Code (Tuesday Week 6)

Dice Example

```
d6 :: [Int]
d6 = [1, 2, 3, 4, 5, 6]
twoD6 :: [(Int, Int)]
twoD6 = (,) < $> d6 < *> d6
game :: [(Int,Int)]
game = twoD6 >>= \setminus (d1, d2) \rightarrow
          if abs (d1 - d2) < 2 then
              d6 >>= \d2' -> pure (d1, d2')
          else
              pure (d1, d2)
game' :: [(Int,Int)]
game' = do
  (d1,d2) < - twoD6
  if abs (d1 - d2) < 2 then do
     d2' <- d6
     pure (d1, d2')
  else do
     pure (d1, d2)
score :: (Int, Int) -> Int
score (d1, d2) = abs (d1 - d2)
scores :: [Int]
scores = fmap score game
```

Student Database Example

Arbitrary Search Trees

```
import Test.QuickCheck
data Tree = Leaf
          | Branch Int Tree Tree
          deriving (Show, Eq)
instance Arbitrary Tree where
 -- arbitrary :: Gen Tree
 arbitrary = arbitrary >>= \min
               -> arbitrary >>= \(Positive max')
                 -> searchTrees min (min + max')
  where
     searchTrees :: Int -> Int -> Gen Tree
     searchTrees min max
           | min < max = oneof [ leafGen</pre>
                                  , branchGen
           | otherwise = leafGen
       where
         leafGen :: Gen Tree
         leafGen = pure Leaf
         branchGen :: Gen Tree
         branchGen = choose (min, max)
                       >>= \n ->
                         Branch n <$> searchTrees min (n -1)
                                  <*> searchTrees n max
{- with do notation:
instance Arbitrary Tree where
  -- arbitrary :: Gen Tree
```

```
arbitrary = do
   min <- arbitrary
   Positive max' <- arbitrary
   searchTrees min (min + max')
  where
     searchTrees :: Int -> Int -> Gen Tree
     searchTrees min max
           | min < max
                         = oneof [ leafGen
                                  , branchGen
           | otherwise = leafGen
       where
         leafGen :: Gen Tree
         leafGen = pure Leaf
         branchGen :: Gen Tree
         branchGen = do
           n <- choose (min, max)</pre>
           Branch n <$> searchTrees min (n -1)
                    <*> searchTrees n max
-}
```

Basic Instances

Most of this code duplicates the standard library and Prelude, so won't compile.

```
maybeMap :: (a -> b) -> Maybe a -> Maybe b
maybeMap f (Just x) = Just (f x)
maybeMap f Nothing = Nothing

instance Functor [] where
  fmap = map

instance Functor Maybe where
  fmap = maybeMap

instance Functor ((->) x) where

-- fmap :: (a -> b) -> f a -> f b
-- so for this type, f is (x ->) so:
-- fmap :: (a -> b) -> (x -> a) -> (x -> b)
  fmap = (.)

instance Functor ((,) x) where
-- fmap :: (a -> b) -> (x,a) -> (x,b)
  fmap f (x,a) = (x, f a)
```

```
-- remember (3,2) == (,) 3 2
-- we can write functions:
-- toString :: Int -> String
-- as
-- toString :: (->) Int String
instance Applicative Maybe where
  -- pure :: a -> Maybe a
  pure a = Just a
  -- (<*>) :: Maybe (a -> b) -> Maybe a -> Maybe b
  Nothing <*> arg = Nothing
  Just f <*> Nothing = Nothing
  Just f <^* > Just a = Just (f a)
{- Proof of Functor Laws for all applicatives
   where
    fmap \ f \ x = pure \ f < *> x
1. fmap id x == id x
2. fmap f (fmap g x) == fmap (f . g) x
-- Proof of 1)
pure id <^*> x == x -- Identity
-- Proof of 2)
pure f < *> (pure g < *> x)
== -- composition (backwards)
pure (.) <*> pure f <*> pure g <*> x
== -- homomorphism
pure ((.) f) <^*> pure g <^*> x
== -- homomorphism
pure (f \cdot g) < *> x
- }
-- This instance is what Haskell actually uses for lists.
instance Applicative [ ] where
         <*> as = []
  (f:fs) <^*> as = map f as ++ (fs <^*> as)
```

```
pure a = [a]
-- This instance is put behind ZipList in the standard library
instance Applicative [ ] where
  <*> as = []
  fs
        <*> [] = []
  (f:fs) <^*> (a:as) = f a : (fs <^*> as)
  pure a = a: pure a
instance Applicative ((->) x) where
  pure :: a \to (x \to a)
  pure a x = a
  -- (<*>) :: f (a -> b) -> f a -> f b
  -- (<*>) :: (x -> a -> b) -> (x -> a) -> (x -> b)
  (f <^*> a) = \x -> f x (a x)
 instance Monad Maybe where
   (>>=) :: Maybe a -> (a -> Maybe b) -> Maybe b
   Just a >>= f = f a
   Nothing >>= f = Nothing
 instance Monad [ ] where
   (>>=) :: [a] -> (a -> [b]) -> [b]
   (>>=) as f = concatMap f as
 instance Monad ((->) x) where
   (>>=) :: (x -> a) -> (a -> x -> b) -> (x -> b)
   (>>=) xa axb = \xspacex -> axb (xa x) x
{ -
We can make an applicative operator given a monad, by writing:
(<^*>) :: m(a -> b) -> ma -> mb
mf <^*> mx = mf >>= \backslash f ->
              pure(fx)
- }
```