Quiz (Week 9)

Logic

Question 1

Which one (or more) of expressions listed below is a possible formalisation of the phrase: *Not all that glitters is gold*.

1. ✗ $\exists x.\ (\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x))$
2. ✗ $\forall x.\ (\neg(\mathsf{Glitter}(x)) \rightarrow \mathsf{Gold}(x))$
3. ✔ $\neg\forall x. (\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x))$
4. ✗ $\forall x.\ (\neg(\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x)))$

Question 2

Which of the expressions listed below is a possible formalisation of the Abraham Lincoln quote: *You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time*.

1. ✗ $\forall p. \forall t. (\mathsf{Fool}(p,t) \land \neg\mathsf{Fool}(p,t))$
2. ✗ $(\forall p. \exists t. \mathsf{Fool}(p,t)) \land (\exists p. \forall t. \mathsf{Fool}(p,t)) \rightarrow \exists p. \exists t. \neg\mathsf{Fool}(p,t)$
3. ✗ $(\forall p. \exists t. \mathsf{Fool}(p,t)) \land (\exists p. \forall t. \mathsf{Fool}(p,t)) \land \neg\forall p. \forall t. \neg\mathsf{Fool}(p,t)$
4. ✔ $(\forall p. \exists t. \mathsf{Fool}(p,t)) \land (\exists p. \forall t. \mathsf{Fool}(p,t)) \land \neg\forall p. \forall t. \mathsf{Fool}(p,t)$
5. ✗ $(\forall p. \exists t. \mathsf{Fool}(p,t)) \land (\exists p. \forall t. \mathsf{Fool}(p,t)) \land (\mathit{False} \rightarrow \forall p. \forall t. \mathsf{Fool}(p,t))$
6. ✔ $(\exists t. \forall p. \mathsf{Fool}(p,t)) \land (\forall t. \exists p. \mathsf{Fool}(p,t)) \land \neg\forall t. \forall p. \mathsf{Fool}(p,t)$

Question 3

Here is a proof of a logical statement in *natural deduction style*:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\overline{A \lor B}^{\alpha} \quad \dfrac{\dfrac{\overline{A \to C}^{\beta} \quad \overline{A}^{\delta_1}}{C}\textcircled{5}}{C \lor D}\textcircled{3} \quad \dfrac{\dfrac{\overline{B \to D}^{\gamma} \quad \overline{B}^{\delta_2}}{D}\textcircled{5}}{C \lor D}\textcircled{4}}{C \lor D}\textcircled{2}^{\delta_1;\delta_2}}{(B \to D) \to C \lor D}\textcircled{1}^{\gamma}}{(A \to C) \to (B \to D) \to C \lor D}\textcircled{1}^{\beta}}{A \lor B \to (A \to C) \to (B \to D) \to C \lor D}\textcircled{1}^{\alpha}
$$

The names of the rules used have been replaced with circled numbers. What is the rule used in each position?

1. ✗ ① is →-E; ② is ∨-I; ③ is ∨-E$_1$; ④ is ∨-E$_2$; ⑤ is →-I
2. ✗ ① is →-I; ② is ∨-E; ③ is ∨-I$_2$; ④ is ∨-I$_1$; ⑤ is →-E
3. ✗ ① is ∨-I; ② is →-E; ③ is →-I$_2$; ④ is →-I$_1$; ⑤ is ∨-E
4. ✗ ① is ∨-I; ② is →-E; ③ is →-I$_1$; ④ is →-I$_2$; ⑤ is ∨-E
5. ✔ ① is →-I; ② is ∨-E; ③ is ∨-I$_1$; ④ is ∨-I$_2$; ⑤ is →-E

Curry-Howard Correspondence

Question 4

Selct all of the following types for which you can write a total, terminating Haskell function.

1. ✔ `(a -> b) -> (b -> c) -> (a -> c)`
2. ✗ `((a, b) -> c) -> (a -> c)`
3. ✔ `(a -> c) -> ((a, b) -> c)`
4. ✗ `((a -> c) -> c) -> a`

Question 5

What is the computational interpretation of the theorem $(A \to (B \to C)) \to ((A \land B) \to C)$?

1. ✔ The function that transforms a *curried* function to an *uncurried* one.
2. ✗ The function that transforms an *uncurried* function to a *curried* one.
3. ✗ The function that creates a tuple of the two given A values and B values.
4. ✗ There is no computational interpretation of this logical formula.

Question 6

Which of the following Haskell programs constitutes a valid proof of the theorem given in Question 3?

1. ✔
```
proof (Left a) f g = Left (f a)
proof (Right b) f g = Right (g b)
```
2. ✗
```
proof (a, b) f g = (f a, g b)
```
3. ✗
```
proof x f g = if x then f x else g x
```
4. ✗
```
proof x f g = x (f x) (g x)
```
5. ✗
```
proof (Left a) f g = Left (g a)
proof (Right b) f g = Right (f b)
```

Question 7

Below is a complicated proof that assuming A and B, we can derive $A \land B$:

$$\cfrac{\cfrac{\cfrac{\cfrac{\overline{B \land A}^{\,\delta}}{A}\land\text{-E}_2 \qquad \cfrac{\overline{B \land A}^{\,\delta}}{B}\land\text{-E}_1}{A \land B}\land\text{-I}}{(B \land A) \to (A \land B)}\to\text{-I}^{\delta} \qquad \cfrac{B \quad A}{B \land A}\land\text{-I}}{A \land B}\to\text{-E}$$

What is the equivalent program to this proof, in typed lambda calculus (using Haskell-style syntax for pairs)? Assume $a : A$ and $b : B$.

1. ✗ (a, b)
2. ✔ $(\lambda x.\ (\mathsf{snd}\ x, \mathsf{fst}\ x))\ (b, a)$
3. ✗ $(\mathsf{snd}\ (b, a), \mathsf{fst}\ (b, a))$
4. ✗ $(\mathsf{fst}\ (a, b), \mathsf{snd}\ (a, b))$
5. ✗ $(\lambda x.\ (\mathsf{fst}\ x, \mathsf{snd}\ x))\ (a, b)$

Question 8

What proof results from applying *proof simplification* as much as possible to the proof from Question 7?

1. ✗

$$\dfrac{\dfrac{}{A} \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\,\wedge\text{-I}}{B}\,\wedge\text{-E}_1}{A \wedge B}\,\wedge\text{-I}$$

2. ✗

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\,\wedge\text{-I}}{A}\,\wedge\text{-E}_2 \qquad \dfrac{}{B}}{A \wedge B}\,\wedge\text{-I}$$

3. ✔

$$\dfrac{\dfrac{}{A} \quad \dfrac{}{B}}{A \wedge B}\,\wedge\text{-I}$$

4. ✗

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\,\wedge\text{-I}}{A}\,\wedge\text{-E}_2 \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\,\wedge\text{-I}}{B}\,\wedge\text{-E}_1}{A \wedge B}\,\wedge\text{-I}$$

5. ✗

$$\dfrac{\dfrac{\dfrac{A \quad B}{A \wedge B}\,\wedge\text{-I}}{A}\,\wedge\text{-E}_1 \qquad \dfrac{\dfrac{A \quad B}{A \wedge B}\,\wedge\text{-I}}{B}\,\wedge\text{-E}_2}{A \wedge B}\,\wedge\text{-I}$$

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