RELATIONAL ALGEBRA

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WHAT IS THIS LECTURE ABOUT?

- Relational Algebra
  - query language for relations
- Basic Operations
  - selection, projection
  - difference, union
  - cross-product, renaming
- Derived Operations
  - join, natural join, equi-join, division

RELATIONAL QUERY LANGUAGES

- allow the *manipulation* and *retrieval* of data from a database
- two types of query languages:
  - Declarative: describe what a user wants, rather than how to compute it
    - Tuple Relational Calculus
    - Domain Relational Calculus
  - Procedural: operational, useful for representing execution plans
    - Relational Algebra

WHAT IS RELATIONAL ALGEBRA?

- **algebra**: mathematical system consisting of
  - operands: variables or values from which new values can be constructed
  - operators: symbols denoting procedures that construct new values from given values
- **relational algebra**: an algebra whose operands are relations or variables that represent relations
  - operators do the most common things that we need to do with relations in a database
  - can be used as a query language for relations

RELATIONAL ALGEBRA: PRELIM

- **Query**:
  - Input: relational instances
  - Output: relational instances
  - specified using the schemas
    - may produce different results for different instances
    - the schema of the result is fixed
- there are two types of notation for attributes:
  - positional (e.g. 2, 4)
  - named-field (e.g. C.name, Person.SSN)

RELATIONAL ALGEBRA: PRELIM

- Basic operations:
  - *Selection* $\{\sigma\}$: selects a subset of rows
  - *Projection* $\{\pi\}$: deletes columns
  - *Cross-product* $\{\times\}$: combines two relations
  - *Set-difference* $\{-\}$
  - *Union* $\{\cup\}$
- When the relations have named fields:
  - *Renaming* $\{\rho\}$
- Additional operations:
  - *Intersection, join, division*

KEEP IN MIND!

- SQL uses multisets, however in Relational Algebra we will consider relations as **sets**

- We will consider the **named perspective**, where every attribute must have a unique name

The attribute order in a relation does not matter!

BASIC OPERATIONS

SELECTION

Notation: $\sigma_C(R)$

- C is a condition that refers to the attributes of R
- outputs the rows of R that satisfy C
- output schema: same as input schema

Example

- $\sigma_{age>24}(Person)$
- $\sigma_{age>24 \text{ and } age\leq 28}(Person)$
- $\sigma_{age>24 \text{ and } name=\text{“Paris”}}(Person)$

```
SELECT *
FROM Person
WHERE age > 24 ;
```

SELECTION: EXAMPLE

Person

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	25	206-473-8221

$$\downarrow \sigma_{age>24}(Person)$$

SSN	name	age	phoneNumber
123123645	John	30	608-321-1163
384475687	Arun	25	206-473-8221

PROJECTION

Notation: $\pi_{A_1,A_2,\ldots,A_n}(R)$

- outputs only the columns $A_1, A_2, \ldots, A_n$
- removes any duplicate tuples
- output schema: $R(A_1, A_2, \ldots, A_n)$

Example

- $\pi_{SSN,age}(Person)$
- $\pi_{SSN,phoneNumber,age}(Person)$

```
SELECT DISTINCT SSN,age
FROM Person ;
```

PROJECTION: EXAMPLE

Person

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	20	206-473-8221

$$\pi_{SSN,name}(Person)$$

SSN	name
934729837	Paris
123123645	John
384475687	Arun

RA OPERATORS ARE COMPOSITIONAL

```sql
SELECT DISTINCT SSN,age
FROM Person
WHERE age > 24 ;
```

Two logically equivalent expressions in RA:

- $\pi_{SSN,age}\left(\sigma_{age>24}(Person)\right)$
- $\sigma_{age>24}(\pi_{SSN,age}(Person))$

UNION

Notation: $R_1 \cup R_2$

- outputs all tuples in R_1 or R_2
- both relations must have the same schema!
- output schema: same as input

A	B
a_1	b_1
a_2	b_1
a_2	b_2

$\cup$

A	B
a_1	b_1
a_3	b_1
a_4	b_4

$=$

A	B
a_1	b_1
a_2	b_1
a_2	b_2
a_3	b_1
a_4	b_4

DIFFERENCE

Notation: $R_1 - R_2$

- outputs all tuples in R_1 and not in R_2
- both relations must have the same schema!
- output schema: same as input

A	B
a_1	b_1
a_2	b_1
a_2	b_2

$-$

A	B
a_1	b_1
a_3	b_1
a_4	b_4

$=$

A	B
a_2	b_1
a_2	b_2

CROSS-PRODUCT

Notation: $R_1 \times R_2$

- matches each tuples in R_1 with each tuple in R_2
- input schema: $R_1(A_1, A_2, \dots, A_n)$, $R_2(B_1, B_2, \dots, B_m)$
- output schema: $R(A_1, \dots, A_n, B_1, \dots, B_m)$

Example

- $Person \times Department$ —

```
SELECT *
FROM Person,Department;
```

CROSS-PRODUCT: EXAMPLE

Person

SSN	name
934729837	Paris
123123645	John

Dependent

depSSN	depname
934729837	Helen
934729837	Bob

$$Person \times Dependent$$

SSN	name	depSSN	depname
934729837	Paris	934729837	Helen
123123645	John	934729837	Bob
934729837	Paris	934729837	Bob
123123645	John	934729837	Helen

RENAMING

Notation: $\rho_{A_1, A_2, \dots, A_n}(R)$

- does not change the instance, only the schema!
- input schema: $R(B_1, B_2, \dots, B_n)$
- output schema: $R(A_1, \dots, A_n)$

Why is it necessary?

named perspective: when joining relations, we need to distinguish between attributes with the same name!

RENAMING: EXAMPLE

Person

SSN	name
934729837	Paris
123123645	John

Dependent

SSN	name
934729837	Helen
934729837	Bob

$$Person \times \rho_{depSSN,depname}\ (Dependent)$$

SSN	name	depSSN	depname
934729837	Paris	934729837	Helen
123123645	John	934729837	Bob
934729837	Paris	934729837	Bob
123123645	John	934729837	Helen

DERIVED OPERATIONS

INTERSECTION

Notation: $R_1 \cap R_2$

- outputs all tuples in R_1 and R_2
- output schema: same as input
- can be expressed as: $R_1 - (R_1 - R_2)$

```
SELECT R.A, R.B
FROM R,S
WHERE R.A = S.A
AND R.B = S.B;
```

R

A	B
a_1	b_1
a_2	b_1
a_2	b_2

$\cap$

S

A	B
a_1	b_1
a_3	b_1
a_4	b_4

$=$

A	B
a_1	b_1

JOIN (THETA JOIN)

Notation: $R_1 \bowtie_\theta R_2 = \sigma_\theta(R_1 \times R_2)$

- cross-product followed by a selection
- θ can be any boolean-valued condition
- might have less tuples than the cross-product!

```
SELECT *
FROM R₁, R₂
WHERE ϑ;
```

THETA JOIN: EXAMPLE

Person

SSN	name	age
934729837	Paris	26
123123645	John	22

Dependent

dSSN	dname	dage
934729837	Helen	23
934729837	Bob	28

$$Person \bowtie_{Person.age > Dependent.dage} Dependent$$

SSN	name	age	dSSN	dname	dage
934729837	Paris	26	934729837	Helen	23

EQUI-JOIN

Notation: $R_1 \bowtie_\theta R_2$

- special case of join where the condition θ contains only equalities between attributes
- output schema: same as the cross-product

Example for $R(A, B), S(C, D)$

- $R \bowtie_{B=C} S$
- output schema: $T(A, B, C, D)$

```
SELECT *
FROM R, S
WHERE R.B = S.C;
```

NATURAL JOIN

Notation: $R_1 \bowtie R_2$

- equi-join on all the **common** fields
- the output schema has one copy of each common attribute

Person

SSN	name	age
934729837	Paris	26
123123645	John	22

Dependent

SSN	dname
934729837	Helen
934729837	Bob

```
SELECT SSN,name,age,dname
FROM Person P,
Department D
WHERE P.SSN = D.SSN ;
```

$Person \bowtie Dependent$

SSN	name	age	dname
934729837	Paris	26	Helen
934729837	Paris	26	Bob

NATURAL JOIN

Natural Join $R \bowtie S$

- Input schema: $R(A, B, C, D), S(A, C, E)$
  - Output schema: *T(A, B, C, D, E)*
- Input schema: $R(A, B, C), S(D, E)$
  - Output schema: *T(A, B, C, D, E)*
- Input schema: $R(A, B, C), S(A, B, C)$
  - Output schema? *T(A, B, C,)*

SEMI-JOIN

Notation: $R_1 \ltimes R_2$

- natural join followed by projection on the attributes of R_1

Example:

- $R(A, B, C), S(B, D)$
- $R \ltimes S = \pi_{A,B,C}(R \bowtie S)$
- output schema: *T(A, B, C)*

```
SELECT A,B,C
FROM R, S
WHERE R.B = S.B ;
```

DIVISION

Notation: R_1/R_2

- suppose $R_1(A, B)$ and $R_2(B)$
- the output contains all values **a** such that for every tuple (**b**) in R_2, tuple **(a, b)** is in R_1
- output schema: *R(A)*

DIVISION: EXAMPLE

A

A	B
a_1	b_1
a_1	b_2
a_1	b_3
a_2	b_1

$\mathbf{B_1}$

B
b_2
b_3
b_1

$\mathbf{B_2}$

B
b_1

A / B_1

A
a_1

A / B_2

A
a_1
a_2

Extending Relational Algebra

GROUP BY AGGREGATE

- is part of the so-called extended RA
- helps us to compute counts, sums, min, max, …

Examples

- What is the average age of the customers?
- How many people bought an iPad?

GROUP BY AGGREGATE

Notation: $\gamma_{X,Agg(Y)}(R)$

- group by the attributes in X
- aggregate the attribute in Y
  - SUM, COUNT, AVG (average), MIN, MAX

- Output schema: X + an extra (numerical) attribute

EXAMPLE

Person

SSN	name	age
934729837	Paris	24
123123645	John	30
384475687	Arun	21

$$\gamma_{AVG(age)}(Person)$$

AVG(age)
25

```
SELECT AVG(age)
FROM Person ;
```

EXAMPLE

Person

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	21	206-473-8221

```
SELECT SSN,
  COUNT(phoneNumber)
FROM Person
GROUP BY SSN;
```

$$\gamma_{SSN,COUNT(phoneNumber)}(Person)$$

SSN	COUNT(phoneNumber)
934729837	2
123123645	1
384475687	1

CONSTRUCTING RA QUERIES

COMBINING RA OPERATORS

- We can build more complex queries by combining RA operators together

  e.g. standard algebra: $(x + 1) * y - z^2$

- There are 3 different notations:
  - sequence of assignment statements
  - expressions with operators
  - expression trees

COMBINING RA OPERATORS

Input schema: $R(B, C), S(A, B)$

- expressions with operators

  $\pi_A(\sigma_{C=1}(R) \bowtie S)$

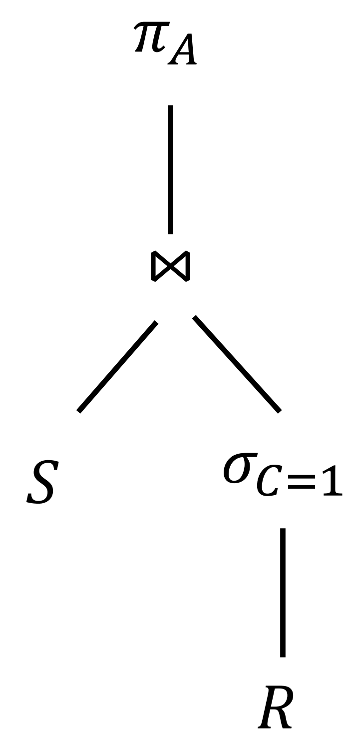
- sequence of assignment statements

  $R' = \sigma_{C=1}(R)$

  $R'' = R' \bowtie S$

  $R''' = \pi_A(R'')$

- expression trees



EXPRESSIVE POWER OF RA

- RA cannot express transitive closure!

Edges

From	To
a	b
b	c
a	d
c	d

Transitive closure computes all pairs of nodes connected by a directed path