Math 208-711 Exam 03 Solutions

1] a) $\lim\limits_{x \to \infty} f(x) = 1$

c) $\lim\limits_{x \to -2^+} f(x) = -\infty$

b) $\lim\limits_{x \to -2^-} f(x) = +\infty$

d) $\lim\limits_{x \to -2} f(x) = DNE$

2] $\lim\limits_{x \to -2} \dfrac{x^2 + 2x - 5}{7 - 7x} = \dfrac{(-2)^2 + 2(-2) - 5}{7 - 7(-2)} = \dfrac{4 - 4 - 5}{7 + 14} = -\dfrac{5}{21}$

3] $\lim\limits_{x \to 7} \dfrac{x^2 - 4x - 21}{x^2 - 49} = \dfrac{49 - 28 - 21}{49 - 49} = \dfrac{0}{0}$ indeterminate (more work)

$\lim\limits_{x \to 7} \dfrac{(x+3)(x-7)}{(x+7)(x-7)} = \lim\limits_{x \to 7} \dfrac{x+3}{x+7} = \dfrac{7+3}{7+7} = \dfrac{10}{14}$

4] $\lim\limits_{x \to -\infty} \dfrac{8 + 2x - 6x^3}{4x^3 + 4} = \lim\limits_{x \to -\infty} \dfrac{-6x^3}{4x^3} = -\dfrac{6}{4} = -\dfrac{3}{2}$

5] $f(x) = x^2 + 3x$

Step 1 $f(x+h) = (x+h)^2 + 3(x+h)$

$\qquad = x^2 + 2xh + h^2 + 3x + 3h$

Step 2 $f(x+h) - f(x) = x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x)$

$\qquad = 2xh + h^2 + 3h$

$\qquad = h(2x + h + 3)$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h}{h}(2x + h + 3) = 2x + h + 3$

Step 4 $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (2x + h + 3) = 2x + 3$

6] $y = 4x^5 - 2x + 3$

$y' = 20x^4 - 2$

7] $f(x) = \dfrac{5}{4x^{11}} = \dfrac{5}{4} x^{-11}$

$f'(x) = -11\left(\dfrac{5}{4}\right) x^{-12} = -\dfrac{55}{4} x^{-12}$

8] $p(t) = t^5 + \dfrac{5}{t^4} - \dfrac{8}{\sqrt{t}} + 200 = t^5 + 5t^{-4} - 8t^{-\frac{1}{2}} + 200$

$p'(t) = 5t^4 - 20t^{-5} + 4t^{-\frac{3}{2}}$

9] $y = (5x-1)^2 = 25x^2 - 5x - 5x + 1$

$\qquad = 25x^2 - 10x + 1$

$\qquad y' = 50x - 10$

10] tangent line is horizontal when $m = 0$, $f'(x)$ is slope
of tangent line

$y = 3x^2 - 7x + 5$

$y' = 6x - 7 \qquad = 0$

$\qquad\qquad 6x = 7$

$\qquad\qquad\quad x = 7$

11] $C(x) = 10\,000 + 200x - .1x^2$

$C'(x) = 200 - .2x$

$C'(100) = 200 - .2(100) = \$180 \qquad\qquad 200 - 20$

12] $R(x) = 20 + 15x - .0005x^2$

$R'(x) = 15 - .001x$

$R'(1000) = 15 - .001(1000) = 15 - 1 = \$14,000$