

Solutions to  
Practice Exam

Math 208

Spring 2021

1. Sonya started an account 12 years ago with a balance of \$2200. The account pays at 4.33% compounded semi-annually. How much does she have now?

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$\begin{aligned} t &= 12 \\ P &= 2200 \\ r &= .0433 \\ m &= 2 \end{aligned}$$

How much Interest is Earned?

$$I = A - P = \$1478.53$$

$$A = 2200 \left(1 + \frac{.0433}{2}\right)^{2(12)} = \$3678.53$$

2. How many years will it take for an investment to triple if interest is 8.3% compounded continuously? Round years to the nearest 0.01.

$$A = Pe^{rt}$$

$$\frac{A}{P} = 3$$

$$t = \frac{1}{.083} (\ln 3)$$

$$\frac{A}{P} = e^{rt}$$

$$\begin{aligned} r &= .083 \\ t &=? \end{aligned}$$

$$= 13.24 \text{ yrs}$$

$$\ln\left(\frac{A}{P}\right) = rt$$

$$t = \frac{1}{r} \ln\left(\frac{A}{P}\right)$$

3. (a) \_\_\_\_\_

One of the matrices  $A$  or  $B$  is singular. Why? Given

$$A = \begin{bmatrix} 2 & -4 \\ -5 & 10 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -3 \\ -6 & 10 \end{bmatrix}$$

$A$  is singular because  $\det A = 0$

$$\det A = 20 - 20 = 0$$

$$\det B = 20 - 18 = 2$$

- (b) \_\_\_\_\_

Find the inverse of the other matrix using Cramer's Rule.

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} 10 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \frac{3}{2} \\ 3 & 1 \end{bmatrix}$$

4. \_\_\_\_\_

Solve the matrix equation  $Ax = b$  for the values of  $x$ .

$$\begin{bmatrix} 2 & -3 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 5 & \frac{3}{2} \\ 3 & 1 \end{bmatrix}$$

$$A x = b$$

$$A^{-1}A x = A^{-1}b$$

$$x = A^{-1}b$$

$$= \begin{bmatrix} 5 & \frac{3}{2} \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 - \frac{3}{2} \\ 3 - 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ 2 \end{bmatrix}$$

5. (a) Write the system of equations as an augmented matrix. Do not solve the augmented matrix.

$$2x_1 + 3x_2 + 5x_3 = 2$$

$$-x_1 - 9x_2 = -4$$

$$3x_1 + x_3 = 12$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 5 & 2 \\ -1 & -9 & 0 & -4 \\ 3 & 0 & 1 & 12 \end{array} \right]$$

(b) \_\_\_\_\_

Solve the linear system corresponding to the following reduced form augmented matrix.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 &= 20 \\ x_2 &= 4 - 4t \\ x_3 &= t \end{aligned}$$

$$\begin{aligned} x_2 + 4t &= 4 \\ x_2 &= 4 - 4t \end{aligned}$$

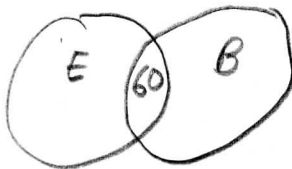
6. At State University there are 360 students taking English or Business. 240 are taking English, 180 are taking Business, and 60 are taking both English and Business. How many are taking English but not Business? (A Venn diagram may help)

(a) 200

(b) 120

(c) 180

(d) 300



$$240 - 60 = 180$$

7. (a)  $2 \cdot 2 \cdot 2 = 8$

What is the size of the sample space for flipping a fair coin 3 times? **Write** the sample space

$$S = \{HHH, HHT, HTH, HTT, \underline{TTH}, \underline{THT}, \underline{TTT}\}$$

- (b)  $\frac{3}{8}$

A fair coin is flipped 3 times. What is the probability that only one of the flips is heads?

$$nE = 3$$

$$nS = 8$$

$$P(E) = \frac{3}{8}$$

8. \_\_\_\_\_

5 thank you notes are written and 5 envelopes are addressed. Accidentally the notes are randomly inserted into the envelopes and mailed without checking the addresses. What is the probability that all the notes will be inserted into the correct envelopes?

$$nE = 1$$

$$nS = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$$

$$P(E) = \frac{1}{120}$$

9.  $\frac{-\frac{1}{2}}{\quad}$

Find

$$\lim_{x \rightarrow \infty} \frac{-x^4 + 5x^3 + 1}{2x^4 - 9} = \lim_{x \rightarrow \infty} \frac{-x^4}{2x^4} = -\frac{1}{2}$$

10.  $\frac{2}{7}$

Find

$$\lim_{x \rightarrow 6} \frac{x^2 - 2x - 12}{x^2 + 6}$$

$$\frac{6^2 - 2(6) - 12}{6^2 + 6} = \frac{36 - 12 - 12}{36 + 6} = \frac{12}{42} = \frac{2}{7}$$

11.  $\underline{\hspace{2cm}}$

Given that the total cost of producing  $x$  smartphones is

$$C(x) = 10,000 + 180x - 0.4x^2$$

What is the marginal cost of producing the 150<sup>th</sup> phone?

$$C'(x) = 180 - .8x$$

$$C'(149) = 180 - .8(149) = 60.8$$

12. The price-demand function is given by  $x = 9000 - 10p$ .

(a)  $p = 900 - .1x$

Express the price  $p$  as a function of the demand  $x$ .

$$\begin{aligned} 10p + x &= 9000 \\ 10p &= 9000 - x \\ p &= 900 - .1x \end{aligned}$$

(b) \_\_\_\_\_

Find the revenue function.

$$R(x) = xp = 900x - .1x^2$$

(c) \_\_\_\_\_

Find the marginal revenue function.

$$R'(x) = 900 - .2x$$

13. Given the Revenue function and the Cost function shown respectively as

$$R(x) = 500x - 0.01x^2$$

$$C(x) = 10x + 1500$$

(a) \_\_\_\_\_ Find the Profit Function,  $P(x)$ .

$$\begin{aligned} P(x) &= R(x) - C(x) = 500x - .01x^2 - (10x + 1500) \\ &= 490x - .01x^2 - 1500 \end{aligned}$$

(b) \_\_\_\_\_ Find the Marginal Profit Function.

$$P'(x) = 490 - .02x$$

In the problems 14-20, find each derivative and simplify (fractional and negative exponents are acceptable).

Power Rule

14.  $y' = 9x^2 - x^{-2/3}$

$$y = 3x^3 - \sqrt[3]{27x} = 3x^3 - 3x^{1/3}$$

Quotient Rule

15.  $\frac{60x}{(x^2+2)^2}$

$$f(x) = \frac{15x^2}{x^2+2}$$

$$f'(x) = \frac{(x^2+2)30x - 2x(15x^2)}{(x^2+2)^2}$$

$$= \frac{30x^3 + 60x - 30x^3}{(x^2+2)^2}$$

$$= \frac{60x}{(x^2+2)^2}$$

$$T = 15x^2$$

$$T' = 30x$$

$$B = x^2 + 2$$

$$B' = 2x$$

Product Rule

16. \_\_\_\_\_

$$h(t) = (2t^3 + 4)(2t^2 - 1)$$

$$h'(t) = (2t^3 + 4)4t + 6t^2(2t^2 - 1)$$

$$= 8t^4 + 16t + 12t^4 - 6t^2$$

$$= 20t^4 - 6t^2 + 16t$$

$$F = 2t^3 + 4$$

$$F' = 6t^2$$

$$S = 2t^2 - 1$$

$$S' = 4t$$

$$17. \frac{18x^2}{6x^3-5}$$

$$g(x) = \ln(6x^3 - 5)$$

$$= \frac{1}{6x^3-5} (18x^2)$$

$$18. \underline{\hspace{2cm}}$$

$$g(x) = 3x^2 e^x \ln x^3$$

$$g'(x) = 3x^2 \left( \frac{3e^x}{x} + e^x \ln x^3 \right) + 6xe^x \ln x^3$$

$$\text{Note: } \ln x^3 = 3 \ln x$$

$$\begin{aligned} g'(x) &= 9xe^x + 9x^2 e^x \ln x + 18xe^x \ln x \\ &= 9xe^x (1 + x \ln x + 2x \ln x) \\ &= 9xe^x (1 + 3x \ln x) \end{aligned}$$

$$19. \underline{\hspace{2cm}}$$

$$f(x) = 2xe^{2x^3}$$

$$\begin{aligned} f'(x) &= 2x(6x^2)e^{2x^3} + 2e^{2x^3} \\ &= 2e^{2x^3}(6x^3 + 1) \end{aligned}$$

General Ln Rule

$$\frac{d}{dx} \ln u = \frac{1}{u} u'$$

Product Rule

$$F = 3x^2$$

$$F' = 6x$$

$$S = e^x \ln x^3$$

$$\text{Product Rule}$$

$$F' = e^x$$

$$S = \ln x^3$$

$$S' = \frac{3x^2}{x^3}$$

General Ln Rule

$$\begin{aligned} S' &= e^x \frac{3x^2}{x^3} + e^x \ln x^3 \\ &= \frac{3e^x}{x} + e^x \ln x^3 \end{aligned}$$

Product Rule

$$F = 2x$$

$$F' = 2$$

$$S = e^{2x^3}$$

$$S' = 6x^2 e^{2x^3}$$

General  $e^u$  Rule

$$\frac{d}{dx} e^u = e^u u'$$



20. \_\_\_\_\_

General Power Rule

$$y = (2t^3 + 4)^{-12}$$

$$y' = -12(2t^3 + 4)^{-13} (6t^2)$$

$$= -72t^2(2t^3 + 4)^{-13}$$

$$= \frac{-72t^2}{(2t^3 + 4)^{13}}$$

21. What does the derivative represent? (circle all that apply)

(A) Slope of the tangent line

(B) Average rate of change

(C) Slope of the secant line

(D) Velocity

(E) Instantaneous rate of change

22. \_\_\_\_\_

Find the equation of the line tangent to the curve  $y = e^{2x+1}$  at  $x = 1$ . Write the result in  $y = mx + b$  form

$$x_0 = 1$$

$$f(x) = y = e^{2x+1}$$

$$f(1) = e^3 = 20.086 = y_0$$

$$f'(x) = 2e^{2x+1} = m$$

$$f'(1) = 2e^3 = 40.171$$

$$y - y_0 = m(x - x_0)$$

$$y - 20.086 = 40.171(x - 1)$$

$$= 40.171x - 40.171$$

$$y = 40.171x - 20.086$$

23. Given the price-demand equation of  $x = f(p) = 40,000 - 400p = 400(100 - p)$   
 $f'(p) = -400$

(a) \_\_\_\_\_ Find  $E(p)$  with  $p = \$40$ .

$$E(p) = - \frac{p f'(p)}{f(p)} = - \frac{p(-400)}{400(100-p)} = \frac{p}{100-p}$$

$$E(40) = \frac{40}{100-40} = \frac{40}{60} = \frac{2}{3}$$

(b)  $\downarrow 6.67\%$  If the \$40 price is raised by 10%, what is the approximate change in demand?

$p \uparrow 10\%$  then demand  $\downarrow \frac{2}{3}(10\%)$

decrease by 6.67%

24. \_\_\_\_\_ What is the elasticity of demand function, given the price-demand equation of

$$x = f(p)$$

$$p^2 + 20p + 10x = 9500$$

$$10x = 9500 - 20p - p^2$$

$$x = f(p) = 950 - 2p - .1p^2$$

$$E(p) = - \frac{p f'(p)}{f(p)}$$

$$f'(p) = -2 - .2p = -2(1 + .1p)$$

$$= \frac{-p(-2)(1 + .1p)}{950 - 2p - .1p^2} = \frac{2(p + .1p^2)}{950 - 2p - .1p^2}$$

25. Given

$$E(p) = \frac{7p}{240 - p}$$

(a) 30

$$E(p) = 1$$

Determine the price which results in unit elasticity of demand.

$$E(p) = 1 = \frac{7p}{240 - p}$$

$$240 - p = 7p$$

$$240 = 8p$$

$$p = 30$$

(b) \_\_\_\_\_

What range of prices results in inelastic demand?

$$E(p) < 1$$

$$0 < p < 30$$

Last question is worth double points.

26. Use the Graphing Strategy and Curve Sketching methods to sketch the curve for  $f(x) = x^3 + 18x^2 + 81x$ . (10 points for a properly completed sign chart)

26

$$f(x) = x^3 + 18x^2 + 81x = x(x^2 + 18x + 81) = x(x+9)^2$$

$$f(x) = 0 \text{ at } x = 0 \text{ and } x = -9$$

$$f'(x) = 3x^2 + 36x + 81 = 3(x^2 + 12x + 27) = 3(x+3)(x+9)$$

$$f'(x) = 0 \text{ at } x = -3 \text{ and } x = -9$$

$$f''(x) = 6x + 36 = 6(x+6)$$

$$f''(x) = 0 \text{ at } x = -6$$

Sign Chart

x	-9	-6	-3	0
f(x)	0	-54	-108	0
f'(x)	+++ 0	- - -	0 +++	
f''(x)	- - -	0	+++	

Max

Min

Worth 10 points on sec 202

Final

Check f'

	(x+3)	(x+9)	
x < -9	-	-	= +
-9 < x < -3	-	+	= -
x > -3	+	+	= +

Check f''

	(x+6)	
x < -6	-	= -
x > -6	+	= +

$$f(-6) = -216 + 648 - 486 = -54$$

$$f(3) = -27 + 162 - 243 = -108$$

