7.7.

Casimir's trick

A typical experiments starts out with a beam of particles of random spins. In this case, the relevant cross section is the average over all initial spin configurations, and sum over all final spin configurations.

$$\frac{1}{2}\frac{1}{2}\cdots\sum_{s_i, s_f} |M|^2$$

$e-\mu$ scattering

$$M = -\frac{g_e^2}{(P_1-P_3)^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)]$$

$$|M|^2 = \frac{g_e^4}{(P_1-P_3)^4}\underbrace{[\bar{u}(3)\gamma^\mu u(1)]}_{1\times1}\underbrace{[\bar{u}(4)\gamma_\mu u(2)]}_{1\times1}\underbrace{[\bar{u}(3)\gamma^\nu u(1)]^*}_{1\times1}\underbrace{[\bar{u}(4)\gamma_\nu u(2)]^*}_{1\times1}$$

$$\| \quad [\bar{u}(3)\gamma^\nu u(1)]^\dagger\,[\bar{u}(4)\gamma_\nu u(2)]^\dagger$$

$$\| \quad [u^\dagger(1)\gamma^{\nu\dagger}\gamma^{0\dagger}u(3)]\,[u^\dagger(2)\gamma_\nu^\dagger\gamma^{0\dagger}u(4)]$$

$\gamma^{0\dagger} = \gamma^0$, $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \Longrightarrow \Downarrow$

$\gamma^\nu\gamma^0 =$

$$[\underbrace{u^\dagger(1)\gamma^0}_{=\bar{u}(1)}\gamma^\nu\underbrace{\gamma^0\gamma^0}_{=1}u(3)]\,[\underbrace{u^\dagger(2)\gamma^0}_{=\bar{u}(2)}\gamma_\nu\underbrace{\gamma^0\gamma^0}_{=1}u(4)]$$

$$\Downarrow$$

$$[\bar{u}(1)\gamma^\nu u(3)]\,[\bar{u}(2)\gamma_\nu u(4)]$$

$$|M|^2 = \frac{g_e^4}{(P_1-P_3)^4}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)][\bar{u}(1)\gamma^\nu u(3)][\bar{u}(2)\gamma_\nu u(4)]$$

$$|M|^2 = \frac{g_e^4}{(P_1-P_3)^4} \underbrace{[\bar{u}(3)\,\gamma^\mu u(1)]\,[\bar{u}(1)\,\gamma^\nu u(3)]}\,[\bar{u}(4)\,\gamma_\mu u(2)]\,[\bar{u}(2)\,\gamma_\nu u(4)]$$

sum over the spins of particle "1"

sum over spins of "2"

use $\sum_{s=1,2} u^s(p)\bar{u}^s(p) = (\not{p}+mc)$

$$\Rightarrow \quad = \frac{g_e^4}{(\quad)^4} [\bar{u}(3)\,\underbrace{\gamma^\mu(\not{P}_1+m_e c)\gamma^\nu}_{\shortparallel \; Q_{ij}\ 4\times4}\,u(3)]$$

$$\times [\bar{u}(4)\,\gamma_\mu(\not{P}_2+m_\mu c)\gamma_\nu\,u(4)]$$

Sum over spins of "3" and "4"

$$\sum_{s_3=1,2}\sum_{i,j=1}^{4}\left[\bar{u}^{(s_3)}_{(3)i}\,Q_{ij}\,u^{(s_3)}_{(3)j}\right] = \sum_{i,j=1}^{4} Q_{ij} \sum u^{(s_3)}_{(3)j}\,\bar{u}^{(s_3)}_{(3)i}$$

$$= \sum_{i,j=1}^{4} Q_{ij}\left\{\sum u^{(s_3)}_{(3)}\,\bar{u}^{(s_3)}_{(3)}\right\}_{ji}$$

$$= \sum_{i,j=1}^{4} Q_{ij}\,(\not{P}_3+m_e)_{ji}.$$

$$= \mathrm{Tr}\left[Q\,(\not{P}_3+m_e)\right].$$

$$\sum_{s_1,s_2,s_3,s_4}|M|^2 = \frac{g_e^4}{(P_1-P_3)^4}\,\mathrm{Tr}\left[\gamma^\mu(\not{P}_1+m_e)\gamma^\nu(\not{P}_3+m_e)\right]\times$$

$$\mathrm{Tr}\left[\gamma_\mu(\not{P}_2+m_\mu)\gamma_\nu(\not{P}_4+m_\mu)\right].$$

average over the initial spins.

$$\frac{1}{4}\sum |M|^2 = \frac{g_e^4}{4(p_1-p_3)^4}\, Tr\left[\gamma^\mu(\not{p}_1+m_e)\gamma^\nu(\not{p}_3+m_e)\right] \times$$

$$Tr\left[\gamma_\mu(\not{p}_2+m_\mu)\gamma_\nu(\not{p}_4+m_\mu)\right].$$

properties of trace

1. $Tr(A+B) = Tr(A) + Tr(B)$
2. $Tr(\alpha A) = \alpha Tr(A)$
3. $Tr(AB) = Tr(BA)$

   $Tr(ABC) = Tr(CAB) = Tr(BCA)$.

$$Tr\left[\gamma^\mu(\not{p}_1+m_e)\gamma^\nu(\not{p}_3+m_e)\right]$$

$$= Tr\left[\gamma^\mu\not{p}_1\gamma^\nu\not{p}_3\right] + m_e\left\{\underbrace{Tr\left[\gamma^\mu\not{p}_1\gamma^\nu\right]}_{=0} + \underbrace{Tr\left[\gamma^\mu\gamma^\nu\not{p}_3\right]}_{=0}\right\}$$

$$+ m_e^2\, \underbrace{Tr\left[\gamma^\mu\gamma^\nu\right]}_{=4g^{\mu\nu}}.$$

use text book page. 252 – 253.

$$Tr\left[\gamma^\mu\gamma^\nu\right] = 4g^{\mu\nu}$$

trace of the product of "odd" number of gamma matrices $= 0$

$$\Rightarrow Tr\left[\gamma^\alpha\gamma^\beta\gamma^\lambda\right] = 0$$

$$Tr\left[\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\right] = 4\left(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}\right).$$

$$\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3] = (p_1)_\lambda (p_3)_\sigma \, \mathrm{Tr}[\gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma]$$

$$= (p_1)_\lambda (p_3)_\sigma \, 4\,(g^{\mu\lambda} g^{\sigma\nu} - g^{\mu\nu} g^{\lambda\sigma} + g^{\mu\sigma} g^{\lambda\nu})$$

$$= 4\,(p_1^\mu p_3^\nu - g^{\mu\nu}\, p_1 \cdot p_3 + p_1^\nu p_3^\mu).$$

$$\Rightarrow \mathrm{Tr}[\gamma^\mu (\not{p}_1 + m_e) \gamma^\nu (\not{p}_3 + m_e)]$$

$$= 4\,[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu + g^{\mu\nu}(m_e^2 - p_1 \cdot p_3)]$$

2nd trace

$$\mathrm{Tr}[\gamma_\mu (\not{p}_2 + m_\mu) \gamma_\nu (\not{p}_4 + m_\mu)]$$

$$= 4\,[p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} + g_{\mu\nu}(m_\mu^2 - p_2 \cdot p_4)]$$

$g_{\mu\nu} g^{\mu\nu} = 4$

$$\frac{1}{4} \sum |M|^2 = \frac{4 g_e^4}{(p_1 - p_3)^4} \times$$

$(p_1 \cdot p_2)(p_3 \cdot p_4)$

$= (p_1 \cdot p_4)(p_2 \cdot p_3)$

$$[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu + g^{\mu\nu}(m_e^2 - p_1 \cdot p_3)]\,[p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} + g_{\mu\nu}(m_\mu^2 - p_2 \cdot p_4)]$$

$$= \frac{4 g_e^4}{(p_1 - p_3)^4} \Big[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) + m_\mu^2 (p_1 \cdot p_3) - \cancel{(p_1 \cdot p_3)}(p_2 \cdot p_4)$$

$$+ (p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_2)(p_3 \cdot p_4) + m_\mu^2 (p_1 \cdot p_3) - \cancel{(p_1 \cdot p_3)}(p_2 \cdot p_4)$$

$$4\,(m_e^2 m_\mu^2 - m_e^2 (p_2 \cdot p_4) - m_\mu^2 (p_1 \cdot p_3) + \cancel{(p_1 \cdot p_3)}(p_2 \cdot p_4) \Big]$$

$$m_e^2 (p_2 \cdot p_4) - \cancel{(p_1 \cdot p_3)(p_2 \cdot p_4)} + m_e^2 (p_2 \cdot p_4) - \cancel{(p_1 \cdot p_3)}(p_2 \cdot p_4)$$

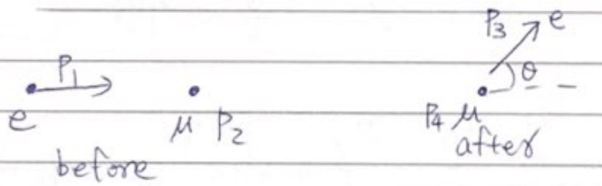
$$\frac{1}{4}\Sigma|M|^2 = \frac{8g_e^4}{(P_1-P_3)^4}\Big\{(P_1\cdot P_2)(P_3\cdot P_4) + (P_1\cdot P_4)(P_2\cdot P_3) - m_e^2(P_2\cdot P_4) - m_\mu^2(P_1\cdot P_3) + 2m_e^2 m_\mu^2\Big\}$$ ※

7.8 cross section

Ex. 7.7 Mott and Rutherford scattering.

An electron (m_e) scatters off a much heavior "muon" (m_μ; $m_\mu \gg m_e$.

Assuming the recoil of m_μ can be neglected, find differential scattering cross section $\frac{d\sigma}{d\Omega}$ in the lab frame



$\Rightarrow |\vec{P_1}| = |\vec{P_3}| = |\vec{P}|$

In this limit, we can use prob. 6.8

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi m_\mu c}\right)^2 \frac{1}{4}\Sigma|M|^2$$

$P_1 = (\sqrt{|\vec{P}|^2+m_e^2}, 0, 0, |\vec{P}|)$, $P_2 = (m_\mu, 0)$, $P_3 = (\sqrt{|\vec{P}|^2+m_e^2}, |\vec{P}|\sin\theta, 0, |\vec{P}|\cos\theta)$

$P_4 = (m_\mu, 0)$

$$P_1\cdot P_3 = |\vec{P}|^2+m_e^2 - |\vec{P}|^2\cos\theta = m_e^2 + |\vec{P}|^2(1-\cos\theta) = m_e^2 + 2|\vec{P}|^2\sin^2\frac{\theta}{2}$$

$$(P_1-P_3)^2 = P_1^2 - 2P_1\cdot P_3 + P_3^2 = 2m_e^2 - 2\left(m_e^2 + 2|\vec{P}|^2\sin^2\frac{\theta}{2}\right)$$

$$= -4|\vec{P}|^2\sin^2\frac{\theta}{2}$$

$$P_2 \cdot P_4 = m_\mu^2$$

$$(P_1 \cdot P_2)(P_3 \cdot P_4) = (P_1 \cdot P_4)(P_2 \cdot P_3) = m_\mu^2 (|\vec{p}|^2 + m_e^2)$$

$$\frac{1}{4}\Sigma |M|^2 = \frac{8 g_e^4}{16 |\vec{p}|^4 \sin^4\frac{\theta}{2}} \left\{ 2m_\mu^2 (|\vec{p}|^2 + m_e^2) - m_e^2 \cancel{m_\mu^2} - m_\mu^2 \left(\cancel{m_e^2} + 2|\vec{p}|^2 \sin^2\frac{\theta}{2} \right) + 2\cancel{m_e^2} m_\mu^2 \right\}$$

$$= \frac{g_e^4}{2|\vec{p}|^4 \sin^4\frac{\theta}{2}} \left\{ 2 m_e^2 m_\mu^2 + 2m_\mu^2 |\vec{p}|^2 \left(1 - \sin^2\frac{\theta}{2}\right) \right\}$$

$$= \frac{g_e^4 m_\mu^2}{|\vec{p}|^4 \sin^4\frac{\theta}{2}} \left(m_e^2 + |\vec{p}|^2 \cos^2\frac{\theta}{2} \right)$$

$$\frac{g_e^2}{4\pi} = \alpha$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi \cancel{m_\mu}} \right)^2 \frac{g_e^4 \cancel{m_\mu^2}}{|\vec{p}|^4 \sin^4\frac{\theta}{2}} \left(m_e^2 + |\vec{p}|^2 \cos^2\frac{\theta}{2} \right).$$

$$= \frac{\alpha^2}{4|\vec{p}|^4 \sin^4\frac{\theta}{2}} \left(m_e^2 + |\vec{p}|^2 \cos^2\frac{\theta}{2} \right) \quad \#$$

unit $[GeV^{-2}]$ or $[cm^2]$.

If the incoming electron is nonrelativistic, $|\vec{p}| \ll m_e$, above equation reduces to Rutherford formula.

$$|\vec{p}| = m_e v$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 m_e^2}{4 m_e^4 v^4 \sin^4\frac{\theta}{2}} = \frac{\alpha^2}{4 m_e^2 v^2 \sin^4\frac{\theta}{2}}. \quad ※$$

Prob. 7.30
7.47

$(\not{p}_1+m)v(1)=0$

$\bar{v}(1)(\not{p}+m)=0$

$(\not{p}_1-m)u(1)=0$

$\bar{u}(1)(\not{p}_1-m)=0$

DATE: $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$

Compton scattering

$$M=\frac{g_e^2}{(p_1-p_3)^2-m^2}\left[\bar{u}(4)\gamma^\mu(\not{p}_1-\not{p}_3+m)\gamma^\nu u(1)\right]\epsilon_\mu(2)\epsilon_\nu^*(3)$$

$$+\frac{g_e^2}{(p_1+p_2)^2-m^2}\left[\bar{u}(4)\gamma^\nu(\not{p}_1+\not{p}_2+m)\gamma^\mu u(1)\right]\epsilon_\mu(2)\epsilon_\nu^*(3)$$

$$(p_1-p_3)^2-m^2=\overset{m^2}{p_1^2}-2p_1\cdot p_3+\overset{0}{p_3^2}-m^2=-2p_1\cdot p_3$$

$$(p_1+p_2)^2-m^2=\overset{m^2}{p_1^2}+\overset{0}{p_2^2}+2p_1\cdot p_2-m^2=2p_1\cdot p_2.$$

$$(\not{p}_1+m)\gamma^\nu u(1)=(-\gamma^\nu\not{p}_1+2p_1^\nu+\gamma^\nu m)u(1)=2p_1^\nu u(1)-\gamma^\nu\underbrace{(\not{p}_1-m)u(1)}_{0}$$

$$=2p_1^\nu u(1).$$

$$(\not{p}_1+m)\gamma^\mu u(1)=2p_1^\mu u(1).$$

$$M=\underbrace{\frac{g_e^2}{-2p_1\cdot p_3}\left[\bar{u}(4)\left(2\gamma^\mu p_1^\nu-\gamma^\mu\not{p}_3\gamma^\nu\right)u(1)\right]\epsilon_\mu(2)\epsilon_\nu^*(3)}_{=M_1}$$

$$+\underbrace{\frac{g_e^2}{2p_1\cdot p_2}\left[\bar{u}(4)\left(2\gamma^\nu p_1^\mu+\gamma^\nu\not{p}_2\gamma^\mu\right)u(1)\right]\epsilon_\mu(2)\epsilon_\nu^*(3)}_{=M_2}.$$

$$\sum|M|^2=\sum(M_1+M_2)(M_1^*+M_2^*)=\sum M_1M_1^*+\sum M_1M_2^*+\sum M_1^*M_2+\sum M_2M_2^*$$

$$\Sigma |M_1|^2 = \frac{g_e^4}{(2P_1\cdot P_3)^2} \left[\bar{u}(4)\left(2\gamma^\mu p_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right) u(1)\right]_{1\times1}$$

$$\left[\bar{u}(1)\left(2\gamma^{\mu'} p_1^{\nu'} - \gamma^{\nu'} \not{P}_3 \gamma^{\mu'}\right) u(4)\right]_{1\times1}.$$

$$\underbrace{\epsilon_\mu(2)\; \epsilon^*_{\nu}(3)\; \epsilon^*_{\mu'}(2)}_{=-g_{\mu\mu'}}\; \epsilon_{\nu'}(3) \qquad \epsilon^*_\nu(3)\,\epsilon_{\nu'}(3) = -g_{\nu\nu'}$$

$$= \frac{g_e^4}{(2P_1\cdot P_3)^2} \mathrm{Tr}\left[(\not{P}_4+m)\left(2\gamma^\mu p_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right)(\not{P}_1+m)\left(2\gamma_\mu P_{1\nu} - \gamma_\nu \not{P}_3 \gamma_\mu\right)\right]$$

Tr [odd number of gamma matrices] = 0

$$\mathrm{Tr}\left[\not{P}_4\left(2\gamma^\mu p_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right)\not{P}_1\left(2\gamma_\mu P_{1\nu} - \gamma_\nu \not{P}_3 \gamma_\mu\right)\right].$$

③ ②

$$+m^2\, \mathrm{Tr}\left[\left(2\gamma^\mu p_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right)\left(2\gamma_\mu P_{1\nu} - \gamma_\nu \not{P}_3 \gamma_\mu\right)\right].$$

⑧ ⑦

$$\mathrm{Tr}\left[\underbrace{\not{P}_4\gamma^\mu}_{=-2\not{P}_4}\not{P}_3\underbrace{\gamma^\nu\not{P}_1\gamma_\nu}_{=-2\not{P}_1}\not{P}_3\gamma_\mu\right] = 4\,\mathrm{Tr}\left[\not{P}_4\not{P}_3\not{P}_1\not{P}_3\right].$$

$$= 4\times 4\left(2P_3\cdot P_4\, P_1\cdot P_3 - P_1\cdot P_4\, \underbrace{P_3\cdot P_3}_{=0}\right) = 32\,(P_1\cdot P_3)(P_3\cdot P_4) \text{ — ①}$$

$$-2\,\mathrm{Tr}[\not p_4\, \gamma^\mu \not p_3\, \gamma^\nu \not p_1\, \gamma_\mu\, p_{1\nu}]$$

$$= -2\,\mathrm{Tr}[\not p_4\, \gamma^\mu \not p_3\, \underbrace{\not p_1 \not p_1}_{p_1^2 = m^2}\, \gamma_\mu] = -2m^2\,\mathrm{Tr}[\not p_4\, \underbrace{\gamma^\mu \not p_3\, \gamma_\mu}_{-2\not p_3}]$$

$$= \underbrace{4m^2\,\mathrm{Tr}[\not p_4 \not p_3]}_{4\, p_3\cdot p_4} = 16\, m^2 (p_3\cdot p_4) \text{ — ②}$$

$$-2\,\mathrm{Tr}[\not p_4\, \gamma^\mu p_1^\nu \not p_1\, \gamma_\nu \not p_3\, \gamma_\mu] = -2\,\mathrm{Tr}[\not p_4\, \gamma^\mu \underbrace{\not p_1 \not p_1}_{p_1^2 = m^2} \not p_3\, \gamma_\mu]$$

$$= 16\, m^2 (p_3\cdot p_4) \text{ — ③}$$

$$4\,\mathrm{Tr}[\not p_4\, \gamma^\mu \underbrace{p_1^\nu \not p_1\, \gamma_\mu\, p_{1\nu}}_{m^2}] = 4m^2\,\mathrm{Tr}[\not p_4\, \underbrace{\gamma^\mu \not p_1\, \gamma_\mu}_{-2\not p_1}]$$

$$= -8m^2\,\mathrm{Tr}[\not p_4 \not p_1] = -32\, m^2 (p_1\cdot p_4) \text{ — ④}$$

$$m^2\,\mathrm{Tr}[\gamma^\mu \not p_3\, \underbrace{\gamma^\nu \gamma_\nu}_{4} \not p_3\, \gamma_\mu] = 16\, m^2\,\mathrm{Tr}[\not p_3 \not p_3]$$

$$= 0 \text{ — ⑤}$$

$$4m^2\,\mathrm{Tr}[\gamma^\mu \underbrace{p_1^\nu \gamma_\mu\, p_{1\nu}}_{m^2}] = 4m^4\,\mathrm{Tr}(\underbrace{\gamma^\mu \gamma_\mu}_{4\cdot 1}) = 64\, m^4 \text{ — ⑥}$$

$$-2m^2\,\mathrm{Tr}[\gamma^\mu \underbrace{p_1^\nu \gamma_\nu}_{\not p_1} \not p_3\, \gamma_\mu] = -8m^2\,\mathrm{Tr}[\not p_1 \not p_3] = -32\, m^2 (p_1\cdot p_3) \text{ — ⑦}$$

$$-2m^2\,\mathrm{Tr}[\gamma^\mu \not p_3\, \gamma^\nu \gamma_\mu\, p_{1\nu}] = -2m^2\,\mathrm{Tr}[\underbrace{\gamma^\mu \not p_3 \not p_1\, \gamma_\mu}_{4}]$$

$$= -8m^2\,\mathrm{Tr}[\not p_3 \not p_1] = -32\, m^2 (p_1\cdot p_3) \text{ — ⑧}$$

$$\sum |M_1|^2 = \frac{g_e^4}{(2P_1\cdot P_3)^2}\Big\{ 32(P_1\cdot P_3)(P_3\cdot P_4) + 32m^2(P_3\cdot P_4) - 32m^2(P_1\cdot P_4) - 64m^2(P_1\cdot P_3) + 64m^4 \Big\}$$

$$\sum |M_2|^2 = \frac{g_e^4}{(2P_1\cdot P_2)^2}\sum \left[\bar{u}(4)\left(2\gamma^\nu P_1^\mu + \gamma^\nu \not{P}_2 \gamma^\mu\right) u(1)\right] \epsilon_\mu(2)\, \epsilon_\nu^*(3)$$

$$\left[\bar{u}(1)\left(2\gamma^{\nu'} P_1^{\mu'} + \gamma^{\mu'} \not{P}_2 \gamma^{\nu'}\right) u(4)\right] \underbrace{\epsilon_{\mu'}^*(2)}_{= -g_{\mu\mu'}}\, \underbrace{\epsilon_{\nu'}(3)}_{= -g_{\nu\nu'}}$$

$$= \frac{g_e^4}{(2P_1\cdot P_2)^2} \mathrm{Tr}\Big[(\not{P}_4 + m)\left(2\gamma^\nu P_1^\mu + \gamma^\nu \not{P}_2 \gamma^\mu\right)(\not{P}_1 + m) \left(2\gamma_\nu P_{1\mu} + \gamma_\mu \not{P}_2 \gamma_\nu\right)\Big]$$

$$\sum |M_2|^2 = \frac{g_e^4}{(2P_1\cdot P_2)^2}\Big\{ 32(P_1\cdot P_2)(P_2\cdot P_4) - 32m^2(P_2\cdot P_4) - 32m^2(P_1\cdot P_4) + 64m^2(P_1\cdot P_2) + 64m^4 \Big\}$$

$$\sum M_1 M_2^* = \frac{g_e^4}{(-2P_1\cdot P_3)(2P_1\cdot P_2)} \left[\bar{u}(4)\left(2\gamma^\mu P_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right) u(1)\right] \epsilon_\mu(2)\, \epsilon_\nu^*(3)$$

$$\left[\bar{u}(1)\left(2\gamma^{\nu'} P_1^{\mu'} + \gamma^{\mu'} \not{P}_2 \gamma^{\nu'}\right) u(4)\right] \underbrace{\epsilon_{\mu'}^*(2)}_{= -g_{\mu\mu'}}\, \underbrace{\epsilon_{\nu'}(3)}_{= -g_{\nu\nu'}}$$

$$= \frac{g_e^4}{(-2P_1\cdot P_3)(2P_1\cdot P_2)} \mathrm{Tr}\Big[(\not{P}_4 + m)\left(2\gamma^\mu P_1^\nu - \gamma^\mu \not{P}_3 \gamma^\nu\right)(\not{P}_1 + m)\left(2\gamma_\nu P_{1\mu} + \gamma_\mu \not{P}_2 \gamma_\nu\right)\Big]$$

$\Sigma M_2 M_1^*$.

$$\Sigma M_1 M_2^* = \frac{g_e^4}{(-2P_1\cdot P_3)(2P_1\cdot P_2)} \Big\{ -32(P_1\cdot P_2)(P_1\cdot P_4) + 32(P_1\cdot P_3)(P_1\cdot P_4) + 32(P_1\cdot P_4)(P_2\cdot P_3) + 16m^2\big[2(P_1\cdot P_2) - 2(P_1\cdot P_3) + (P_1\cdot P_4) - (P_2\cdot P_3) + (P_2\cdot P_4) - (P_3\cdot P_4)\big] + 16\,m^4$$

$$P_1 + P_2 = P_3 + P_4$$

$$\Rightarrow P_1\cdot P_2 = P_3\cdot P_4.$$

$$S = (P_1+P_2)^2 = (P_3+P_4)^2 = 2P_1\cdot P_2 + m^2 = 2P_3\cdot P_4 + m^2$$

$$t = (P_1-P_4)^2 = (P_2-P_3)^2 = -2P_1\cdot P_4 + 2m^2 = -2P_2\cdot P_3$$

$$u = (P_1-P_3)^2 = (P_2-P_4)^2 = -2P_1\cdot P_3 + m^2 = -2P_2\cdot P_4 + m^2$$

$$s+t+u = 2m_e^2 \Rightarrow t = 2m_e^2 - s - u$$

google search : "FORM" trace of gamma matrix.

$$-su + m^2u + m^2s - m^4$$

$$\Sigma |M_1|^2 = \frac{g_e^4}{(m^2-u)^2} 8 \left\{ -su + 3m^2u + m^2s + m^4 \right\}$$

$$= \frac{g_e^4}{(m^2-u)^2} \cdot 8 \left[-(s-m^2)(u-m^2) + 2m^2u + 2m^4 \right]$$

$$= \frac{g_e^4}{(m^2-u)^2} 8 \left[-(s-m^2)(u-m^2) + 2m^2(u-m^2) + 4m^4 \right]$$

$$\Sigma |M_2|^2 = \frac{g_e^4}{(s-m^2)^2} 8 \left[-su + m^2u + 3m^2s + m^4 \right]$$

$$\left[-(s-m^2)(u-m^2) + 2m^2(s-m^2) + 4m^4 \right]$$

$$= \frac{g_e^4}{(s-m^2)^2} 8 \left[-(s-m^2)(u-m^2) + 2m^2(s-m^2) + 4m^4 \right]$$

$$\Sigma M_1 M_2^* = \Sigma M_1^* M_2 = \frac{g_e^4}{(s-m^2)(u-m^2)} 8 \left[m^2u + m^2s + 2m^4 \right]$$

$$m^2(u-m^2) \quad m^2(s-m^2) + 4m^4$$

$$= \frac{g_e^4}{(s-m^2)(u-m^2)} 8 \left[m^2(u-m^2) + m^2(s-m^2) + 4m^4 \right]$$

$$\frac{1}{4}\Sigma|M_1+M_2|^2$$

$$\frac{1}{4}\Sigma|M|^2 = \frac{1}{4}\left(\Sigma|M_1|^2 + \Sigma M_1 M_2^* + \Sigma M_1^* M_2 + \Sigma|M_2|^2\right)$$

$$= \frac{g_e^4\, 8}{4}\Bigg\{ -\frac{(s-m^2)}{(u-m^2)} + 2m^2\frac{1}{(u-m^2)} + 4m^4\frac{1}{(u-m^2)^2}$$

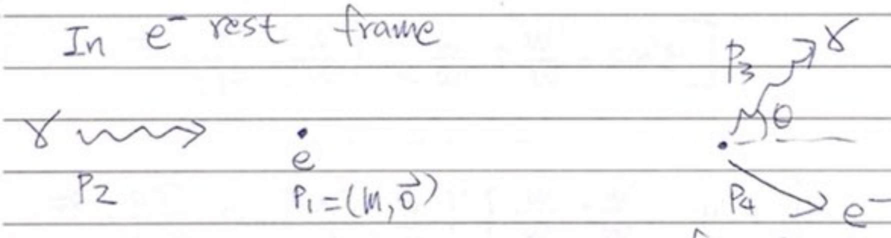
$$-\frac{(u-m^2)}{(s-m^2)} + 2m^2\frac{1}{(s-m^2)} + 4m^4\frac{1}{(s-m^2)^2}$$

$$2m^2\frac{1}{s-m^2} + 2m^2\frac{1}{u-m^2} + 8m^4\frac{1}{(u-m^2)(s-m^2)}\Bigg\}$$

$$= 2g_e^4\Bigg\{-\frac{(s-m^2)}{(u-m^2)} - \frac{(u-m^2)}{(s-m^2)} + 4m^2\left[\frac{1}{(u-m^2)} + \frac{1}{(s-m^2)}\right]$$

$$+ 4m^4\left[\frac{1}{(u-m^2)} + \frac{1}{(s-m^2)}\right]^2\Bigg\}$$

In e^- rest frame



before   after

$$P_1\cdot P_2 = m\omega, \quad P_1\cdot P_3 = m\omega'$$

Prob. 3.27

$$P_2 = (\omega, 0, 0, \omega), \quad P_3 = (\omega', \omega'\sin\theta, 0, \omega'\cos\theta)$$

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1-\cos\theta) \Rightarrow \omega' = \frac{\omega}{1+\frac{\omega}{m}(1-\cos\theta)}$$

$$(s-m^2) = 2P_1\cdot P_2 = 2m\omega, \quad (u-m^2) = -2P_1\cdot P_3 = -2m\omega'$$

$$\frac{1}{4}\Sigma|M|^2 = 2g_e^4\left\{+\frac{\omega}{\omega'}+\frac{\omega'}{\omega}+2m\underbrace{\boxed{\left(-\frac{1}{\omega'}+\frac{1}{\omega}\right)}}_{\frac{1}{m}(1-\cos\theta)} + m^2\underbrace{\left(\frac{-1}{\omega'}+\frac{1}{\omega}\right)^2}_{=\frac{(1-\cos\theta)^2}{m^2}}\right\}$$

$$-2(1-\cos\theta)+1-2\cos\theta+\cos^2\theta$$

$$= -\sin^2\theta$$

$$\Rightarrow \frac{1}{4}\Sigma|M|^2 = 2g_e^4\left\{\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-\sin^2\theta\right\}$$

Prob. 6.10 (b)

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi m}\right)^2\left(\frac{\omega'}{\omega}\right)^2\cdot 2g_e^4\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-\sin^2\theta\right]$$

$$= \frac{\alpha^2}{2m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-\sin^2\theta\right] \qquad \frac{g_e^2}{4\pi}=\alpha$$

$$d\Omega = d\cos\theta\, d\phi.$$

$$0\le\phi\le 2\pi.$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-\sin^2\theta\right]}$$

Klein – Nishina formula. for Compton scattering.