4) Explain 'image frequency' in heterodyne receivers and how it results in the mixer converting different input frequencies to the same IF frequency.

Consider an RF signal having frequency f_1 and the receiver having a mixer of LO frequency f_{LO}. Let $f_1 > f_{LO}$. Then the output of the mixer will have the intermediate frequency (IF) f_2 equal to $f_1 - f_{LO}$. .Now consider another scenario where the input frequency is equal to $2f_{LO} - f_1$. This time the output of the mixer will be $f_{LO} - (2f_{LO} - f_1) = f_1 - f_{LO}$, which is equal to the output of the mixer in the previous case. This frequency $2f_{LO} - f_1$ which gives the same IF frequency as f_1 is called the 'image frequency'.

Similarly if in the first case instead $f_1 < f_{LO}$, the output of the mixer will be same for another frequency equal to $2f_{LO} - f_1$ which is again the image frequency. This means that when one band of frequencies is down-converted to IF frequencies, another band of image frequencies is converted to the same IF band. So as far as the receiver is concerned, it cant distinguish between those 2 bands of input frequencies which creates a problem for spectroscopic measurements.

5) Given its not possible to maintain a constant phase relationship between all the input frequencies as the input signal is processed, what should be the ideal phase response of the components of the processing line like amplifiers and filters?

Any signal is a sum of its fourier components; sinusoidal waves of different frequencies. The passage of a signal through the signal processing line can be thought of as the passage of these individual waves through the various components like filters and amplifiers. Now, everytime a signal component passes through a filter or amplifier, the output is just a time-delayed version of the same wave (neglecting any amplitude change). But the amount of delay that a component experiences is dependent on its frequency. A delay in time means a corresponding phase shift for the wave, hence the different phase shifts caused by a filter/amplifier for different input frequencies is called the *phase response.*

Now for a general phase response, the fourier components will get delayed by different amounts of time. But this will change the shape of the original signal itself which is totally undesirable. An ideal phase response of a filter/amplifier would be such that all the fourier components are shifted by the *same amount of time*. This will preserve the overall signal shape despite causing phase differences between the components. Amount of phase shift is proportional to the frequency for a fixed time delay. In the same time delay, the components with higher frequencies will have a higher phase shift while longer frequency components will have lower phase shift. Hence the plot between the phase shift and frequency ie phase response, in an ideal case should be a straight line ie linear.

6) Explain the Wiener-Khinchin theorem and its use in noise characterization in signal processing.

The cross-correlation of 2 functions *f* and *g* denoted by f * g, is defined to be -

$$(f*g)(t)=\int_{-\infty}^{\infty} f(x)g(x-t)\,dx$$

Similar to the convolution theorem, the cross-correlation theorem says that the fourier transform of the cross-correlation of 2 functions is the product of the fourier transforms of the 2 functions, where one of them has been complex conjugated. In other words -

$$f*g \Leftrightarrow \overline{F}\cdot G$$

When g is taken to be f itself, then the cross-correlation is called auto-correlation and the statement of the cross-correlation theorem becomes -

$$f*f \Leftrightarrow \overline{F}\cdot F=|F|^2$$

This is the Wiener-Khinchin theorem. $|F|^2$ is called the power spectrum of f and it gives the corresponding strengths of various frequencies comprising f. Thus the theorem says that the auto-correlation and the power spectrum of a function f form a fourier pair. This theorem is used in the *auto-correlation spectrometer* to calculate the power spectrum of the signal.

The noise generated in the receiver is totally uncorrelated ie one portion of the noise waveform is totally independent of another portion at a different time. So the auto-correlation of the noise would be zero everywhere except at t = 0, where it would proportional to the mean-squared voltage. So the ACF of the noise is a dirac delta function. Applying the Wiener-Khinchin theorem on this, the power spectrum of the noise will be the fourier transform of the ACF. The fourier transform of a dirac delta function is a straight line parallel to the x-axis ie the power spectrum takes the same values at all frequencies.