

# Selected Solutions 11.2

$$\begin{array}{lll}
 31 \quad f(x) = x^4 - 24x^2 & f'(x) = 4x^3 - 48x & f''(x) = 12x^2 - 48 \\
 = x^2(x^2 - 24) & = 4x(x^2 - 12) & = 12(x^2 - 4) \\
 & = 4x(x - \sqrt{12})(x + \sqrt{12}) & = 12(x - 2)(x + 2)
 \end{array}$$

Concavity depends on  $f''$

$$f''(x) = 0 \text{ at } x = 2, x = -2$$

$$+++ (-2) --- (2) +++$$

Concave down  $(-2, 2)$

Concave up  $\{(-\infty, -2), (2, \infty)\}$

$$\begin{aligned}
 f(-2) &= 16 - 24(4) \\
 &= 16 - 96 = -80
 \end{aligned}$$

$$\begin{aligned}
 f(2) &= 16 - 24(4) \\
 &= -80
 \end{aligned}$$

Inflection points at  $\{(-2, -80), (2, -80)\}$

$$63 \quad f(x) = (x^2 - 4)^2$$

$$\begin{aligned}
 f'(x) &= 2(x^2 - 4) \cdot 2x \\
 &= 4x(x^2 - 4) \\
 &= 4x(x - 2)(x + 2)
 \end{aligned}$$

$$\begin{aligned}
 f''(x) &= 4(x^2 - 4) + 4x(2x) \\
 &= 4x^2 - 16 + 8x^2 \\
 &= 12x^2 - 16 = 12\left(x^2 - \frac{4}{3}\right)
 \end{aligned}$$

$$f'(x) = 0 \text{ at } x = \pm 2 \text{ and } x = 0$$

$$f''(x) = 0 \text{ at } x = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$$

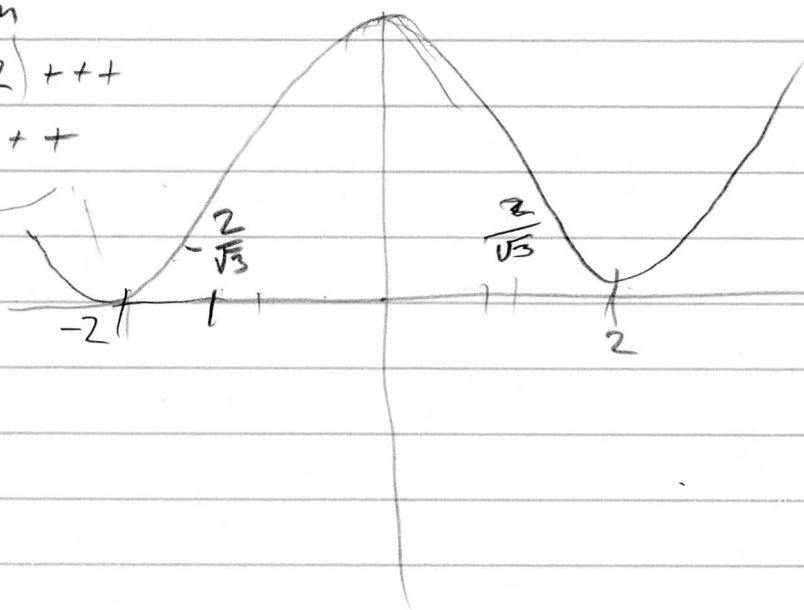
$$\begin{array}{lll}
 f' & \begin{array}{c} m \\ --- (-2) ++ (0) --- (2) +++ \end{array} & \\
 f'' & \begin{array}{c} m \\ +++ (-\frac{2}{\sqrt{3}}) --- (\frac{2}{\sqrt{3}}) +++ \end{array} & 
 \end{array}$$

$$f(-2) = 0$$

$$f(2) = 0$$

$$f(0) = 16$$

$$f\left(\pm \frac{2}{\sqrt{3}}\right) = \frac{64}{9} = 7.11$$



$$65 \quad f(x) = 2x^6 - 3x^5$$

$$f'(x) = 12x^5 - 15x^4$$

$$= 3x^4(4x - 5)$$

$$f''(x) = 60x^4 - 60x^3$$

$$= 60x^3(x - 1)$$

$$f'(x) = 0 \text{ at } x = 0, x = \frac{5}{4}$$

$$f''(x) = 0 \text{ at } x = 0, x = 1$$

$$f' \quad \text{---} (0) \text{---} \overset{m}{\left(\frac{5}{4}\right)} \text{+++}$$

$$f'' \quad \text{+++} (0) \text{---} (1) \text{+++}$$

$$f(0) = 0$$

Minimum at  $x = \frac{5}{4}$

$$f(1) = -1$$

$$f\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)^5 \left(2\left(\frac{5}{4}\right) - 3\right)$$

$$= 1.53$$

