

Math 208-711 Exam 1

Solutions

$$\begin{aligned} 1 \quad A &= P \left(1 + \frac{r}{m} \right)^{mt} \\ &= 22000 \left(1 + \frac{.0275}{12} \right)^{12(3)} \\ &= 22000 (1.08589) \\ &= \$23889.72 \end{aligned}$$

$$\begin{aligned} 2 \quad A &= P e^{rt} \\ &= 5500 e^{.06(20)} \\ &= 5500 (3.32) \\ &= \$18260.64 \end{aligned}$$

$$\begin{aligned} 3 \quad A &= P \left(1 + \frac{r}{m} \right)^{mt} \\ &= 18000 \left(1 + \frac{.035}{365} \right)^{365(10)} \\ &= 18000 (1.4119) \\ &= \$25542.79 \end{aligned}$$

$$\begin{aligned} 4 \quad A &= P \left(1 + \frac{r}{m} \right)^{mt} \\ P &= \frac{A}{\left(1 + \frac{r}{m} \right)^{mt}} \\ &= (63500) \left(\frac{1}{\left(1 + \frac{.03}{4} \right)^{4(5)}} \right) \\ &= 63500 \left(\frac{1}{1.6118} \right) \\ &= \$54685.56 \end{aligned}$$

$$\begin{aligned}
 5 \quad APY &= \left(1 + \frac{r}{m}\right)^m - 1 \\
 &= \left(1 + \frac{.035}{4}\right)^4 - 1 \\
 &= .03546 = 3.55\%
 \end{aligned}$$

$$\begin{aligned}
 6 \quad APY &= e^r - 1 \\
 &= e^{.034} - 1 \\
 &= 3.46\%
 \end{aligned}$$

$$7 \quad \begin{bmatrix} 3 & 7 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

8 Not defined for addition

$$9 \quad 2 \times 3 \quad 3 \times 2 \quad \checkmark \rightarrow 2 \times 2$$

$$\begin{aligned}
 \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ -6 & 3 \\ 9 & 0 \end{bmatrix} &= \begin{bmatrix} -8-6+0 & -10+3+0 \\ 0+6+36 & 0-3+0 \end{bmatrix} \\
 &= \begin{bmatrix} -14 & 7 \\ 42 & -3 \end{bmatrix}
 \end{aligned}$$

10 Misprint

$$2x - y = 9$$

$$x + 3y = 1$$

$$y = 2x - 9$$

$$x + 3(2x - 9) = 1$$

$$x + 6x - 27 = 1$$

$$7x = 28$$

$$\underline{x = 4}$$

$$\underline{z = -8}$$

$$y = 8 - 9$$

$$\underline{y = -1}$$

11

$$B I^T = B = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

12

$$\left[\begin{array}{cc|cc} -3 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array} \right] \xrightarrow{\substack{1/2 R_2 \\ R_1}} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1/2 \\ -3 & 2 & 1 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 \\ R_2 + 3R_1 \end{array} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1/2 \\ 0 & 8 & 1 & 3/2 \end{array} \right] \xrightarrow{\substack{R_1 \\ 1/8 R_2}} \begin{array}{l} R_1 \\ 1/8 R_2 \end{array} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1/2 \\ 0 & 1 & 1/8 & 3/16 \end{array} \right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_2 \end{array} \left[\begin{array}{cc|cc} 1 & 0 & -3/8 & 2/16 \\ 0 & 1 & 1/8 & 3/16 \end{array} \right] \quad M^{-1} = \begin{bmatrix} -1/4 & 1/8 \\ 1/8 & 3/16 \end{bmatrix}$$

13

$$\begin{bmatrix} -6 & 6 & 0 \\ 12 & -4 & 8 \\ 9 & 0 & 1 \end{bmatrix}$$

14

$$\begin{aligned} \det &= 4(-9) - (2)(-20) \\ &= -36 + 40 \\ &= 4 \end{aligned}$$

$$15 \quad \det = 0 = 3(4x+3) - (4x-1)$$

$$12x + 9 - 4x + 1 = 0$$

$$8x + 10 = 0$$

$$8x = -10$$

$$x = -\frac{10}{8} = -\frac{5}{4}$$

16

$$a) \quad A^{-1} = \frac{1}{-55+56} \begin{bmatrix} -11 & -8 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} -11 & -8 \\ 7 & 5 \end{bmatrix}$$

$$b) \quad A A^{-1} = I$$

$$17 \quad \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6-2 \\ -2+4 \end{bmatrix} = \begin{bmatrix} -8 \\ 2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$18 \quad \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{4-3} \begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 36-18 \\ -9+6 \end{bmatrix} = \begin{bmatrix} 18 \\ -3 \end{bmatrix}$$

19 $A = \begin{bmatrix} -3 & 4 \\ 6 & -8 \end{bmatrix}$, $\det(A) = 24 - 24 = 0$
singular

20 $\left[\begin{array}{cc|c} -3 & 4 & 7 \\ 6 & -8 & -14 \end{array} \right] \xrightarrow[-\frac{1}{3}R_1]{} \left[\begin{array}{cc|c} 1 & -\frac{4}{3} & -\frac{7}{3} \\ 0 & 0 & 0 \end{array} \right]$

Infinite solution

$x_2 = t$

$x_1 - \frac{4}{3}t = -\frac{7}{3}$

$x_1 = \frac{4}{3}t - \frac{7}{3}$