cup B) = 5/6$

$P(A) + P(B) = P(A \cup B) + P(A \cap B) = 1$

#

A = { odd number } = $\{1, 3, 5\}$

B = { even number } = $\{2, 4, 6\}$

$A \cap B = \phi \qquad A \cup B = S$

$P(A) = 3/6 = 1/2$, $P(B) = 3/6 = 1/2$

$P(A) + P(B) = 1$

$P(A \cup B) = 6/6 = 1$

$P(A \cap B) = 0/6 = 0$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and

- $P(A \cup B) = P(A) + P(B)$ if A and B are independent

$P(S) = 1$

$P(0) = 0$

- $P(A) = 1 - P(B)$ for independent event

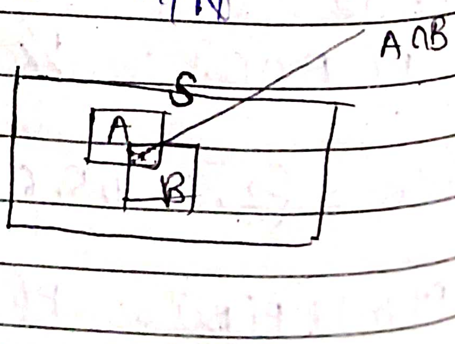
Conditional Probability :-

event A = Ariving late in the school

B = clock malfunctioned.

$P(A) = m/N$ $P(B) = n/N$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$



Que Throw a dice ~~twice~~ twice

~~the~~ event $A = \{$ sum of outcomes is 8 $\}$

$B = \{$ both are even $\}$

total sample space is 36

$P(A) = 3/6 = 1/2$, $P(B) = 3/6 = 1/2$

$P(A) + P(B) = 1$

$P(A \cup B) = 6/6 = 1$

$P(A \cap B) = 0/6 = 0$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and

- $P(A \cup B) = P(A) + P(B)$ if A and B are independe

$P(S) = 1$

$P(0) = 0$

- $P(A) = 1 - P(B)$ for independent event

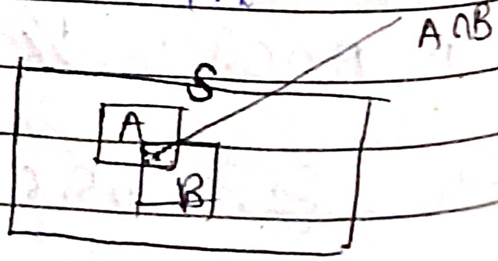
Conditional Probability :-

event A = Ariving late in the School

B = clock malfunctioned.

$P(A) = m/N$ $P(B) = n/N$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$



Qu Throw a dice ~~twice~~ twice

~~the~~ event $A = \{$ sum of outcomes is 8 $\}$

$B = \{$ both are even $\}$

total sample space is 36

$P(A) = 3/6 = 1/2$; $P(B) = 3/6 = 1/2$

$P(A) + P(B) = 1$

$P(A \cup B) = 6/6 = 1$

$P(A \cap B) = 0/6 = 0$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  and
- $P(A \cup B) = P(A) + P(B)$ if A and B are independent

$P(S) = 1$

$P(0) = 0$

- $P(A) = 1 - P(B)$ for Independent event

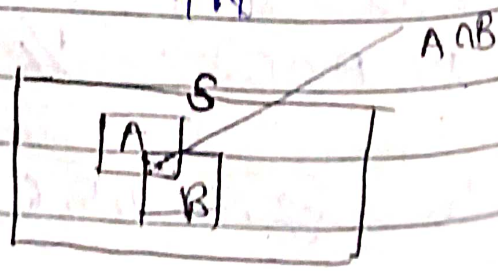
Conditional Probability :-

event A = Ariving late in the school

B = Clock malfunctioned.

$P(A) = m/N$ $P(B) = n/N$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



Qu Throw a dice ~~thrice~~ twice

the event $A = \{$ sum of outcomes is $8\}$

$B = \{$ both are even $\}$

total sample space is 36

$A = \{(2,6),(4,4),(6,2),(3,5)(5,3)\} = 5/36$

$B = \{(2,2)(2,4)(2,6),(4,2)(4,4),(4,6),(6,2)(6,4),(6,6)\}$

$B = 9/36 = 1/4$

$(A \cap B) = \{(2,6),(4,4),(6,2)\}$

$P(A \cap B) = 3/36 = 1/12$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{3/36}{9/36} = \dfrac{1}{3}$ $\boxed{\checkmark \quad P(B) \neq 0}$

Que coin is flipped 5 times we get tail on 1st flip given exactly three heads are flipped.

$P(A) = \dfrac{2^4}{2^5} = 1/2$

$P(B) = \dfrac{5C_3}{2^5} = \dfrac{10}{32}$

$P(A \cap B) = \dfrac{4C_3}{2^5} = \dfrac{4}{32} = \dfrac{1}{8}$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{4/32}{10/32} = 2/5$

Que coin is flipped 5 times we get Tail on 1st flip and Head on the Last flip.

$P(A) = \dfrac{2^4}{2^5} = \dfrac{1}{2}$

$P(A \cap B) = \frac{1}{4} = \frac{6}{25} = \frac{1}{4}$

$P(B) = \frac{24}{25} = \frac{1}{2}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2} = P(A)$$

$\Rightarrow$ A is independent of B

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(A/B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$ Independence.

- Sample space $\longrightarrow (S, P)$

  $P \rightarrow$ Probability

  $S \rightarrow P$, $0 \leq P \leq 1$, $S \in N$

  S = set of all possible outcomes

  Exhaustive.

- Trial $\rightarrow$ performance of an experiment is called a trial.
- Event $\rightarrow$ An outcome of a trial.
- Mutually exclusive events $\longrightarrow$
- Dependent event $\rightarrow \frac{13}{52}, \frac{12}{51}$ without replacement

  Independent $\rightarrow \frac{13}{52}, \frac{13}{52}$ with replacement

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- $P(A \text{ or } B) = P(A \cup B) = P(A+B) = P(A)+P(B)$

for mutually exclusive events.

Throw of dice.

$A = \{2,4,6\}$ even $\quad P(A) = 3/6 = 1/2$

$B = \{1,3,5\}$ odd $\quad P(B) = 3/6 = 1/2$

$A = \{2,4,6\}$

$B = \{2,3,5\}$

$$P(A+B) = P(A)+P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} \Rightarrow 1 - \frac{1}{6} = 5/6$$

$$P(A+B) = 5/6$$

$P(A) = \frac{m_1}{n}$ $\quad P(B) = \frac{m_2}{n}$ $\quad P(AB) = r/n$

$$\frac{m_1}{n} + \frac{m_2}{n} = 1.$$

if A & B are mutualy exclusive.

$$\boxed{\frac{\alpha_1+r}{n} + \frac{\alpha_2+r}{n} > 1 \Rightarrow \frac{\alpha_1+\alpha_2+2r}{n} > 1} \Rightarrow 1+r > 1$$

$$\frac{\alpha_1+\alpha_2+2r}{n} > 1 \Rightarrow 1+r > 1$$

$P(A-B) = \frac{\alpha_1}{n}$ $\quad P(AB) = r/n$

$P(B-A) = \frac{\alpha_2}{n}$

$$P(A-B) + P(B-A) + P(AB) = 1$$

$$\frac{\alpha_1}{n} + \frac{\alpha_2}{n} + \frac{r}{n} = 1$$

- Simple Event → An event is a simple event if there is only one outcome.

Toss of coin → H or Trial

Throw of a dice → 1 or 2 or 3 or 4 or 5 or 6

- Compound event → An event in which you get more then one outcomes.

Throw of dice → even number

$A = \{2, 4, 6\}$

Multiplication Theorem :-

$P(A \cap B)$

$P(A \text{ and } B) = P(AB) = P(A) . P(B)$ for independent event

let. $P(A) = \frac{m_1}{n_1}$ $P(B) = \frac{m_2}{n_2}$

$$P(AB) = \frac{m_1 m_2}{n_1 . n_2} = \frac{m_1}{n_1} . \frac{m_2}{n_2} = P(A) P(B)$$

Dependent event :-

$$P(A \cap B) = P(AB) = P(B) \times P(A/B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \frac{r}{n} = \frac{r}{n} . \frac{m}{m} = \frac{r}{m} . \frac{m}{n} = P(A/B) . P(B)$$

$P(B) = m/n$

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$\Rightarrow \quad P(A \cap B) = P(B) * P(A/B)$

or $\quad P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

For independents events $\quad P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)P(B)}{P(B)}$

$= P(A)$

Random Variable :-

Let (S, P) is a sample space then

$X(s)$, $\forall s \in S$ is a random variable defined

as $\quad X : S \rightarrow R$

Throw of a dice. The outcomes as random variable

$X(s)$ is the number appearing on dice

$S = \{1, 2, 3, 4, 5, 6\}$

$X(1) = 1$, $\quad X(2) = 2 \quad X(3) = 3 \; \cdots \; X(6) = 6$

- Throw of a dice the out comes as random variable

$X(s) = $ sum of number on dice $= 8$

$S = \{(1,1)(1,2).(1,3) \cdots (6,6)\}$

$\cancel{X(8) = \{(2,6),(3,5)(4,4)(5,3)(6,2)\}}$

$X(s) = X_1(s) + X_2(s)$

$X(1) = X_1(1) + X_2(1) = 2 + 6$

$$E(X) = \sum_{s \in S} X(s) P(X)$$

$$Var = \sum_{s \in S} (x-\mu)^2 P(x)$$

$$X(s) = x_1(s) - x_2(s)$$

Conditional probability:-

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q 10 red balls, 10 blue balls = 20 balls
Pick 2 balls probability of two red balls with replacement.

total no of outcomes = 400

A = first ball is red. ⇒ 10×20 $\quad P(A) = \frac{10 \times 20}{20 \times 20} = \frac{1}{2}$

B = second ball is red. ⇒ 20×10 $\quad P(B) = \frac{10 \times 20}{20 \times 20} = \frac{1}{2}$

$$P(A \cap B) = \frac{10 \times 10}{20 \times 20} = \frac{1}{4} = P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).\cancel{P(B)}}{\cancel{P(B)}} = P(A) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Que

$S = 20 \times 19 = 380$

A = first ball is red $\quad 10 \times 19 \quad P(A) = \frac{10 \times 19}{20 \times 19} = 1/2$

B = ~~second ball~~ ball is not red $\quad 20 \times 10 \quad P(B) = \frac{20 \times 10}{20 \times 19} = \frac{10}{19}$

10×10

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$$P(B) = \frac{10\times 9}{20\times 19} + \frac{10\times 10}{20\times 19} = \frac{190}{380} = \frac{1}{2}$$

$$P(A\cap B) = \frac{10\times 9}{20\times 19} = \frac{90}{380} = \frac{9}{38}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \frac{9/38}{1/2} = \frac{18}{38} = \frac{9}{19} \neq \frac{1}{2}$$

Bernouli Trial:-

we a toss a coin $\rightarrow$ H or T

Event

(i) success of event (2) failure of event.

p(success of event) = p
q(failure of event) = q $\qquad p+q=1$

Binomial distribution:-

Probability of r successes in n trials will be

$$P(x=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

mean = np
Varience = npq

Que Toss the coin for 10 times $P(x=2$ heads$)$ in "tosses.

$p = 1/2$, $q = 1/2$

$$P(x=2) = {}^{10}C_{2}\, p^{2} q^{10-2}$$

$$\frac{10\times 9}{1\times 2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{10-2} \Rightarrow \left(\frac{1}{2}\right)^{10}\times 45$$

$$P(0) = {}^{10}C_{0}\, p^{0} q^{10-0} = 1\left(\tfrac{1}{2}\right)^{10}$$

$P(1) = {}^{10}C_{1}\,(1/2)^{1}(1/2)^{10-1} \ldots \times 1 \Rightarrow \frac{10}{2^{10}}$

P(atmost 2 Heads)

$= P(0) + P(1) + P(2)$

$\Rightarrow \frac{1}{2^{10}}[1 + 10 + 45] \Rightarrow \frac{56}{2^{10}} \Rightarrow \frac{56}{1024}$

P(atleast 2 Heads)

$P(2) + P(3) + \dots P(10)$

$1 - (P(0) + P(1)) = 1 - \frac{1}{1024}[1 + 10]$

$= 1 - \frac{11}{1024} = \frac{1013}{1024}$

Que A dice is thrown 10 times ~~P(1 or 6)~~ find Probability of getting 1 or 6 for atleast two time

total outcomes of $= 6^{10}$

$P(1 \text{ or } 6) = \frac{2}{6} = \frac{1}{3} = p \qquad q = 1 - 1/3 = 2/3$

$P(x = r) = {}^{10}C_r\, p^r (q)^{n-r} = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{n-r}$

$= \left(\frac{1}{3}\right)^{10} {}^{10}C_r (2)^{10-r}$

$P(r \geq 2) = 1 - (P(0) + P(1)) \Rightarrow 1 - \left(\frac{2}{3}\right)^{10}[1+5] = 1 - \left(\frac{2}{3}\right)^{10} \times$

$P(0) = \left(\frac{1}{3}\right)^{10} {}^{10}C_0\, 2^{10} = \left(\frac{2}{3}\right)^{10}$

$P(1) = \left(\frac{1}{3}\right)^{10} {}^{10}C_1\, 1^1\, 2^9 = \left(\frac{1}{3}\right)^{10} \times 10 \times 2^9 = 5 \times \left(\frac{2}{3}\right)^{10}$

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Que 5 balls red 5 balls white = 10 balls in a box p(2 red and two white) when you pick 5 balls out of 10, with replacement.

total no of out comes $= 10 \times 10 = 100$

P(2

Que

x_i No of mistake per page	No of pages f_i	$f_i x_i$	$f p_i$
0	211	0	210
1	90	90	92
2	19	38	20
3	5	15	03
4	0	0	0
	$\boxed{N = 325}$	$\boxed{\Sigma f_i x_i = 143}$	$N = 325$

$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{N} =$$

$$\lambda = \bar{x} = \frac{143}{325} = 0.44$$

$$p(0) = \frac{\lambda^0 e^{-\lambda}}{0} = e^{-0.44} = 0.6440$$

$$f(0) = NP(0) = 325 \times 0.6440 = 210$$

$$f(1) = \frac{\lambda^1 e^{-\lambda}}{1!} = \lambda e^{-\lambda} = 0.44 \times e^{-0.44} =$$

$$0.44 \times 0.6440$$

$$f(1) = NP(1)$$

$$f(r) = \frac{\lambda}{r} f(r-1)$$

$$f(1) = \frac{0.44}{1} f(0) = 0.44 \times 210 = 92.15$$

$f(2) = \frac{0.44}{2} f(1) = \frac{0.44}{2} \times 92.15$

$= 20.27$

$f(3) = \frac{0.44}{3} f(2) = \frac{0.44}{3} \times 20.27$

$= 2.97$

$f(4) = \frac{0.44}{4} f(3) = \frac{0.44}{4} \times 2.97$

$= 0.33$

Poisson's Distribution :-

$(P+q)^n \longrightarrow$ n is finite $\Rightarrow$ Binomial distribution.

$\lim_{n \to \infty} (P+q)^n \rightarrow$ P is negligible & n is large.

$P \to 0, \quad q \to 1, \quad n \to \infty$

$$\boxed{P_r = P(x=r) = \frac{\lambda^r e^{-\lambda}}{r!}}$$

λ = is the mean of distribution = np

Q. Imp. characteristics of poisson dist $\Rightarrow$ mean = variance = $np = \lambda$

standard deviation $\sigma = \sqrt{\lambda}$

Ex 1 out of 1000 homes have fire in a certain district $n = 2000$ of theres houses. Find the $P(n)$ houses that exactly 5 houses have fire in a cer

$$p = \frac{1}{1000}, \quad \lambda = np = 2000 \times \frac{1}{1000} = 2$$

$$P_5 = P(x=5) = \frac{2^5 e^{-2}}{5!} = \frac{32 \times e^{-2}}{120} = \frac{4}{15} e^{-2} = 0.036$$

$$\lambda(0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} \quad e^{-2} = 0.13534$$

$$\lambda(1) = \lambda^1 e^{-\lambda}/1! = \lambda e^{-\lambda} = \frac{\lambda}{1} P(0)$$

$$\lambda(2) = \lambda^2 e^{-\lambda}/2! \Rightarrow \lambda^2 e^{-\lambda}/2 = \frac{\lambda}{2}(P(1))$$

$$\lambda(3) = \frac{\lambda^3}{3!} e^{-\lambda} = \frac{\lambda}{3} P(2)$$

Let $N \rightarrow$ total no of trials.

$$f(0) = N \cdot P(0) = 2000 \times e^{-\lambda} = 2000 \times e^{-2}$$

$$e^{-2} = 0.13534$$

270.68

$$f(1) = N \cdot P(1) = N \frac{\lambda}{1} P(1) = \frac{\lambda}{1} N P(0) = \frac{\lambda}{1} f(0)$$

$$f(2) = N P(2) = N\lambda/2 \cdot P(1) = \frac{\lambda}{2} N P(1) = \frac{\lambda}{2} f(1)$$

$$f(3) = \frac{\lambda}{3} f(2)$$

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$$\boxed{f(r) = \frac{\lambda}{r} f(r-1)} \qquad \boxed{P(r) = \frac{\lambda}{r} P(r-1)}$$

Que $3P(x=2) = P(x=4)$ $\cancel{P(x=4)}$ find λ

$$3P(x=2) = \frac{\lambda}{4} P(x=3) = \frac{\lambda}{4} \times \frac{\lambda}{3} P(x=2)$$

$$\Rightarrow \left(3 - \frac{\lambda^2}{12}\right) P(x=2) = 0$$

$\lambda^2 = 36$ $\qquad \lambda = \pm 6$ $\qquad \lambda = 6$

$$P(x=2) \neq 0$$

Que 2% of the bulbs produced are defective

Sample = 200

1) less than 2 defective.

2) more than 3 defective.

Sol^n $n = 200$, $p = 1/100$ $\lambda = \text{mean} = np = \frac{200 \times 2}{100} = 4$

(I) $P_0 + P_1 = P(r=0) + P(x=1) = \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} =$

$$\frac{e^{-\lambda}}{1} + 4e^{-\lambda} = 5e^{-4}$$

(II) $1 - (P_0 + P_1 + P_2 + P_3)$

$$\Rightarrow 1 - \left(P_0 + \frac{\lambda}{1} P_0 + \frac{\lambda^2}{2} P_0 + \frac{\lambda^3}{6} P_0\right) = 1 - 4e^{-4}\left(1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right)$$

$$= 1 - e^{-4}\left(1 + 4 + 8 + \frac{32}{3}\right)$$

Sample data

final poisson's distribution

Qty. No of mistake per page (x_i)	No of ~~proofer~~ pages (f_i)	$f_i x_i$	$f p_i$
0	211	0	210
1	90	90	92
2	19	38	20
3	5	15	03
4	0	0	01
	$n = 325$	$\Sigma f_i x_i = 143$	

Que generate poisson's distribution.

$$\text{mean} = \bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{143}{325} = 0.44 = \lambda$$

$$P_0 = P(x=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-0.44} = 0.6440$$

$$f(0) = NP(0) = 325 \times 0.6440 = 210$$

$$P(1) = P(x=1) = \frac{\lambda^1 e^{-\lambda}}{1!} = \frac{0.44\, e^{-0.44}}{1}$$

$$= f(1) = NP_1 = 325\,\lambda = 92.15$$

$$P_4 = P(x=4) = \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{(0.44)^4 e^{-0.44}}{4!} =$$

$$f(4) = NP_4 = \qquad =$$

or use formula $\boxed{f(r) = \frac{\lambda}{r} f(r-1)}$

i) find σ for both $\sigma \approx 0.44$

(ii) $\lambda = \text{mean} = \frac{\Sigma f_i x_i}{N}$ $\quad \sigma^2 = \frac{\Sigma (x_i - M)^2}{N}$

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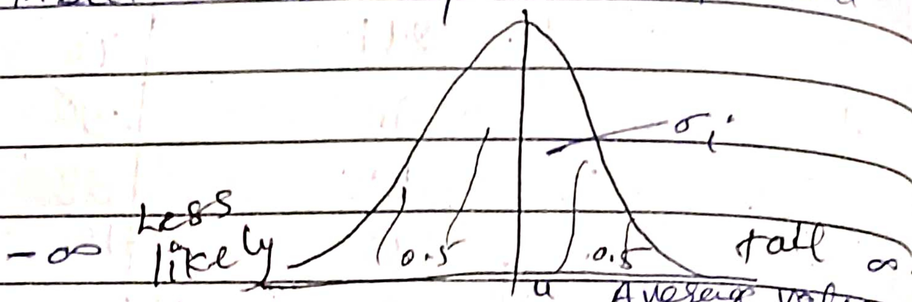
Normal distribution:-

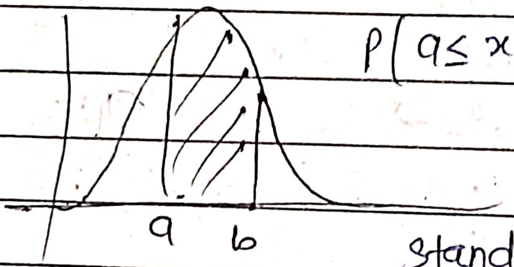
Normal distribution is symmetrical, well shaped



Bell curve = Median μ = mean
= mode

Gaussian curve
Normal curve.

f(x)

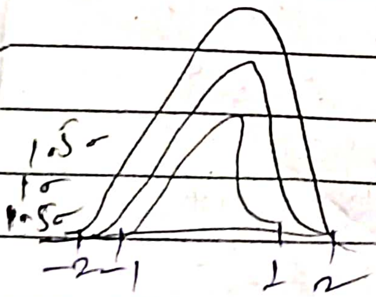


$P(a \le x \le b)$

Standard Normal Distrib.

$\boxed{Z = x - \mu}$

x = Random variable
μ = mean
σ = standard deviation



$(z<1) = 84\%$

Qus daily income of worker fallows Normal distrib. with mean = 1000 rs and standar Devi = 100 find the probality of income less than 1100.

$\frac{1100-10}{1000}$ $\mu = 1000$ (iii) find the prob less than 79..
$\sigma = 100$ (i) -2.1 Ans
$X = 1100$

$\frac{1450}{554}$

$P(x < 1100)$

$= P(z < 1)$

$0.5 + P(0 < z - 1)$

(iii) $P(x < 790)$

$P(z < -2.1)$

Que what is the prob random the table 550 hrs and 650 hrs when the average of 500 hrs and SD = 100.

$\to 0.5$ $\to 1.5$

mean = 500   S.D = 100

$P(x = 550)$

0.5 $z = 550 - 500 = \frac{50}{100} = 0.5$

$z \quad \frac{150}{100} = 1.5$