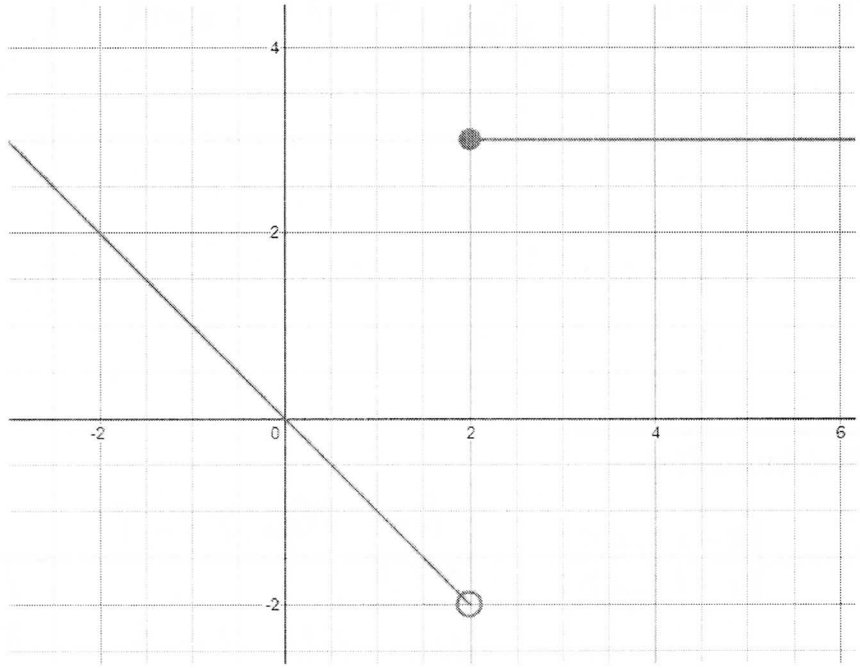
Math 208-202 Exam 03 Solutions

Exam Begins Here

1. Using the graph of $f(x)$, find the limits



(a) $\lim_{x\to 2^-} f(x) =$ -2

(b) $\lim_{x\to 2^+} f(x) =$ 3

(c) $\lim_{x\to 2} f(x) =$ DNE

2. ______

Find

$$\lim_{x\to -2} \frac{x^2+x-2}{10-3x}$$

$$\frac{4-2-0}{10+6} = 0$$

3. ________________

Find

$$\lim_{x\to\infty} \frac{-4x^4+5x^3+1}{2x^4-9} = \lim_{x\to\infty} \frac{-4}{2} \frac{x^4}{x^4} = \lim_{x\to\infty} -2 = -2$$

4. ________________

Find $\lim_{x\to 1} f(x)$

$$f(x) = \begin{cases} 2-x & ,x<1 \\ x-1 & ,x\geq 1 \end{cases}$$

$\lim_{x\to 1^-} f(x) = 2-1 = 1$

$\lim_{x\to 1} f(x) = \text{DNE}$

$\lim_{x\to 1^+} f(x) = 1-1 = 0$

5. ________________

Find

$$\lim_{x\to -7} \frac{x^2+5x-14}{x^2-49} = \frac{49-35-14}{49-49} = \frac{0}{0}$$ indeterminate

$$\lim_{x\to -7} \frac{(x+7)(x-2)}{(x+7)(x-7)} = \lim_{x\to -7} \frac{(x-2)}{x-7} = \frac{-7-2}{-7-7} = \frac{-9}{-14} = \frac{9}{14}$$

6. Worth double points

7. Using the four step process, find the derivative of $x^2 - 2x$.

(a) $f(x+h) = (x+h)^2 - 2(x+h)$

$= x^2 + 2xh + h^2 - 2x - 2h$

(b) $f(x+h) - f(x) = x^2 + 2xh + h^2 - 2x - 2h \; - (x^2 - 2x)$

$= 2xh + h^2 - 2h$

$= h(2x + h - 2)$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h}{h}(2x+h-2)$

$= 2x + h - 2$

(d) $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 2x - 2$

In the problems 8-11, use the rules for computing derivatives. Find each derivative and simplify (fractional and negative exponents are acceptable).

8. $4x^3-36x$

$y = x^4 - 18x^2 + 3$

$y' = 4x^{4-1} - 2(18)x^{2-1} + 0$

9. $-9x^{-4}$

$f(x) = \frac{3x}{x^4} = 3x^{-3}$

$f'(x) = -3(3)x^{-3-1}$

10. $2t + 12t^{-5} - t^{-4/3}$

$h(t) = t^2 - \frac{3}{t^4} + \frac{3}{\sqrt[3]{t}} = t^2 - 3t^{-4} + 3t^{-1/3}$

$h'(t) = 2t^{2-1} - (-4)(3)t^{-4-1} + (-\frac{1}{3})(3)t^{-1/3-1}$

11. $1-3x^{-2}-8x^{-3}$

$f(x) = \frac{3x^3 + 9x + 12}{3x^2} = \frac{3x^3}{3x^2} + \frac{9x}{3x^2} + \frac{12}{3x^2}$

$= x + 3x^{-1} + 4x^{-2}$

$f'(x) = 1 + 3(-1)x^{-1-1} + 4(-2)x^{-2-1}$

12. What does the derivative represent? (circle all that apply) A, D, E

(A) Slope of the tangent line

(B) Average rate of change

(C) Slope of the secant line

(D) Velocity

(E) Instantaneous rate of change

13. Find the values of x where the tangent lines are horizontal for the curve

$$y = x^4 - 18x^2 + 3$$

(a) $4x^3 - 36x = y'$

Find the derivative. (Same as question 8)

Horizontal tangent line means $f'(x) = 0$

(b) $x=0,\ x=-3,\ x=3$

Find all values of x where the slope is zero.

$0 = 4x^3 - 36x = 4x(x^2-9) = 4x(x+3)(x-3)$

14. $C'(98) = 120 - .2(98) = \$100.40$

Given that the total cost of producing x smartphones is

$$C(x) = 12,000 + 120x - 0.1x^2$$

What is the marginal cost of producing the 99^{th} phone?

$C'(x) = 120 - .2x$

15. ____________________

The price demand function is given by

$$p = 50 - 0.2x$$

for x units of production. What is the Revenue function?

$R(x) = x\,p$

$= 50x - .2x^2$

16. Given the Revenue Function of

$$R(x) = 4x^2 - x + 10000$$

and a Cost Function of

$$C(x) = x^2 + 100$$

Find:

(a) $3x^2 - x - 9900$

The Profit Function, $P(x)$.

$P(x) = R(x) - C(x) = 4x^2 - x + 10\,000 - x^2 - 100$

$= 3x^2 - x - 9900$

(b) $P(x) = 6x - 1$

The Marginal Profit Function.