**Efficiency of an Air-Core Induction Linear Accelerator**

**Abstract:**

The purpose of this paper is to compare the input energy of an Air-Core Induction Linear Accelerator (commonly known as a Coil-gun or Gauss Cannon) to the output energy of the projectile. The specific accelerator that was tested in this paper was constructed by Kyle Werner and Peter Yaworsky at Bowdoin College in the modern electronics lab. The input energy is supplied from a voltage discharge from a capacitor which is calculated from the equation E = ½ C\*V2, where C is the capacitance and V is the voltage across the capacitor. Output energy is calculated from the velocity and the mass of the projectile (E = ½ m\*v2). This paper presents a “photo-gate” apparatus which consists of an infrared diode and a photodiode with infrared sensitivity used to determine the muzzle velocity of the accelerator.

**Introduction:**

An Air-Core Induction Linear Accelerator (which from this point forward will be referred to as a Gauss Cannon or Coil-gun) uses the *Lorentz* force to accelerate a conducting projectile.[[1]](#endnote-1) A good source for simple coil-gun background information is a paper presented by A. Balikci from the Gebze Institute of Technology in Turkey entitled *High Velocity Linear Induction Launchers.* His explanation deals with a multi-stage coil gun used to launch materials surrounded by a conducting “sleeve” as a replacement to more expensive launching methods such as rockets. This is more complicated than the coil-gun that is researched by this paper but the physics behind the propulsion of the projectile is the same. When current is passed through the coils which make up the barrel of the gun, current is induced in the projectile (in his case the conducting sleeve.) The interaction between these currents and the inducing magnetic fields results in propulsive and centering forces acting on the projectile. In his multi-stage example, the pulse of current through each successive barrel inductor must be carefully timed in order to continue to propel the projectile. If the timing is off, the magnetic field produced by an inductor could potentially produce a force acting in the direction opposite the motion of the projectile, attempting to pull it back into the inductor. This would hurt performance of the device.[[2]](#endnote-2)

There are many other factors which affect the performance of Gauss Cannons as well. The material of the projectile and its dimensions have an effect on the magnetic flux in the projectile and the mutual inductance of the system. This affect is presented in a paper titled *Optimization of parameters acting on a projectile velocity within a four stage induction coil-gun,* by a group of physicists from Turkey.i The magnetic flux and the mutual inductance determine the amount of force which is exerted on the armature. A paper from the University of Texas at Austin examines the conversion of energy from magnetic to kinetic for a single stage Coil-gun. This paper titled *Coil Gun Technology at the Center for Electromechanics, The University of Texas at Austin* presents a mathematical representation of the firing process which applies to a Gauss Cannon similar to the one examined by this paper.[[3]](#endnote-3) This paper examines the efficiency of the Coil-gun in a much simpler context by simply looking at the relation of the input energy to the system from the output energy of the projectile. There are many variables which affect the performance of a Coil-gun and therefore the efficiency found by this paper is expected to be very small. The paper from the University of Texas shows how the maximum efficiency of a pulsed induction accelerator is around 50% with optimized parameters.iii Most of the materials used in the Gauss Cannon presented by this paper were used because they were readily available and therefore were not carefully selected to optimize the cannon. The main focus of the build was to produce a functioning product in a limited amount of time. A potential area for future research would be optimizing the performance of the device.

Data collection for this experiment was conducted using a programming environment developed by National Instruments called LabVIEW. This application is a “highly productive development environment” which allows users to “rapidly design and deploy measurement and control systems.”[[4]](#endnote-4) In this research, a LabVIEW program was used to control the input voltage across the infrared diode of the photo-gate as well as to monitor the output voltage across the photodiode. The program took as an input the number of samples to be taken, the desired sampling rate, the length of the projectile, the voltage to be placed across the infrared diode, and a voltage threshold which would be used to determine when there was an object in the gate. The program gave the time in the gate, the muzzle velocity, as well as a graph of the voltage measured across the photodiode as a function of time. This graph was used to determine an appropriate voltage threshold level as well as provide the user with a visual representation of the run. Images of the block view diagram and the front view from the LabVIEW program are available in Appendix A. This paper will not comment on the programming behind the operation of this program, however a general explanation of how it works follows.

The muzzle velocity is found from detecting the amount of time the projectile spends in the photo-gate and by measuring the length of the projectile that will be crossing through the gate. During data collection, a voltage is applied across the infrared diode in the photo-gate and the voltage across the voltage diode is measured with a sampling rate of 500 kHz. When an object passes through the photo-gate and blocks the infrared radiation produced by the diode from reaching the detector, the voltage level across the photodiode in the gate drops. For every sample that the voltage across the photodiode is below the voltage threshold inputted by the researcher, a variable representing the number of samples the projectile is in the gate is incremented. At the end of the run the number of samples is divided by the sampling rate which results in the time that the projectile spent in the gate. The length of the projectile can be divided by the time spend in the gate to give the velocity. From here, the measured mass can be combined with this velocity to determine the output energy of the Coil-gun.

**Experimental Set-up:**

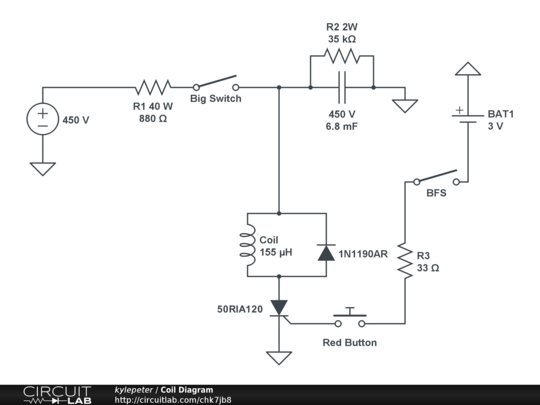
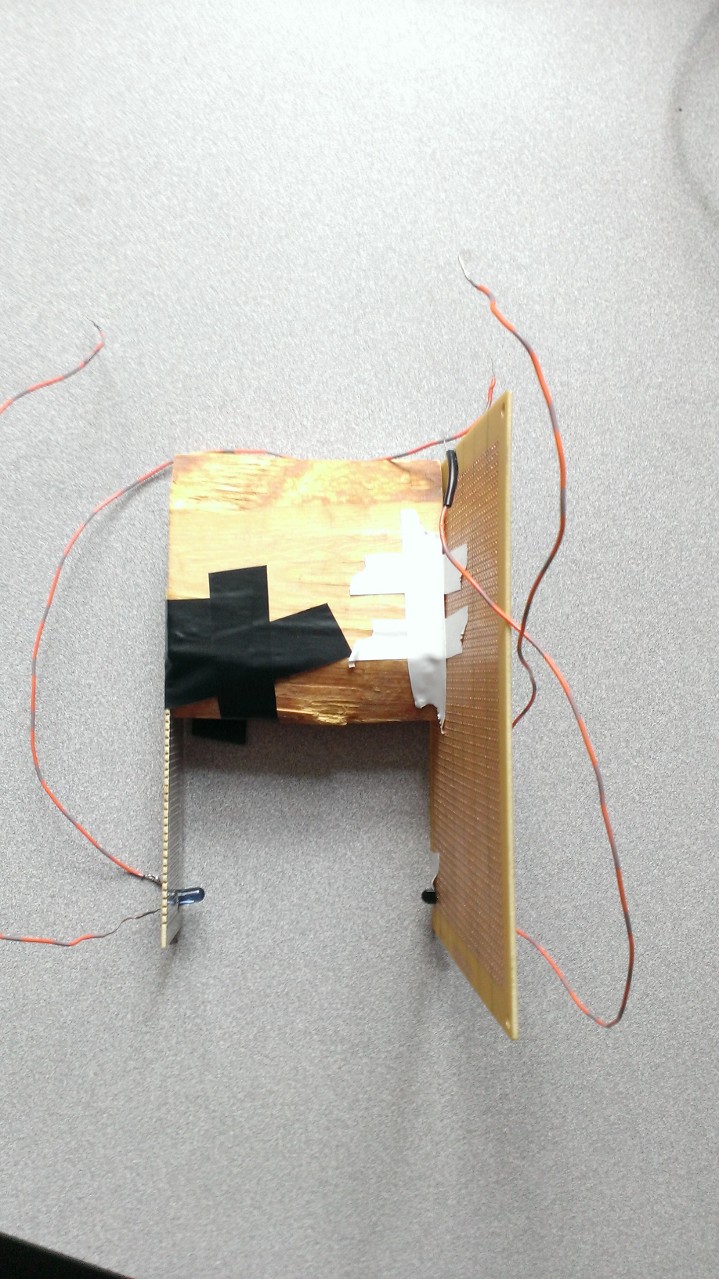
 The photo-gate detector was designed incrementally from testing the concept with the LED and the photodiode in a breadboard to the final product which was a standalone detector. The final product can be seen in Figure 1 with additional images of the apparatus available in Appendix A. The computer interface was upgraded from a standard USB data acquisition card capable of collecting data at 40 kHz to a modular one capable of data collection rates of 500 kHz. While the concept and the actual set-up of the photo-gate detector didn’t change much, this upgrade in computer interface made data collection much more accurate and made the detector capable of recording objects moving at very fast velocities. The theoretical maximum velocity this device can detect is an object moving at a speed at the length of the object multiplied by the 500 kHz. For the projectile fired from the gauss cannon analyzed in this paper which had a length of 2.6 cm, the detector could theoretically detect it traveling at approximately 12.5 km/s. However there are a few limitations of the device which need to be considered when doing error analysis and therefore the actual maximum velocity that it can detect may not be on this magnitude. For the scope of this research however the detector is more than capable of detecting the projectile traveling on the order of tens of meters per second which is the scale that this gauss cannon operates on.

Figure : Image of the photo-gate detector. The LED is located on the board to the left and the photodiode on the right.

The Gauss Cannon itself has a fairly simple construction. It is a single stage coil gun using an air-core inductor made from 16-gauge copper wire wrapped around a piece of PVC piping (which serves as the barrel), a capacitor, and a SCR switch. Figure 2 displays the circuit diagram for the gauss cannon and the various safety switches and the firing mechanism. Located at the middle of the diagram is the coil-gun labeled coil in series with a LED oriented backwards to combat back EMF. The inductance of the coil is 155 μH. The capacitor is located above and to the right of the coil apparatus with the charging mechanism located to the left of it. The SCR is below the coil-gun with the various switches branching off to the right preventing an accidental firing of the cannon. The capacitance of the capacitor is 6900 μF and has a voltage rating of 450 Volts. (The manufacturer spec on the capacitance was 6800 μF +/- 20%. This method for determining the more precise value of 6900 μF is described in detail in the Determining Uncertainty Section). The SCR is used as a switch because it allows a large amount of current to flow very quickly through the coil gun. When a small input voltage reaches a threshold point the SCR allows the capacitor to dump a large current across the coil-gun thereby producing the necessary magnetic field to propel the projectile.

Figure : Circuit Diagram for the Gauss Cannon. The charging circuit and the triggering circuit are also included

**Methods:**

Data collection methods for the gauss cannon trials involved a very simple set-up. The photo-gate was placed within a centimeter from the muzzle of the coil-gun with the “beam” centered with the barrel. The following process was repeated for five trials at input voltages of 100V, 150V, 200V, 300V and 430V:

1. The capacitor was charged to approximately 5V above the desired Voltage. For example the capacitor would be charged to 105 Volts for the 100V trials. This is because the voltage would decay once the charging voltage source was disconnected.
2. The voltage across the capacitor was monitored using a DMM and when the voltage was 1V above the target voltage the LabVIEW program was initiated with a sample rate of 500 kHz and taking 1,000,000 samples. This would have the program take data for two seconds which would ensure that it would record the shot fired at the desired voltage.
3. When the desired voltage was reached, the necessary input voltage required for firing the device was given to the SCR resulting in the firing of the projectile. Trials where the device failed to fire or fired backwards were discarded.
4. The values calculated by the program were recorded as data to be manipulated into output energy of the gauss cannon.

This procedure was conducted for two projectiles of different sized. One had a mass of 0.01028 kg and a length of 0.025m and the other had a mass of 0.01447kg and a length of 0.026m. This data was loaded into an excel spreadsheet which then calculated the output energy and input energy of the cannon. This data is presented in the data section and the raw data for these trials is available in Appendix C.

**Determining Uncertainty:**

One source of error to the velocity calculation is due to the uncertainty of the actual position of the projectile when the photo-gate registers that it is inside. It is possible for the projectile to have moved slightly into the gate in-between the collection of samples. However, assuming that the projectile is moving on the order of 20 m/s, the change of position in 1/500k of a second is very small and therefore is a very small contribution to the error. Another issue is the “beam” of infrared photons between the LED and the photodiode isn’t perfectly collimated, and therefore it is difficult to determine how far into the gate the projectile has to be in order for the photodiode to register a voltage drop. A brief experiment was performed to see if the values gathered from the device were reasonable considering error in other measurements. To perform this test a projectile was dropped from a set height and its time in the gate as well as the velocity were recorded from the LabVIEW Program. The time the projectile should be registered in the gate was calculated using X(t) = X0 + V0 \*t + ½ a\*t2, where X is the position of the center of mass of the projectile, V is the velocity, and a is the acceleration due to gravity. The times when the projectile should have entered and left the gate were determined using this equation and the data collected from LabVIEW was consistent with the expected values given the uncertainty in the height measurement. Therefore I determined that my measurements were reasonable and I would use the standard deviation in the time of several trials as my uncertainty in the time. The data and sample calculations associated with this test are available in Appendix B.

Another source of error in the velocity measurement is the length of the projectile. The length was measured using a pair of calipers and the uncertainty in that measurement determined to be 0.2 mm. Another source of error in the “length” is dependent on the geometry of the projectile and its orientation as it goes through the gate. If the projectile doesn’t pass through the gate with the entirety of the measured length, the measured velocity will be higher than it should be. This is because the time in the gate would be smaller than it would be for the whole length, and therefore dividing the whole length value by this time results in a higher velocity. However, given the geometry of the experimental set-up coupled with the proximity of the detector used in determining the muzzle velocity of the gauss cannon, the source of error in the length is simply the error in the measurement using the calipers. The projectile used in these tests was cylindrical in shape making vertical displacements insignificant as opposed to using a spherical projectile.

The major source of error in the calculation of the input energy to the system was the value of the capacitance. The factory specs on the capacitor gave the capacitance as 6800 μF +/- 20%, which results in scientifically insignificant data if this value is used during calculation. Therefore, a simple RC circuit was constructed in order to determine a more accurate value for the capacitance. A HP 3225B function generator was used to provide an AC square wave to the circuit at a rate of 9 mHz. The resistor in the circuit had a resistance of 1500 Ω with a tolerance of 1%. This resulted in a RC circuit that took 100 seconds to complete one period of its oscillation allowing for an accurate measurement of the RC time constant using an oscilloscope. The main equation used in the calculation of the capacitance was VC (t) = VS (1 – e-t/RC). VS was measured from the oscilloscope trace and determined to be 5.10 V. The uncertainty in the voltage measurement was estimated to be 0.05V. The time for VC to rise from 0 to equal to 0.63\*VS was recorded for the t value. It was determined from the oscilloscope trace that t was 10.0 +/- 0.2 seconds. Using this value for t, C was calculated to be 6900 μF with an uncertainty of 3.4% from the equation stated above. Therefore the final value for the capacitance with uncertainty was 6.9 +/- 0.2 mF. Detailed calculations of this process are available in Appendix B.

The uncertainty in the voltage and the mass of the projectile were taken from the manufacturer specs of the precision of the equipment. The DMM used to determine the voltage has an uncertainty of +/- 1% + 1 dgt and the OHAUS AS60-5 scale used to measure the mass of the projectile has an uncertainty of 0.01g associated with the measurement.

**Data:**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Vin (V)** | **Ein (J)** | **δEin** | **Tin gate (s)** | **vout (m/s)** | **Eout (J)** | **δEout** | **%Eff** | **%Efferr** | **%err** |
| 100 | 34.5 | 0.6 | 0.00132 | 18 | 1.83 | .09 | 5.3 | .3 | 5.3 |
| 150 | 78 | 1 | 0.00102 | 25 | 3.1 | .2 | 4.0 | .3 | 7.1 |
| 200 | 138 | 2 | 0.00089 | 28 | 4.1 | .1 | 2.94 | .09 | 3.2 |
| 300 | 311 | 5 | 0.00076 | 33 | 5.5 | .2 | 1.77 | .06 | 3.5 |
| 430 | 640 | 10 | 0.00069 | 36 | 6.8 | .2 | 1.07 | .04 | 3.3 |

Table 1 : Relevant data for the coil-gun firing the 10.28g projectile. In order to be concise some of the data such as the length (2.50 +/- 0.02cm) and some of the uncertainties are left out. The values presented are an average of several trials. These values can be found in Appendix C along with sample calculation.

Figure : Comparison of the Natural Log of the Input Voltage to the Muzzle Velocity of the Gauss Cannon for the 10.28g projectile. The differences in the vertical error bars are mainly caused by differences in the standard deviation of the time used to calculate the velocity.

Figure : Comparison of the efficiency of the system to the natural log of the input voltage. There is a negative linear relationship between the two values.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Vin (V)** | **Ein (J)** | **δEin** | **Tin gate (s)** | **vout (m/s)** | **Eout (J)** | **δEout** | **%Eff** | **%Efferr** | **%err** |
| 100 | 34.5 | 0.6 | 0.00153 | 17.0 | 1.49 | 0.06 | 4.3 | 0.1 | 4.1 |
| 150 | 78 | 1 | 0.00107 | 24.3 | 3.04 | 0.07 | 3.9 | .1 | 2.8 |
| 200 | 138 | 2 | 0.00096 | 27.1 | 3.78 | 0.09 | 2.73 | 0.08 | 2.8 |
| 300 | 311 | 5 | 0.00084 | 31 | 4.9 | 0.1 | 1.58 | 0.05 | 3.1 |
| 430 | 640 | 10 | 0.00075 | 35 | 6.2 | 0.4 | 0.97 | 0.07 | 7.0 |

Table : Relevant data for the coil-gun firing the 14.47g projectile, again some data has been left out. The numbers present are an average of several trials Raw data is available in Appendix C.

Figure : Comparison of the natural log of the input voltage for the 14.47g projectile. There is a fairly good fit of a linear line to this data suggesting a logarithmic relationship between Input Voltage and the Muzzle Velocity. A large standard deviation in the time used to calculate velocity was responsible for the large error bar on the last point.

Figure : Comparison of the efficiency of the system to the natural log of the input voltage of the 14.47g projectile. There are small fluctuations in the data points but the trend is still linear

**Analysis:**

Tables 1 and 2 above provide a condensed version of the raw data which displays the input and output energies as well as the efficiency at different input voltages. It can be seen from these tables that the efficiency of the gauss cannon decreases as the input energy increases. This suggests that the muzzle velocity of the system doesn’t scale with the input voltage as would be expected in an idealized system. The mass of the projectile and the capacitance of the capacitor do not change between different input voltage trials. Therefore, comparing the equation for calculating the input Energy (E = ½ C V2) to the equation used to calculate the output Energy (E = ½ m v2), a constant ratio is expected between the input voltage and the output velocity in an idealized system. Obviously this system is not ideal and therefore there are forces which are not accounted for in this model. These unaccounted for forces are responsible for limiting the performance of the coil-gun.

The series of graphs presented in Figures 3 through 6 suggest that there is a logarithmic relationship between the muzzle velocities and the input voltages as well as the efficiency of the system to the input voltages. The graphs show how these values have a fairly linear relationship to the natural log of the input voltage. Figures 3 and 4 show there is a positive linear relationship between the muzzle velocity and the natural log of the input voltage; this means that increasing the input voltage to the coil-gun will only result in an increase on the order of ln(∆V) in the velocity of the projectile. For the 10.28g projectile, the equation describing the relationship between the output velocity and the input voltage is v = 11.97\*ln(V)-35.74. In order to boost the velocity of the projectile to say 45 m/s (approximately 100mph), the input voltage would have to be increased to around 850 Volts. For the 14.47g projectile, the voltage would have to be increased to almost 1 kV in order to achieve this velocity. This shows that in order to increase the velocity by approximately 10 m/s (about 33%), the input voltage would at least have to be doubled.

It is interesting that there is a negative logarithmic relationship between the efficiency of the system and the input voltage. This can be seen in figures 5 and 6. The increase in the voltage causes the efficiency of the coil-gun to decay rapidly. For the 10.28g projectile, the equation for the relationship between the input voltage and the efficiency suggests that the efficiency will approach 0% around 575 Volts. Similarly, for the 14.47g projectile, the efficiency will approach 0% around 620V. The combination of the relationships suggests that there is a maximum velocity which is the coil-gun can fire at. Using the figures 3 and 4, this velocity can be approximated as the velocity associated with the values for the input voltage with zero efficiency from figures 5 and 6. For both of the projectiles the maximum velocity is approximately 40 m/s. After this velocity is reached, the increase in the voltage doesn’t help the performance and the ratio of the input to output energies approaches 0. Overall the data suggests that there are factors which are limiting the performance of the Coil-gun as the input voltage is increased.

**Conclusions:**

The Coil-gun examined by this paper has very low efficiency as was expected. Despite this, the development of this device using materials around the lab is an achievement. The photo-gate detector which was developed to measure the muzzle velocity of this cannon did so with sufficient accuracy to provide meaningful data to be analyzed. The fractional uncertainty in the percent efficiency is typically between 3 and 5 percent with a couple outliers in the 7 percent range. Therefore the majority of the data is scientifically relevant. The trials which had this high fractional uncertainty had a high standard deviation in the time used to calculate the velocity. This could be due to any number of factors during the testing including the temperature of the coil and the starting position of the projectile in the barrel to name a few. Potential future modifications to the photo-gate could be to add another “beam” a known distance away and determine the speed from measuring the time difference between first contacts.

Reasons for the inefficiency of the Gauss Cannon are quite numerous. One source is the resistive losses in the coil of wire that makes up the cannon. Another is that the projectile isn’t made of super conducting material and therefore the induced currents are not 100% efficient. The logarithmic nature of the data suggests that there may be a velocity dependant force resisting the motion of the projectile. This would explain why there is a maximum velocity the cannon can fire at. Another source of inefficiency is the magnetic saturation of the projectile. This the when all of the available dipoles in a material are aligned with the magnetic field and the magnetic energy stored in the material is at its maximum.[[5]](#endnote-5) This would explain how the input of more energy to the system improves the output energy less and less. These are just some of the potential losses of efficiency in the Gauss Cannon. Overall, considering the scope of the research, the Coil-gun and the photo-gate detector were able to sufficiently calculate the efficiency of the system.

1. http://www.sciencedirect.com/science/article/pii/S0263224110002162 [↑](#endnote-ref-1)
2. http://www.sciencedirect.com/science/article/pii/S0734743X08001358 [↑](#endnote-ref-2)
3. http://www.utexas.edu/research/cem/IEEE/PR%20158%20Bresie%20Publications.pdf [↑](#endnote-ref-3)
4. <http://www.ni.com/labview/whatis/>

   <http://www.britannica.com/EBchecked/topic/357334/magnetism/71543/Induced-and-permanent-atomic-magnetic-dipoles?anchor=ref113953>

   **Appendix A**

   **Appendix B**

   **Appendix C** [↑](#endnote-ref-4)
5. [↑](#endnote-ref-5)