計算物理概論

Homework 3

Release date: 2023/05/01

Due: 2023/05/15 at 15:30

(Submit your solution to google classroom)

1. The Stefan-Boltzmann constant can be evaluated by calculating the integral

$$\frac{\sigma_B T^4}{\pi} = \int_0^\infty B_\upsilon\,(T) d\nu \quad ,$$

where

$$B_\upsilon(T) = \frac{2h\upsilon^3}{c^2}\frac{1}{e^{h\upsilon/kT}-1},$$

Is the Planck function (assume the temperature T = 6000 K). h = 6.626e-34 (m^2 kg/s) is the Planck constant, c=2.9979e8 (m/s) is the light speed, ν is frequency.

(a) Do not use numpy. Use the midpoint method in the Integrator class we developed to evaluate the Stefan-Boltzmann constant with N = 10_000_000.
(b) Repeat part (a) but use the trapezoidal method.
(c) Repeat part (a) but use numpy.sum().
(d) Repeat part (a) but use numpy.trapz().
(e) Repeat part (a) but use numpy.random for the Monte Carlo method.
(f) Repeat part (e) but with a different sample size (N). Try N= 1_000, 10_000, 100_000, 1_000_000, 10_000_000, and 100_000_000. Measure the numerical errors as a function of the sample size N. Check if it follows the Monte Carlo integration's 1/sqrt(N) law.

The Stefan-Boltzmann constant is

$$\sigma_B = \frac{2\pi^5 k^4}{15c^2h^3} = 5.670374419\ldots \times 10^{-8} \text{ W/m}^2/\text{k}^4$$

(Hints: how to handle infinity in the integral?)