Math 208 Exam 1

Show ALL of your work, and clearly label each problem

Some Formula

- $I = Prt$
- $A = P + I$
- $A = P(1 + \frac{r}{m})^{mt}$
- $A = Pe^{rt}$
- $APY = (1 + \frac{r}{m})^m - 1$
- $APY = e^r - 1$
- $\det(M) = ad - bc$
- $M^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

---

1. A payday lender charges \$60 for a loan of \$500 for 15 days. Find the annual interest rate. (Use a 360-day year.)

$I = Prt$

$60 = 500\, r\, \frac{15}{360}$

$r = \frac{60}{500}\left(\frac{360}{15}\right) = 2.88 = 288\%$

2. Suppose \$30,000 is invested at an annual interest rate of 1.2%. Compute the future value after 10 years if the interest rate is compounded monthly. Round to the nearest cent.

$A = P(1 + \frac{r}{m})^{mt}$

$= 30000\left(1 + \frac{.012}{12}\right)^{12(10)}$

$A = 30000(1.001)^{120}$

$= \$33822.88$

3. Suppose an investment account has an annual interest rate of 4.3%. How long does it take an investment to double if interest is compounded continuously? Round to the nearest tenth of a year.

$A = Pe^{rt}$

$\frac{A}{P} = 2 = e^{rt}$

$\ln(2) = rt$

$t = \frac{\ln(2)}{.043} = 16.12$ yrs

4. If I wish to retire in 35 years with \$500,000, how much must I invest now at 2.1% nominal interest rate compounded continuously. Round to the nearest cent.

$A = Pe^{rt}$

$500000 = Pe^{.021(35)}$

$P = \frac{500000}{e^{.735}}$

$= \$239,752.73$

5. How much must be invested now at UW Credit Union so that 6 years from now the amount would be \$44,500? Assume the nominal interest rate is 3% compounded quarterly. Round to the nearest cent.

$A = P\left(1 + \frac{r}{m}\right)^{mt}$ $\quad P = \frac{44500}{\left(1 + \frac{.03}{4}\right)^{4(6)}} = \frac{44500}{(1.0075)^{24}}$

$P = \frac{A}{\left(1 + \frac{r}{m}\right)^{mt}}$ $\quad P = \$37194.50$

6. Two banks have investment accounts that compound their interest differently. Which of the two accounts has a better effective rate?

   A) Bank A has an interest rate of 5.3% compounded monthly

   B) Bank B has an interest rate of 5.2% compounded continuously

$APY_A = \left(1 + \frac{.053}{12}\right)^{12} - 1$ $\quad APY_B = e^{.052} - 1$

$= .0543 = 5.43\%$ $\quad = .0534 = 5.34\%$

Bank A is better

For all matrix operations, compute the result or if the operation is not possible, state why.

7. Solve the following system of equations using substitution

$$4x - y = 12$$
$$-3x + 4y = 6$$

$y = 4x - 12$

$-3x + 4(4x - 12) = 6$

$-3x + 16x - 48 = 6$

$13x = 54$

$x = 4.1538$

$y = 4.615$

8. Put the following system of equations into augmented matrix form.

$$\begin{aligned} 3x_1 + x_2 - x_3 + 2x_4 &= 2 \\ 2x_1 + 2x_3 &= 1 \\ -x_1 - x_2 + x_4 &= 0 \\ x_1 + 3x_2 - 3x_3 &= 9 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 3 & 1 & -1 & 2 & 2 \\ 2 & 0 & 2 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 \\ 1 & 3 & -3 & 0 & 9 \end{array}\right]$$

9. Solve the augmented matrix using Gauss-Jordan elimination

$$\begin{bmatrix} 3 & 1 & | & 6 \\ 2 & -2 & | & -8 \end{bmatrix} \quad \begin{matrix} R1(1/3) \\ R2(1/2) \end{matrix} \begin{bmatrix} 1 & 1/3 & | & 2 \\ 1 & -1 & | & -4 \end{bmatrix}$$

$$\begin{matrix} R1 \\ R2-R1 \end{matrix} \begin{bmatrix} 1 & 1/3 & | & 2 \\ 0 & -4/3 & | & -6 \end{bmatrix} \quad \begin{matrix} R1 \\ -3/4\,R2 \end{matrix} \begin{bmatrix} 1 & 1/3 & | & 2 \\ 0 & 1 & | & 9/2 \end{bmatrix} \quad \begin{matrix} R1 - 1/3\,R2 \\ R2 \end{matrix} \begin{bmatrix} 1 & 0 & | & 1/2 \\ 0 & 1 & | & 9/2 \end{bmatrix}$$

$x_1 = 1/2$

$x_2 = 9/2$

10. Solve the equation $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. Using the inverse matrix method

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1} = \frac{1}{5-4} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

11. Solve the following for x, y, and z

$$\begin{bmatrix} 3x & 2y \\ 4z & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ z & -6 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 25 & 0 \end{bmatrix} = \begin{bmatrix} 3x+2 & 2y+5 \\ 5z & 0 \end{bmatrix}$$

$3x+2=5$ $\quad$ $2y+5=11$ $\quad$ $5z=25$ $\quad$ $0=0$

$3x=3$ $\quad$ $2y=6$ $\quad$ $z=5$

$x=1$ $\quad$ $y=3$

12. Compute the resulting matrix.

$$\begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$$

$A - 2A = -A$

13. Find AB, and BA, given that $A = \begin{bmatrix} 2 & -2 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$

AB is not defined

3×2 $\quad$ 3×3

$$BA = \begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4+0+0 & -4+6+0 \\ 8+0-3 & -8+1-1 \\ 0+0-6 & 0+1-2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 5 & -8 \\ -6 & -1 \end{bmatrix}$$

3×3 $\quad$ 3×2 $\quad$ 3×2

14. Find

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 0 & 3 & 5 \end{bmatrix}$$

15. Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find $I^{20}AI^3$. $= A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

16. Use the row operations to find A^{-1}, or explain why it doesn't exist if

Augment $A = \left[\begin{array}{ccc|ccc} 4 & 8 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 5 & 10 & 1 & 0 & 0 & 1 \end{array}\right]$

$$\begin{matrix} R2 \\ 1/4\,R1 \\ 1/5\,R3 \end{matrix}\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 1/4 & 1/4 & 0 & 0 \\ 1 & 2 & 1/5 & 0 & 0 & 1/5 \end{array}\right]$$

$$\begin{matrix} R1 \\ R2-R1 \\ R3-R1 \end{matrix}\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -1/4 & 1/4 & -1 & 0 \\ 0 & 0 & -14/5 & 0 & -1 & 1/5 \end{array}\right] \quad \begin{matrix} R1 \\ -4/11\,R2 \\ -5/14\,R3 \end{matrix}\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/11 & 4/11 & 0 \\ 0 & 0 & 1 & 0 & 5/14 & -1/14 \end{array}\right]$$

$$\begin{matrix} R1 \\ R2 \\ R3-R2 \end{matrix}\left[\begin{array}{ccc|c} 1 & 2 & 3 & \sim \\ 0 & 0 & 1 & \sim \\ 0 & 0 & 0 & \sim \end{array}\right]$$

No Inverse

17. Solve the system of equations

$$3x_1 + 2x_2 = 1$$
$$-x_1 - x_2 = -1$$

$Ax = b$

$x = A^{-1}b$

$A = \begin{bmatrix} 3 & 2 \\ -1 & -1 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$A^{-1} = \frac{1}{-3+2} \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

18. Find x, so that

$$\begin{bmatrix} 3 & 4x-1 \\ -1 & 4x+3 \end{bmatrix}$$ is singular $\det = 0$

$\det = 3(4x+3) - (-1)(4x-1) = 0$

$12x + 9 + 4x - 1 = 16x + 8 = 0$

$16x = -8$

$x = -1/2$

19. Give an example of matrices A, and B such that $AB = BA$, but $AB \neq I$

Let $B = I$

then $AI = IA = A$ ✓

20. Give an example of matrices A, B, C, D such that

(a) AB is undefined, why is your example undefined?

$A_{1\times 3} = [1\ 2\ 3]$ $B_{2\times 1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$3 \neq 2$

(b) $C + D$ is undefined, why is your example undefined?

$C_{2\times 1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $D_{1\times 2} = [1\ 2]$ not same dimension