**Computational Physics Lab**

# Homework 4

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## 1 Writing Assignments

### 1.1 Finite difference method for solving 2D Poisson equation

The $3 \times 3$ grids with grid space $h$ and the source term $\rho(x, y)$ can be described as:

$$u_{xx} + u_{yy} = \rho(x, y)$$

where the subscripts mean second partial derivatives of the u with $x$ and $y$, respectively. Besides, we annotate $(x, y)$ with $(i, j)$ where $i$ and $j$ are from $0 \sim (3 + 1)$ (including boundaries, which are $(i, j) = (i, 0),\ (i, 4),\ (0, j)$, and $(4, j)$), and implement the Euler method for derivative; then we can get:

$$\begin{aligned}
&\frac{\partial}{\partial x}\Big(\frac{u_{i,j} - u_{i-1,j}}{h}\Big) + \frac{\partial}{\partial y}\Big(\frac{u_{i,j} - u_{i,j-1}}{h}\Big) = \rho(x, y) \\
\Rightarrow &\frac{1}{h}\Big(\frac{u_{i+1,j} - u_{i,j}}{h} - \frac{u_{i,j} - u_{i-1,j}}{h}\Big) + \frac{1}{h}\Big(\frac{u_{i,j+1} - u_{i,j}}{h} - \frac{u_{i,j} - u_{i,j-1}}{h}\Big) = \rho(x, y) \\
\Rightarrow &\big(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\big) + \big(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}\big) = \rho(x, y)h^2 \\
\Rightarrow &\ 4_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1} = \rho(x, y)h^2 \qquad \text{(I)}
\end{aligned}$$

In the wake of knowing the general form of the solution, we apply the boundary conditions; here boundaries are all zeros (i.e., $u_{0,j},\ u_{i,0},\ u_{4,j},$ and $u_{i,4} = 0$). Exploiting all $i$ and $j$, we can get LHS of eq.(I) in the matrix form $\mathbf{A}$ and RHS in the vector form $\mathbf{b}$ with source term $g_{ij}$ which represents the splitting of the source function $\rho(x, y)$:

$$\mathbf{A} = \begin{bmatrix}
\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}
\end{bmatrix}$$

and

$$\mathbf{b} = -h^2 \begin{bmatrix} g_{11} \\ g_{12} \\ g_{13} \\ \\ g_{21} \\ g_{22} \\ g_{23} \\ \\ g_{31} \\ g_{32} \\ g_{33} \end{bmatrix}$$

# 2 Programming Assignments

## 2.1 (2) Leapfrog method

## 2.2 (3) Energy comparison

# 3 Codes

All the codes are transferred from jupyterlab or python codes; hence, if you want to re-run them, please see the source code in the attached files or my GitHub repository:

<https://github.com/gary20000915/Comphyslab-HW4.git>

## 3.1 `particles.py`