Given we are observing a pulsar in the bandwidth $[\nu_1, \nu_2]$ calculate the time delay due to dispersion between the frequencies ν_1 and ν_2. Verify this on gptool for one pulsar each for Band 3 and 4.

While travelling through the interstellar plasma, different frequencies f in the range $[\nu_1,\nu_2]$ experience different refractive index. The relation is given by -

$$\mu = \sqrt{1 - \left(\frac{f_p}{f}\right)^2} \tag{1}$$

where f_p is the plasma frequency and is given by $f_p = \sqrt{\frac{e^2 n_e}{\pi m_e}}$. So the speed of frequency f in the plasma will be $v = c\mu$. Then assuming that the distance to the pulsar is d, the time delay t experienced by frequency f due to the plasma will be given by -

$$t = \left(\int_0^d \frac{dl}{v}\right) - \frac{d}{c} \tag{2}$$

Substituting v and f_p and approximating μ considering $f >> f_p$ -

$$t = \left(\int_0^d \left[1 + \frac{f_p^2}{2f^2}\right] \frac{dl}{c}\right) - \frac{d}{c} \Rightarrow t = D \times \frac{DM}{f^2} \tag{3}$$

where DM called the *Dispersion Measure* is given by $DM = \int_0^d n_e dl$ and D called the *Dispersion constant* is given by $D = \frac{e^2}{2\pi m_e c}$. From this we can calculate the delay *between* the arrival of the 2 given frequencies ν_1 and ν_2, Δt by -

$$\Delta t = D \times DM \times \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) \tag{4}$$

To verify this on gptool, the program was run on data for pulsar PSR B0329+54 in Band 4 (550 to 750 Mhz). The time delay between the frequencies 550 Mhz and 750 Mhz was observed and measured for 5 cycles (5 Periods) and their average was taken to estimate Δt. This comes out to be 150.0 ms. The theoretical value of Δt from (4) is equal to about 169.7 ms. There is a sizeable difference between the 2 values, but it is to be noted that the measured value was dependent on the pulse width at the frequency 550 Mhz and this measurement couldnt be taken very reliably.

For Band 3, gptool was run on the data for PSR J1645 -0317. It was seen that the there is a dispersion delay of the entire period of the pulsar (387.73 ms) between the frequencies 380 to 475 Mhz. For this frequency range, the Δt we get is equal to 401.8 ms which is close to the period of the pulsar.

Given the bandwidth $[\nu_1,\ \nu_2]$ calculate the amount of residual (smearing of the pulse) resulting from incoherent dedispersion due to the finite width of the spectral channels. Verify this on gptool for one pulsar each for Band 3 and 4.

Let the number of spectral channels be N. Then the width of each spectral channel will be $\Delta\nu = \frac{\nu_2 - \nu_1}{N}$. Even after incoherent dedispersion, there will be a time delay betweent the frequencies within each spectral channel. The maximum time delay between 2 frequencies in a spectral channel $[\nu_i, \nu_f]$ will be between the 2 end frequencies of the channel, given by -

$$\Delta t = D \times DM \times \left(\frac{1}{\nu_i^2} - \frac{1}{\nu_f^2} \right) = D \times DM \times \frac{(\nu_f - \nu_i)(\nu_f + \nu_i)}{\nu_i^2 \nu_f^2} \tag{5}$$

This will also be the amount of smearing caused by a spectral channel. We know $\nu_f - \nu_i = \Delta\nu$ as defined earlier. Its clear that Δt is maximum for the first spectral channel, since the denominator in Eqn(5) is minimum for that channel. So the overall smearing caused by the incoherent dedispersion will also be equal to the smearing caused by the first channel $[\nu_1, \nu_1 + \Delta\nu]$, given by -

$$\Delta t = D \times DM \times \frac{\Delta\nu(2\nu_1 + \Delta\nu)}{\nu_1^2(\nu_1 + \Delta\nu)^2} \tag{6}$$

Since typically $\Delta\nu << \nu_1$, this can be approximated to -

$$\Delta t \approx D \times DM \times \frac{2\Delta\nu}{\nu_1^3} \tag{7}$$

In terms of phase fraction $\frac{\Delta\phi}{\phi}$, the smearing can be written as -

$$\frac{\Delta\phi}{\phi} \approx D \times DM \times \frac{2\Delta\nu}{\nu_1^3} \times \frac{1}{P} \tag{8}$$

where P is the period of the pulsar.

Explain coherent dedispersion.

Let $S(t)$ and $S(\nu)$ be the signal emitted by the pulsar in the time and frequency domain respectively. Then the component of frequency of ν can be written as $S(\nu)e^{-i2\pi\nu t}$. On travelling a distance L between the source and the telescope, the phase of the frequency components changes such that a component of frequency ν now becomes $S(\nu)e^{-i2\pi\nu t}e^{i\phi_P} - (i)$ where -

$$\phi_P = 2\pi\nu\frac{L}{c}\left(1 - \frac{\nu_P^2}{2\nu^2}\right) \tag{9}$$

The frequency ν is downconverted to ν_b by the mixer so (i) becomes $S(\nu)e^{-i2\pi\nu_b t}e^{i\phi_P} - (ii)$. Assuming the bandwidth being used is $[\nu_1, \nu_1 + \Delta\nu]$, ϕ_P can be expanded by a Taylor series about ν_1, where the constant term ϕ_{p0} is simply $\phi(\nu_1)$. This phase factor is the same throughout the bandwidth and causes no time delay so not relevant for dedispersion.

The linear term in the series will be equal to $2\pi\nu_b\tau_1$, where τ_1 is the total time taken by the frequency ν_1 to reach the telescope. This causes a time shift (delay) for each frequency by the same amount τ_1. Again this doesnt cause relative shifts between the frequencies so not relevant for dedispersion. The sum of rest of the higher order terms in the Taylor series turns out to be -

$$\phi_D = -2\pi\nu_b D_1\left(\frac{\nu_b/\nu_1}{1 + \nu_b/\nu_1}\right) \tag{10}$$

where D_1 is the time delay caused by dispersion in frequency ν_1. This factor results in the relative time shifts between the frequencies. So if we reverse this phase factor $\phi_D(\nu_b)$ for every frequency ν_b then we will get back our dedispersed signal. So now our signal is the sum of frequency components given by (ignoring amplitude factor) -

$$e^{-i2\pi\nu_b(t-\tau_1)} \cdot e^{i\phi_{p0}} \cdot e^{i\phi_D} \tag{11}$$

So if we multiply each frequency component by $e^{-i\phi_D}$, this will remove the relative time delays. But this multiplication is happening in the frequency domain. Hence for coherent dedispersion, we first do a fourier transform of our voltage signal. Then we multiply it with $e^{-i\phi_D}$ and and do inverse fourier transformation. This will yield our dedispered signal.

What are the effects on the gptool analysis if the software is given a slightly incorrect range of frequencies than the ones actually recorded.

While running gptool on the data for pulsar PSR B0329+54, the frequency range given for the dedispersion was changed from the correct range [550,750] Mhz to [500,700] Mhz. It was observed that the individual as well as the average folded profiles were smeared to the left. The amount of smear was about 34.9 ms. In terms of $\Delta\phi/\phi$ it was about 0.049. The smear remains constant and doesnt change with the duration of the data.

The smear arises because of the following reason. since the frequencies given to the dedispersion tool are less than the true ones, the amount of shift required to counteract the dispersion delay is overestimated for every frequency. So every frequency is shifted to the left more than what was required. This overshooting is different for different frequencies which again leads to their misalignment causing smearing. Similarly if instead of [500,700] we had [600,800], each frequency's required shift for dedispersion would be *underestimated* causing smearing.

How will the observed period of a pulsar change as we observe it (i) over an entire day, (ii) over a year?

Due to the relative motion between a source and a receiver, the frequency of a signal changes, the effect being called Doppler shift. This arises due to the change in the separation of individual pulses of the EM waves emitted by the source because of the relative motion. But this effect will arise from any kind of pulsating behaviour by the source. If the pulsar is moving towards Earth then the time of separation between 2 pulses is slightly reduced. The apparent time period of the pulsar will decrease similarly if the telscope itself has a velocity component relative to the direction of pulsar. This can arise due to both the rotation of Earth and its revolution around the sun.

(i) We will assume that the pulsar and center of mass of earth have no relative motion. Due to rotation, different places on Earth have varying speeds relative to an inertial frame. Let the latitude of the place of observation of the pulsar be λ. To simplify things, lets also assume that the pulsar lies in the equitorial plane of Earth (ie declination of the pulsar $\delta = 0$). Then the maximum possible relative speed of the telescope (when the pulsar is just on the horizon), v_r wrt to the pulsar is $\omega R_E \cos\lambda$, where the first two terms are the angular velocity of rotation and the radius of the Earth respectively. But this v_r vector changes direction with Earth's rotation, becoming 0 when the pulsar is on the 'meridian' of the observer. So we measure this variation by the sin of the hour angle (H) of the pulsar at the time of observation, giving us $v_r = \omega R_E \cos\lambda \cdot \sin H$. Now if instead of $\delta = 0$, the pulsar were close to the direction of Earth's axis ($\delta = \pi/2$), then there will wont be any relative motion because of rotation. This again gives us a factor, finally yielding $v_r = \omega R_E \cos\lambda \cdot \sin H \cdot \cos\delta$ -(*).

Now let the true period of the pulsar be P (corresponding frequency of pulsar rotation f_P) and the apparent period due to rotation be P' (f'_P). Then

$$\Delta P/P = \frac{1/f'_P - 1/f_P}{1/f_P} = f_P/f'_P - 1 \tag{12}$$

The doppler shift formula is - $f' = \frac{c-v_r}{c} f$ where c is speed of light. So plugging the ratio of the 2 frequencies in Eqn(12) gives -

$$\Delta P/P = \frac{v_r}{c - v_r} \tag{13}$$

v_r is too small compared to c so it can be ignored in the denominator. Now using the expression (*), eqn(13) finally becomes -

$$\Delta P/P = \frac{\omega R_E}{c} \cos\lambda \cdot \sin H \cdot \cos\delta \tag{14}$$

(ii) Lets assume that Earth's orbit around the Sun is circular with constant speed v. Also let θ be the angle between Earth's and the pulsar's position with respect to the sun. Then the relative speed between the source and observer is $v \sin\theta$. If the pulsar is at an angle δ wrt the plane of the solar system, then similar to the effect of declination in the (i) case, v_r is modified as $v_r = v \sin\theta \cdot \cos\delta$. Using this again in eqn (13) yields -

$$\Delta P/P = \frac{v}{c} \sin\theta \cdot \cos\delta. \tag{15}$$

In principle, why cant we have an arbitrarily large number of spectral channels?