

Base Case

$$\begin{aligned} & ([] ++ yS) ++ zS \\ &= yS ++ zS \end{aligned} \quad (1)$$

$$= yS ++ zS \quad (1)$$

$$= [] ++ (yS ++ zS)$$

Inductive Case

$$1.H \quad (xs' ++ yS) ++ zS = xs' ++ (yS ++ zS)$$

$$\begin{aligned} & ((x : xs') ++ yS) ++ zS \\ &= (x : (xs' ++ yS)) ++ zS \end{aligned} \quad (2)$$

$$= (x : ((xs' ++ yS) ++ zS)) \quad (2)$$

$$= x : (xs' ++ (yS ++ zS)) \quad (1.H)$$

$$= x : (xs' ++ (yS ++ zS)) \quad (2)$$

$$= (x : xs') ++ (yS ++ zS)$$

$++$ is assoc.

Base Case

$$\begin{aligned} & [] ++ [] \\ &= [] \quad (1) \\ &= [] \end{aligned}$$

Right identity
for lists

Inductive Case

$$\text{I.H. } xs' ++ [] = xs'$$

$$\begin{aligned} & x : xs' ++ [] \\ \equiv & x : (xs' ++ []) \quad (2) \\ \equiv & x : xs' \quad (\text{I.H. } \rightarrow) \end{aligned}$$

Base Case

$$\begin{aligned} & \text{reverse}(\text{reverse} []) \\ &= \text{reverse} [] \quad (A) \\ &= [] \quad (A) \end{aligned}$$

Reverse is
an involution

Inductive Case

$$\text{I.H. } \text{reverse}(\text{reverse } l s') = l s'$$

$$\begin{aligned} & \text{reverse}(\text{reverse}(x : l s')) \\ &= \text{reverse}(\text{reverse } l s' ++ [x]) \quad (B) \\ &= \text{STUCK!} \\ &= x : \text{reverse}(\text{reverse } l s') \\ &= x : l s' \end{aligned}$$

Use lemma
on next page
(I.H.)

Base case

$$\begin{aligned} & \text{reverse} ([] ++ [x]) \\ &= \text{reverse} [x] & (1) \\ &= (\text{reverse} [] ++ [x]) & (B) \\ &= [] ++ [x] & (A) \\ &= [x] & (1) \\ &= x : \text{reverse} [] & (A) \end{aligned}$$

Helper lemma

$$[x] \equiv x : []$$

Inductive Case

$$\begin{aligned} & \text{I.H. } \text{reverse} (ys' ++ [x]) = x : \text{reverse } ys' \\ & \text{reverse} ((y : ys') ++ [x]) \\ &= \text{reverse} (y : (ys' ++ [x])) & (2) \\ &= \text{reverse} (ys' ++ [x] ++ [y]) & (B) \\ &= (x : \text{reverse } ys') ++ [y] & (1.H) \\ &= (x : \text{reverse } ys') ++ [y] & (B) \\ &= x : \text{reverse} (y : ys') \end{aligned}$$