Selected Solutions 11.2

31 $f(x) = x^4 - 24x^2$ $\quad f'(x) = 4x^3 - 48x$ $\quad f''(x) = 12x^2 - 48$

$= x^2(x^2 - 24)$ $\quad = 4x(x^2 - 12)$ $\quad = 12(x^2 - 4)$

$= 4x(x - \sqrt{12})(x + \sqrt{12})$ $\quad = 12(x-2)(x+2)$

Concavity depends on f''

$f''(x) = 0$ at $x = 2, x = -2$

$+++ \ (-2) \ --- \ (2) \ +++$

Concave down $(-2, 2)$ Concave up $\{(-\infty, -2), (2, \infty)\}$

$f(-2) = 16 - 24(4)$ $\quad f(2) = 16 - 24(4)$

$= 16 - 96 = -80$ $\quad = -80$

Inflection points at $\{(-2, -80), (2, -80)\}$

63 $f(x) = (x^2 - 4)^2$

$f'(x) = 2(x^2 - 4)2x$ $\quad f''(x) = 4(x^2 - 4) + 4x(2x)$

$= 4x(x^2 - 4)$ $\quad = 4x^2 - 16 + 8x^2$

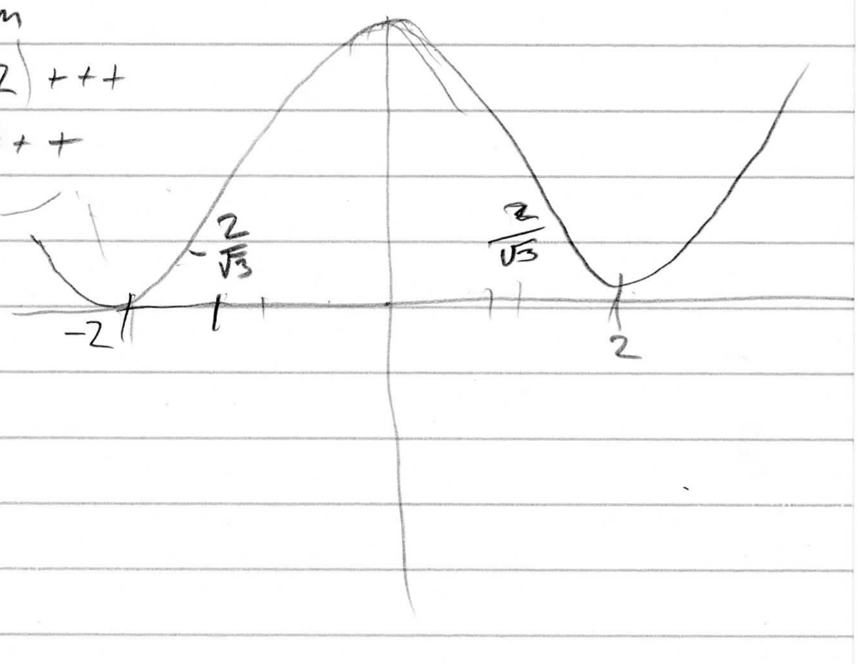
$= 4x(x-2)(x+2)$ $\quad = 12x^2 - 16 = 12(x^2 - \frac{4}{3})$

$f'(x) = 0$ at $x = \pm 2$ and $x = 0$ $\quad f''(x) = 0$ at $x = \pm\sqrt{4/3} = \pm \frac{2}{\sqrt{3}}$

m M m

f' $---\ (-2)\ +++\ (0)\ ---\ (2)\ +++$

f'' $+++\ (-\frac{2}{\sqrt{3}})\ ---\ (\frac{2}{\sqrt{3}})\ +++$



$f(-2) = 0$

$f(2) = 0$

$f(0) = 16$

$f(\pm \frac{2}{\sqrt{3}}) = \frac{64}{9} = 7.11$

65 $f(x) = 2x^6 - 3x^5$

$f'(x) = 12x^5 - 15x^4$

$= 3x^4(4x-5)$

$f''(x) = 60x^4 - 60x^3$

$= 60x^3(x-1)$

$f'(x) = 0$ at $x=0$, $x = \frac{5}{4}$

$f''(x) = 0$ at $x=0$, $x=1$

f' $- - - \ (0) \ - - - \ \overset{m}{\left(\frac{5}{4}\right)} \ + + +$

f'' $+ + + \ (0) \ - - - \ (1) \ + + +$

$f(0) = 0$

$f(1) = -1$

$f(5/4) = \left(\frac{5}{4}\right)^5 \left(2\left(\frac{5}{4}\right) - 3\right)$

$= 1.53$

Minimum at $x = 5/4$