

$$A \wedge B \rightarrow B \wedge A$$

$$\frac{A \in \Gamma}{\Gamma \vdash A}$$

$$\begin{array}{c}
 \begin{array}{c}
 \alpha \\
 \wedge\text{-E}_2 \\
 \frac{A \wedge B \vdash A \wedge B}{A \wedge B \vdash B}
 \end{array}
 \quad
 \begin{array}{c}
 \alpha \\
 \wedge\text{-E}_1 \\
 \frac{A \wedge B \vdash A \wedge B}{A \wedge B \vdash A}
 \end{array}
 \\
 \wedge\text{-I} \\
 \frac{A \wedge B \vdash B \quad A \wedge B \vdash A}{A \wedge B \vdash B \wedge A}
 \\
 \rightarrow\text{I} \alpha \\
 \hline
 \vdash A \wedge B \rightarrow B \wedge A
 \end{array}$$

$$A \vee \perp \rightarrow A$$

$$\begin{array}{c}
 \begin{array}{c}
 \textcolor{orange}{\alpha} \quad \overline{\quad} \\
 A \vee \perp \vdash A \vee \perp
 \end{array}
 \quad
 \begin{array}{c}
 \overline{\quad} \textcolor{orange}{\beta_1} \\
 A \vee \perp, A \vdash A
 \end{array}
 \quad
 \begin{array}{c}
 \overline{\quad} \textcolor{orange}{\beta_2} \\
 A \vee \perp, \perp \vdash \perp \\
 \hline
 A \vee \perp, \perp \vdash A \\
 \downarrow \textcolor{blue}{\neg E}
 \end{array}
 \quad
 \textcolor{blue}{V-E} \textcolor{orange}{\beta_1} \textcolor{orange}{\beta_2} \quad \textcolor{blue}{i}
 \end{array}
 \quad
 \begin{array}{c}
 A \vee \perp \vdash A \\
 \hline
 \vdash (A \vee \perp) \rightarrow A \quad \textcolor{blue}{\rightarrow I} \textcolor{orange}{\alpha}
 \end{array}$$

$$A \rightarrow \neg\neg A$$

$$\neg\neg A \rightarrow A$$

$$A \rightarrow ((A \rightarrow \perp) \rightarrow \perp)$$

$$((A \rightarrow \perp) \rightarrow \perp) \rightarrow A$$

$$A \vdash ((A \rightarrow \perp) \rightarrow \perp)$$

$$(A \rightarrow \perp) \rightarrow \perp \vdash A$$

$$A, A \rightarrow \perp \vdash \perp$$

