國立清華大學
NATIONAL TSING HUA UNIVERSITY

11110PHYS401300
Computational Physics Lab
計算物理實作

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Due: 2023.1.09
(submit to google classroom)

Homework 4

Reading Assignments

1. Read the wikipedia article on linear equations `https://en.wikipedia.org/wiki/System_of_linear_equations`.
2. In Lab3, we use `scipy.linalg.solve()` as a black box and use it directly. We have also shown an example that we could gain significant speedup if we use the correct solver (`spsolve()`) for a spare matrix. Therefore, it would be better if you know the underline algorithm of the solver. LU decomposition is the first and the base concept of the solver `https://en.wikipedia.org/wiki/LU_decomposition`.

Written Assignments

1. Consider a 2D Poisson equation on a unit squre and is discretized using 3×3 grids,

$$u_{xx} + u_{yy} = \rho(x, y). \tag{1}$$

Use finite difference method to establish 9 linear equations on the below grid in Figure 1. The variables u and ρ are distreted to $u_{i,j}$ and $\rho_{i,j}$, and h is the cell space. Assuming a zero boundary condition, the system can be described by the below linear equation,

$$\boldsymbol{A} \cdot \begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \\ u_{12} \\ u_{22} \\ u_{32} \\ u_{13} \\ u_{23} \\ u_{33} \end{bmatrix} = \boldsymbol{b} \tag{2}$$

Derive and show the matrix **A** and vector **b**.

Programming Assignments

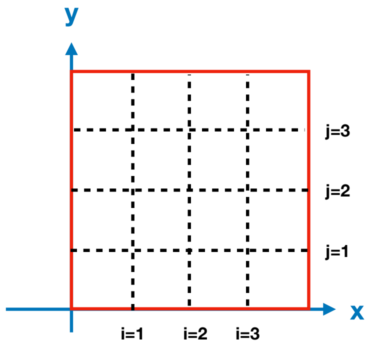

Figure 1: the finite difference grid on au unit square.

1. Following the first problem in the written assignment, set $\rho_{22} = 1$ and zeros in elsewhere, solve the system u.

2. Consider a 2D Poisson's equation in a domain $[-5 < x < 5] \times [-5 < y < 5]$ and is discretized using 128×128 grids. Given a source function,

$$\rho(x, y) = e^{-\frac{5}{4} r_1^2} + \frac{3}{2} \times e^{-r_2^2}, \tag{3}$$

   where $r_1 = (x + 1.5)^2 + y^2$ and $r_2 = (x - 1.5)^2 + y^2$. Assume a periodic boundary condition, solve the corresponding potential of this source function. Draw color and contour plots of the source function and the solution potential.

3. Following the above question, plots errors as functions of iterations for (1) Jacobi method, (2) Gauss-Seidel method, and (3) Successive Over-Relaxation method with $w = 1.2, 1.5$, and 2.0 (plot them on the same figure).

4. Now, consider an error tolerance $\epsilon = 10^{-7}$, measure the total computing time with 32×32, 64×64, and 128×128 grids, using (1) sparse matrix solver, (2) Jacobi method, (3) Gauss-Seidel method, (4) SOR with $w = 1.2$ and 1.5. Plot the computing times as functions of grid resolutions with different numerical methods in log-log scale. Describe and discuss how error converge.