Data Structures and Algorithm

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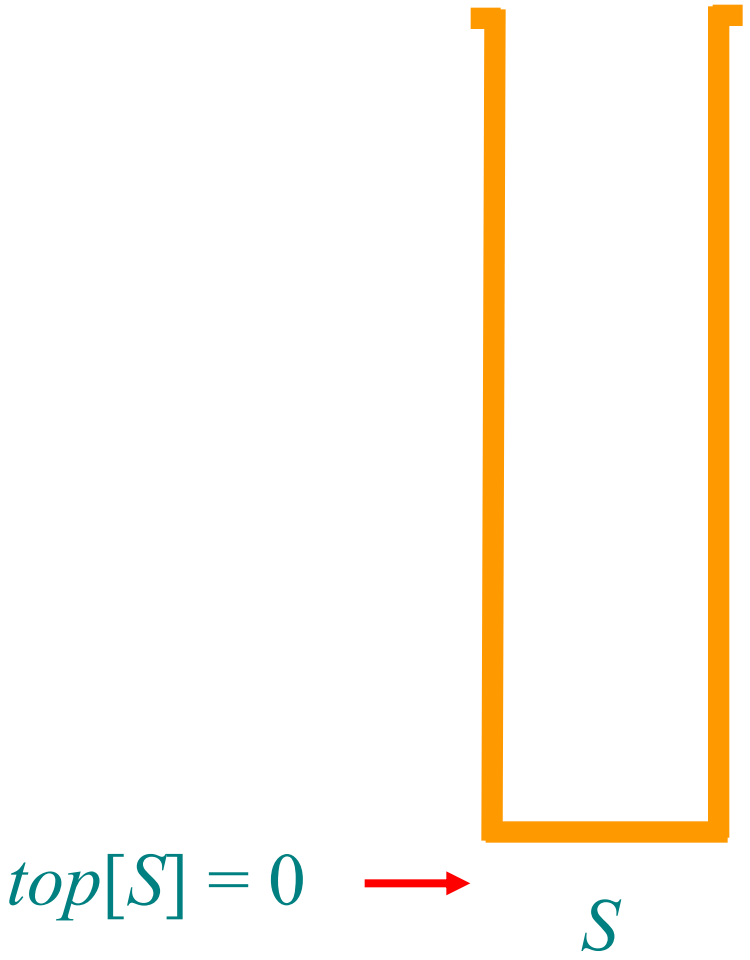
What is data structure?

A data structure is a way to store and organize data in order to facilitate access and modifications.

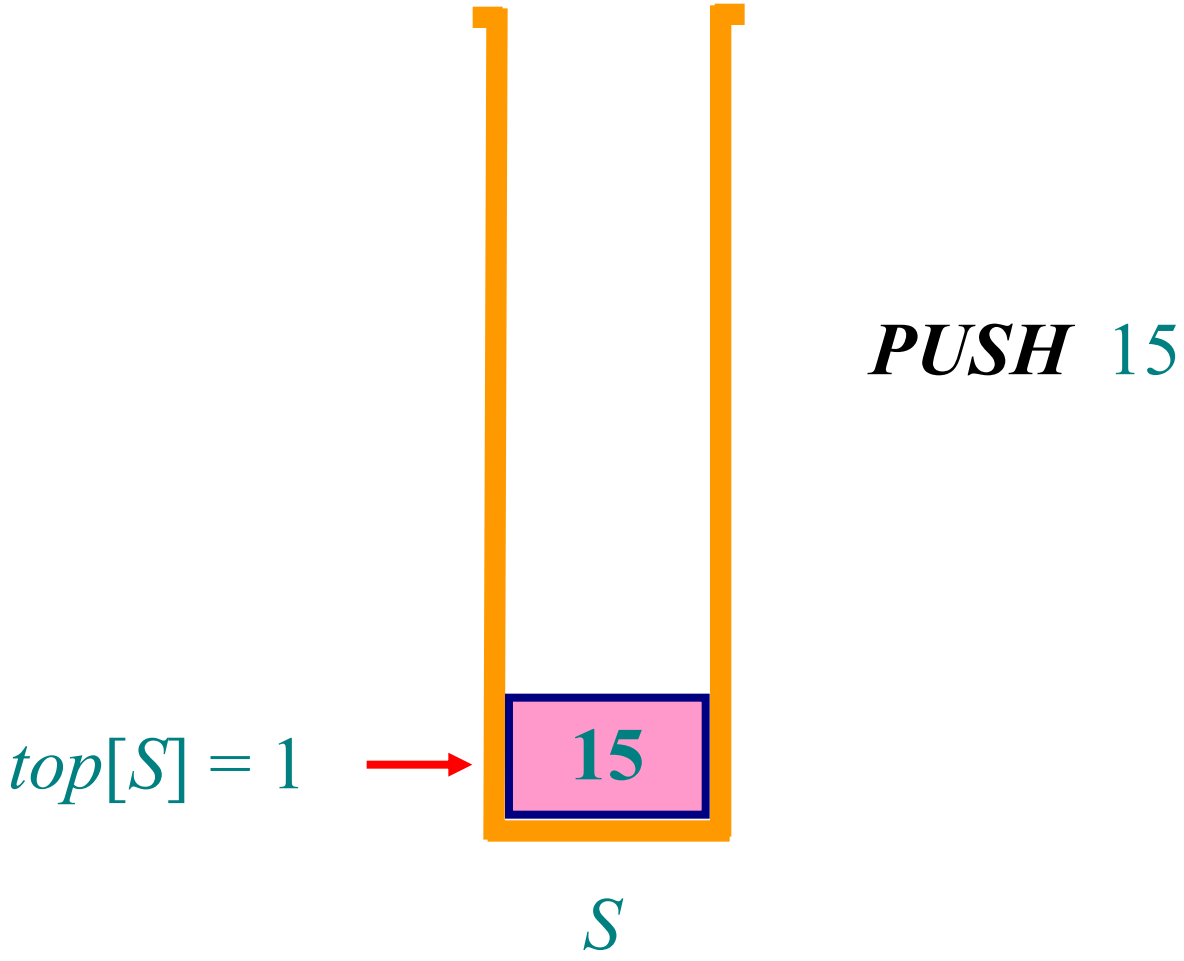
Elementary data structures

- ***Stacks***
- ***Queues***
- ***Linked lists***
- ***Trees***

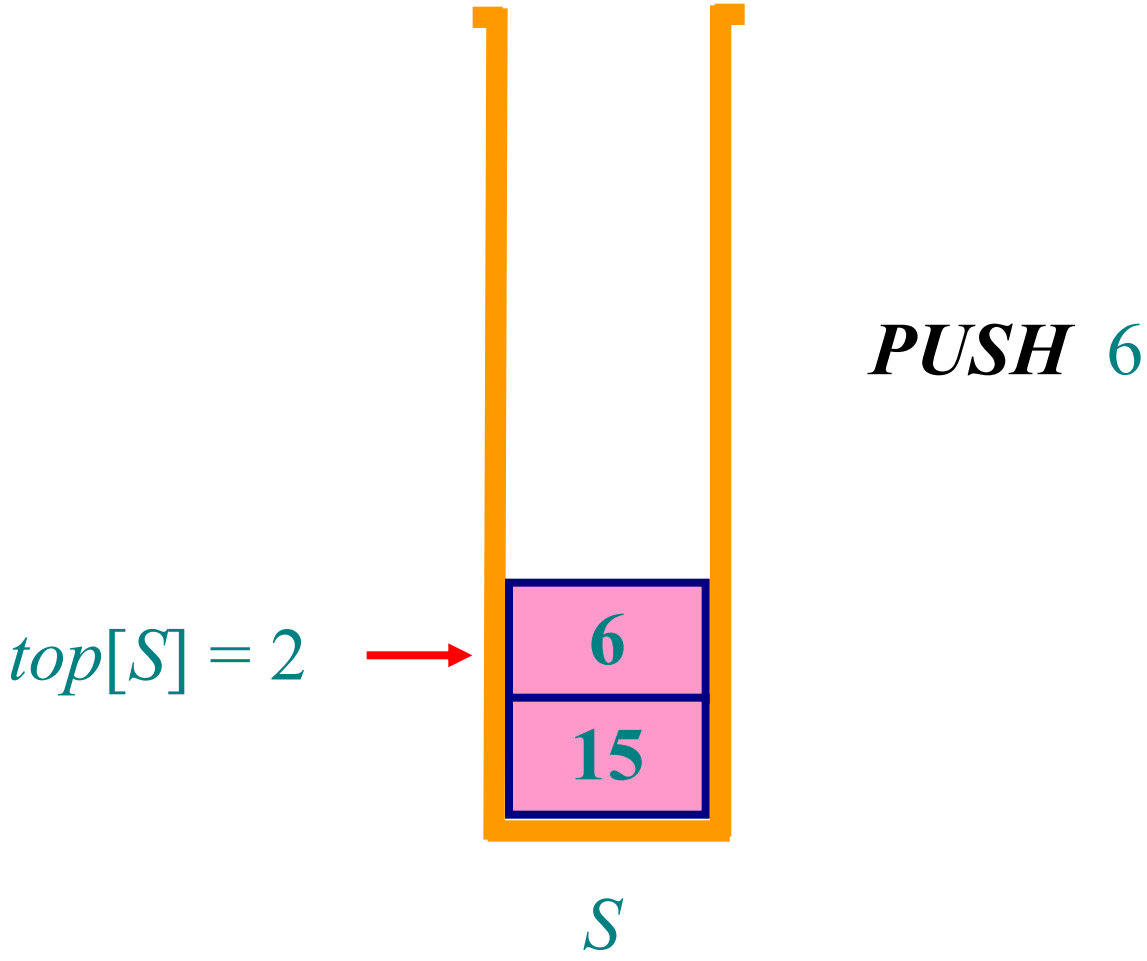
Stacks



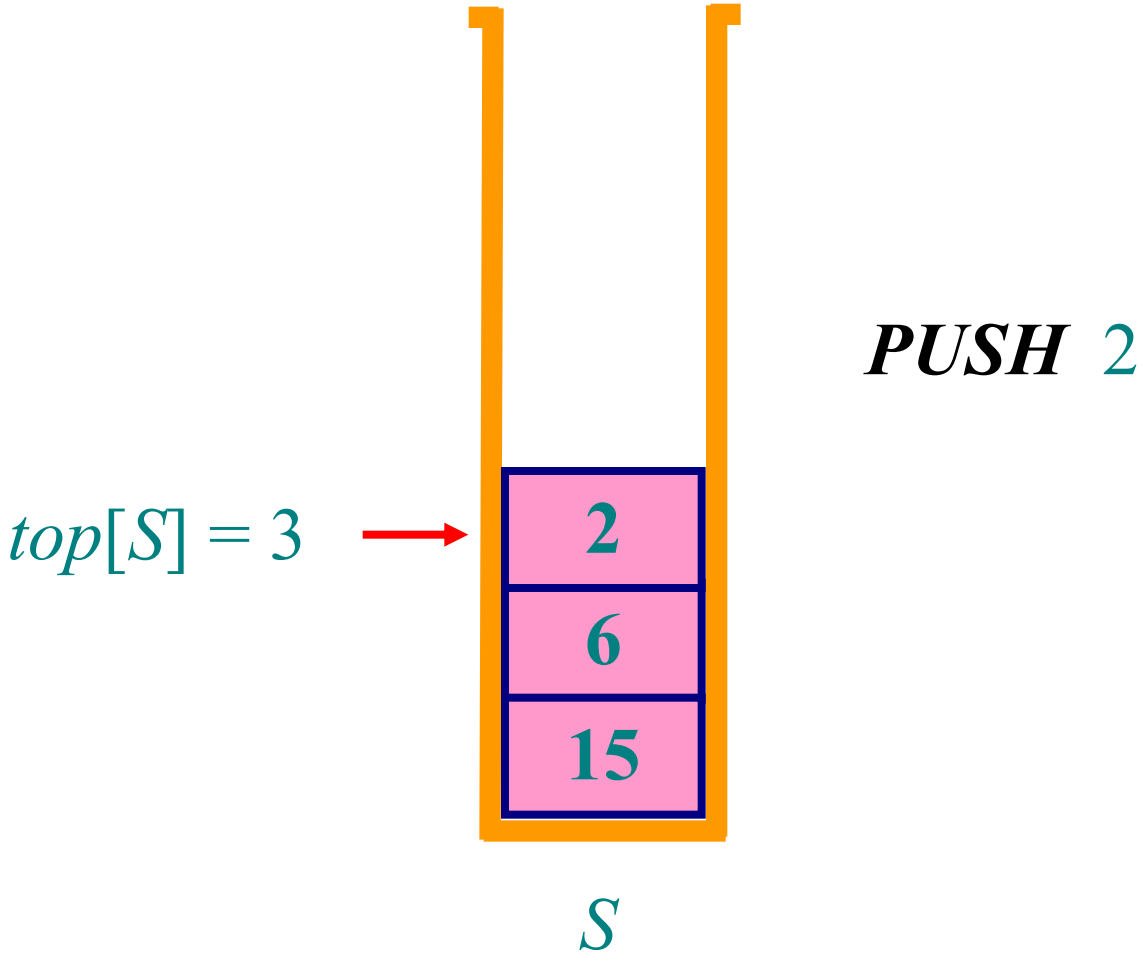
Stacks



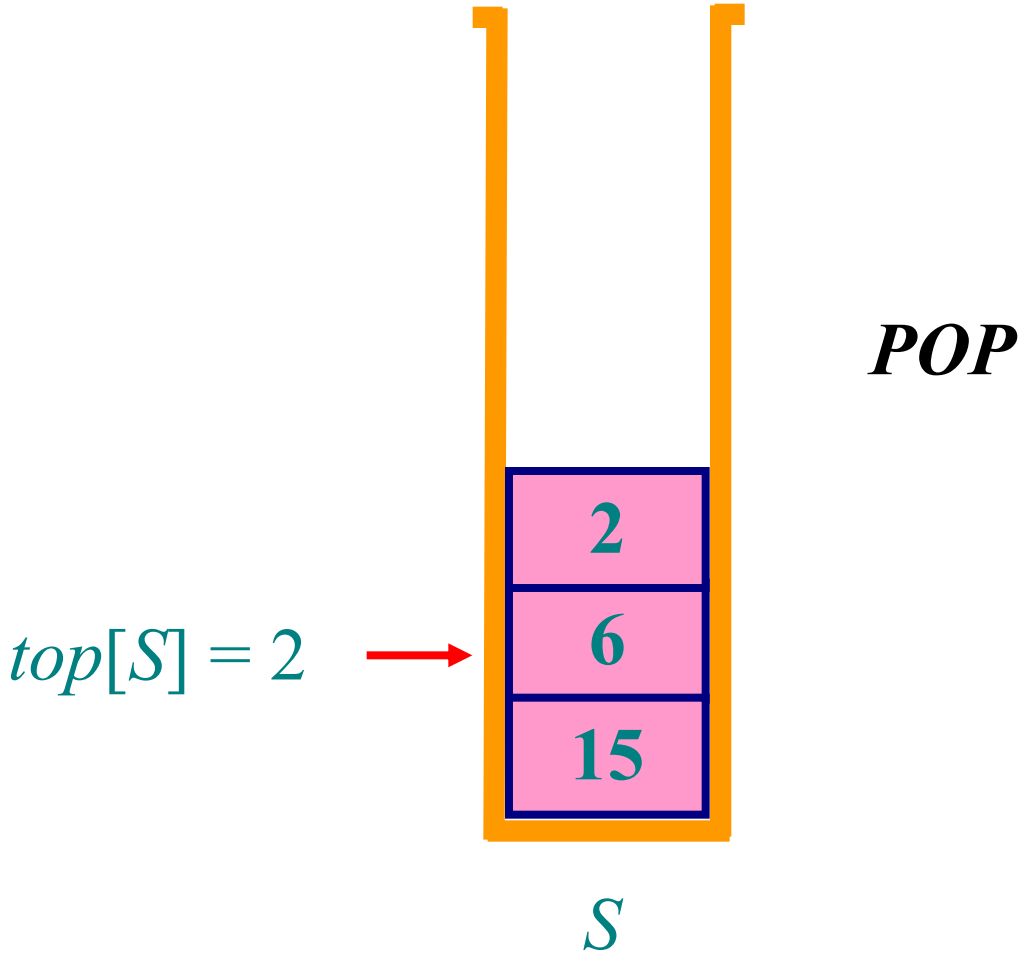
Stacks



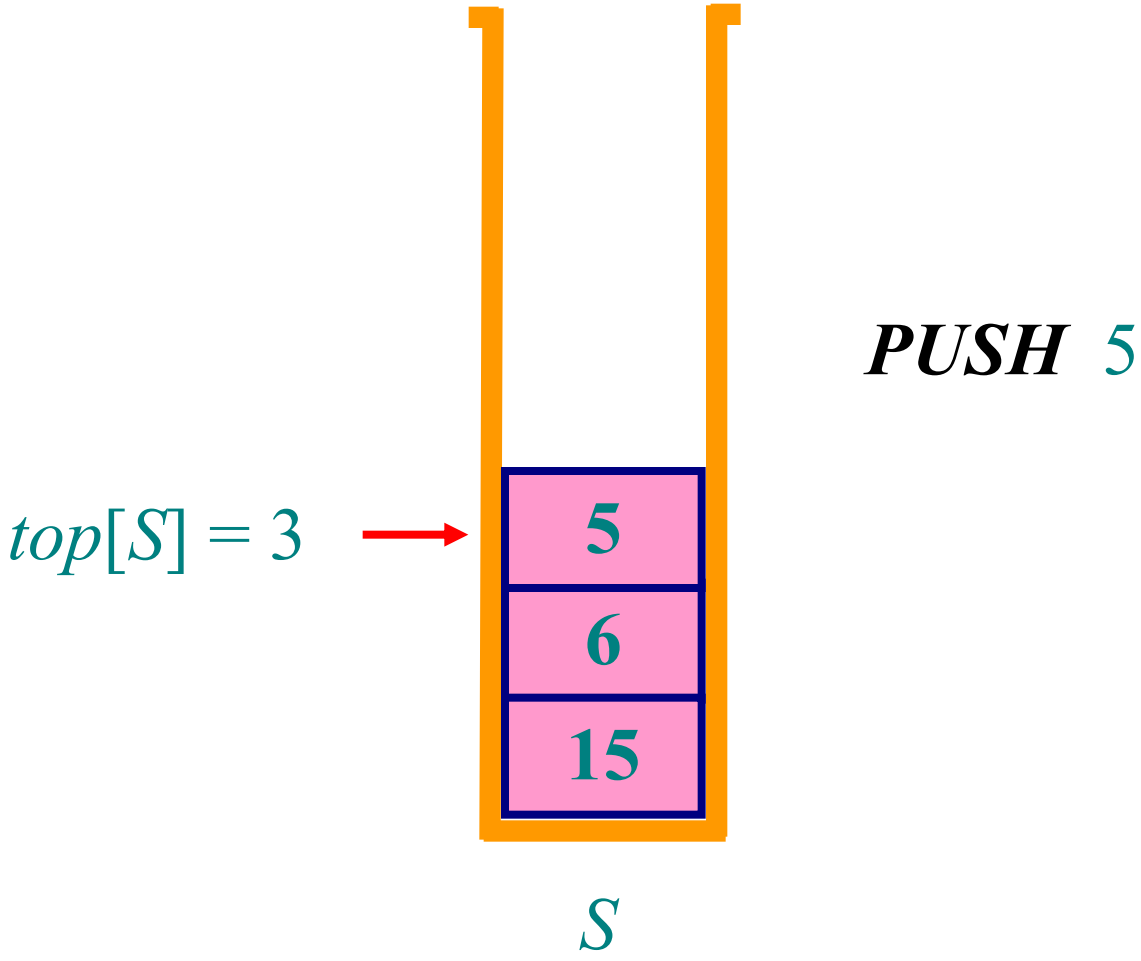
Stacks



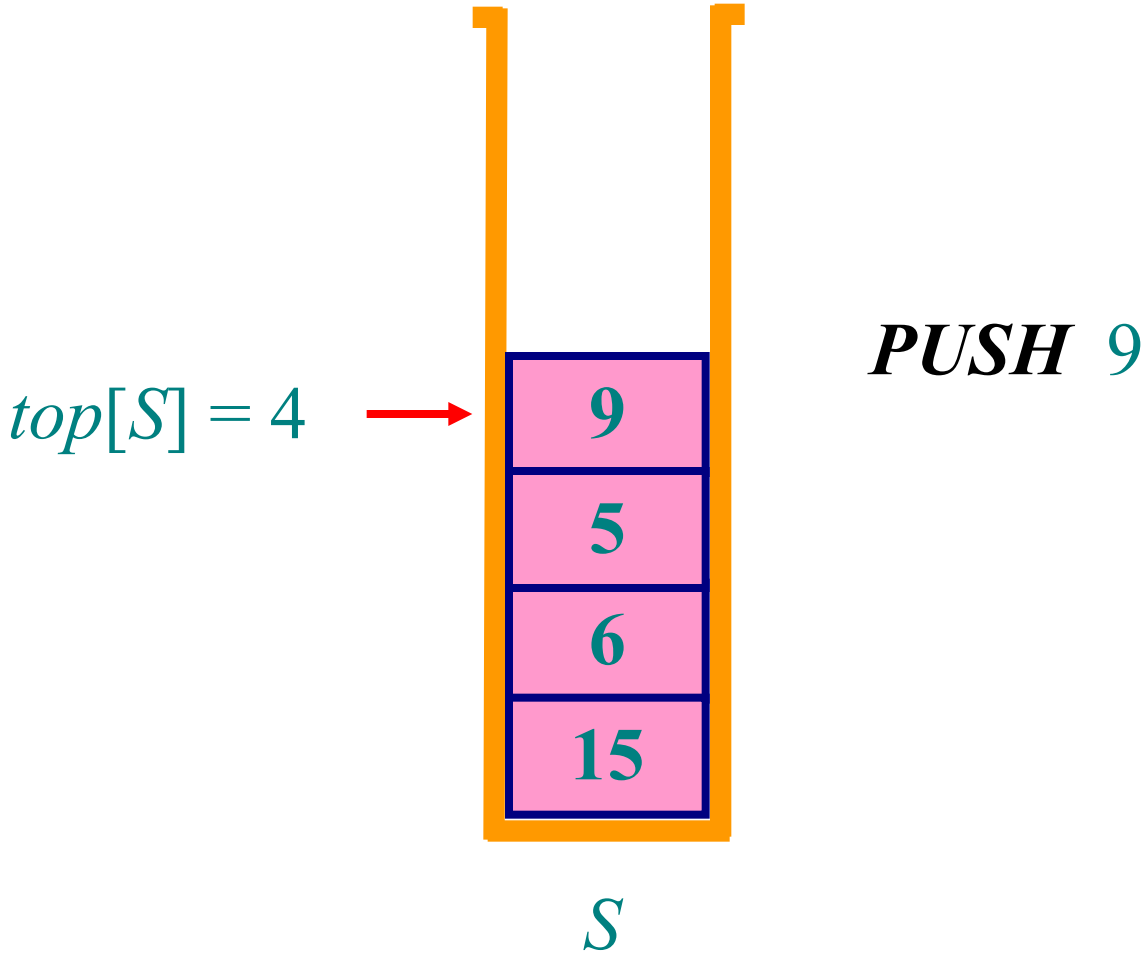
Stacks



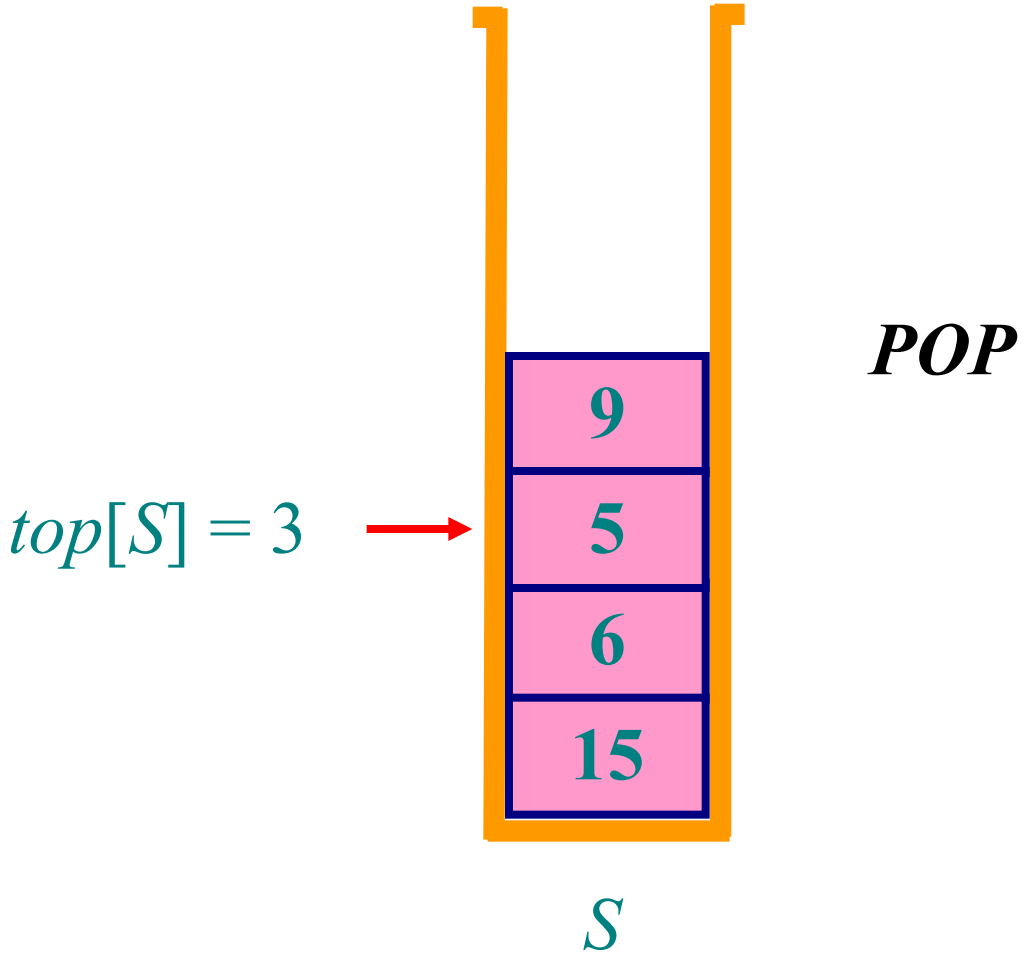
Stacks



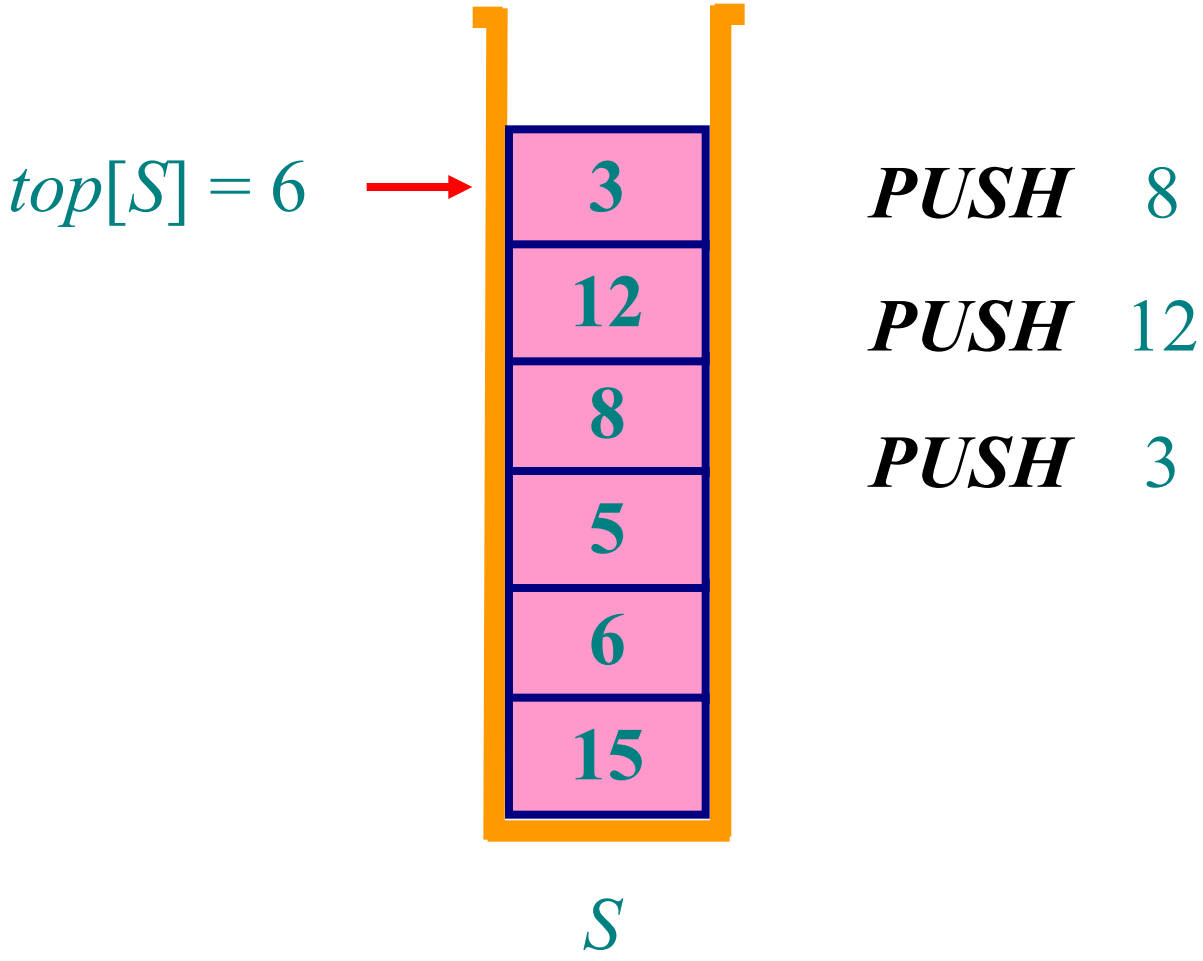
Stacks



Stacks



Stacks

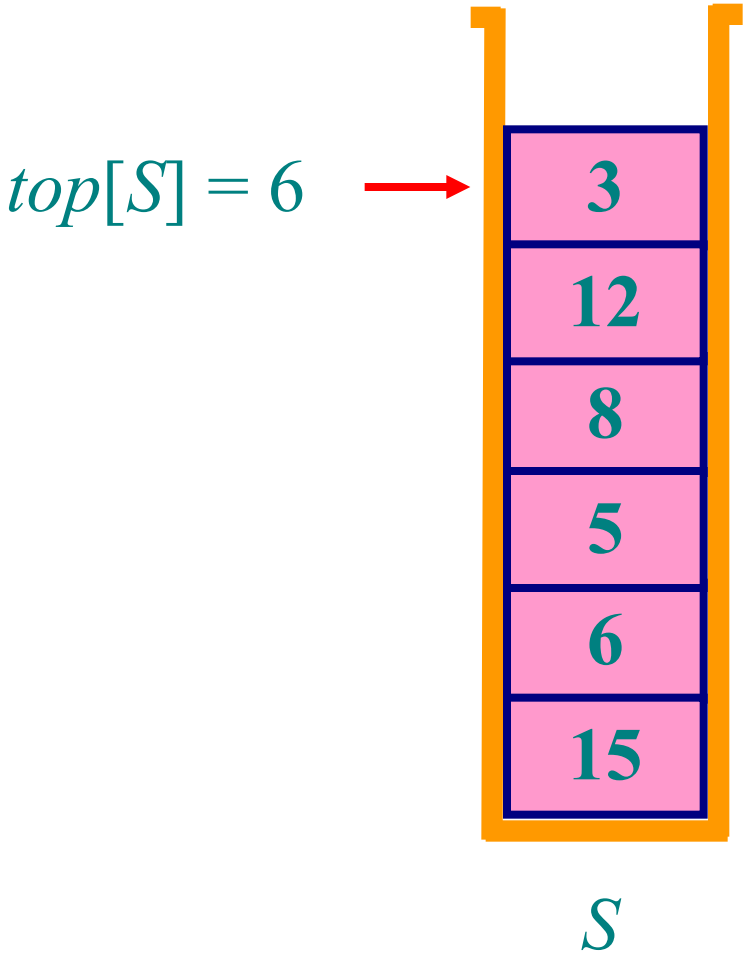


PUSH 8

PUSH 12

PUSH 3

Stacks



LIFO

last-in, first-out

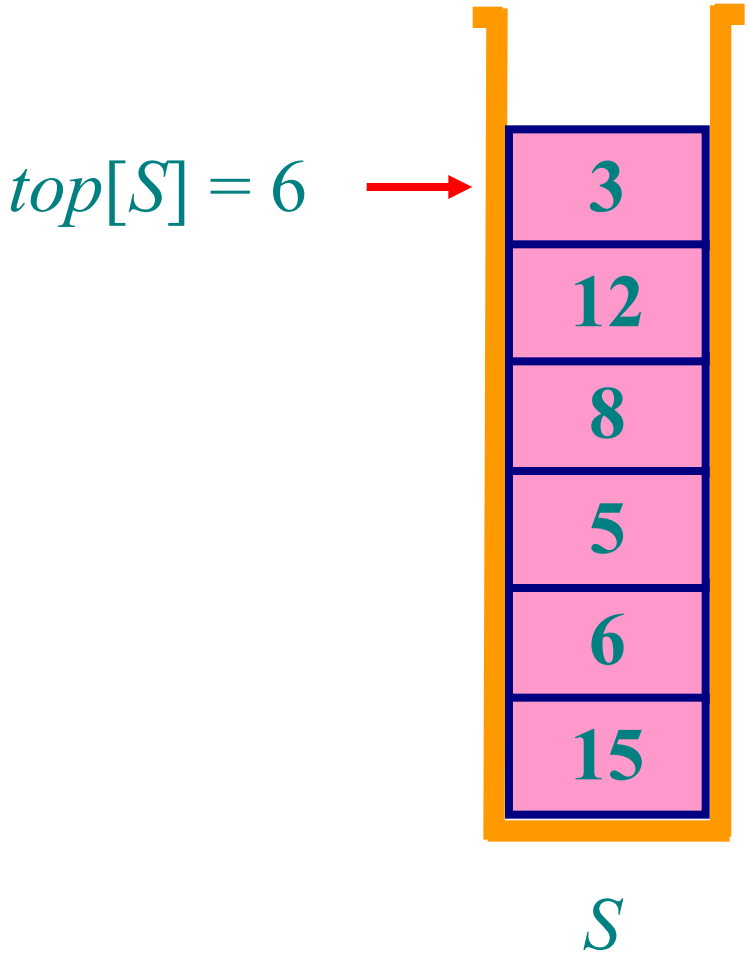
Underflows

If an empty stack is popped.

Overflows

If *top*[*S*] exceeds *n*.

Stacks

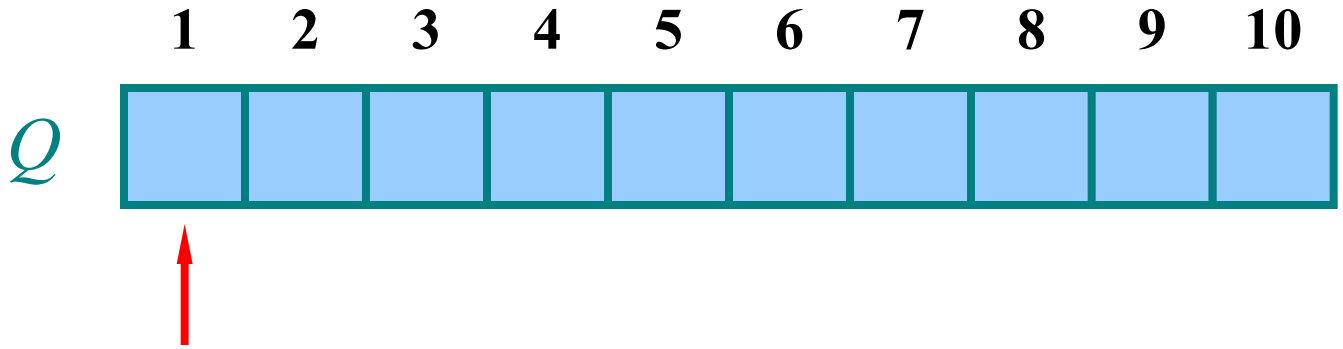


STACK-EMPTY(*S*)
1. **if** *top*[*S*] = 0
2. **then return** TRUE
3. **else return** FALSE

PUSH(*S*, *x*)
1. *top*[*S*] ← *top*[*S*] + 1
2. *S*[*top*[*S*]] ← *x*

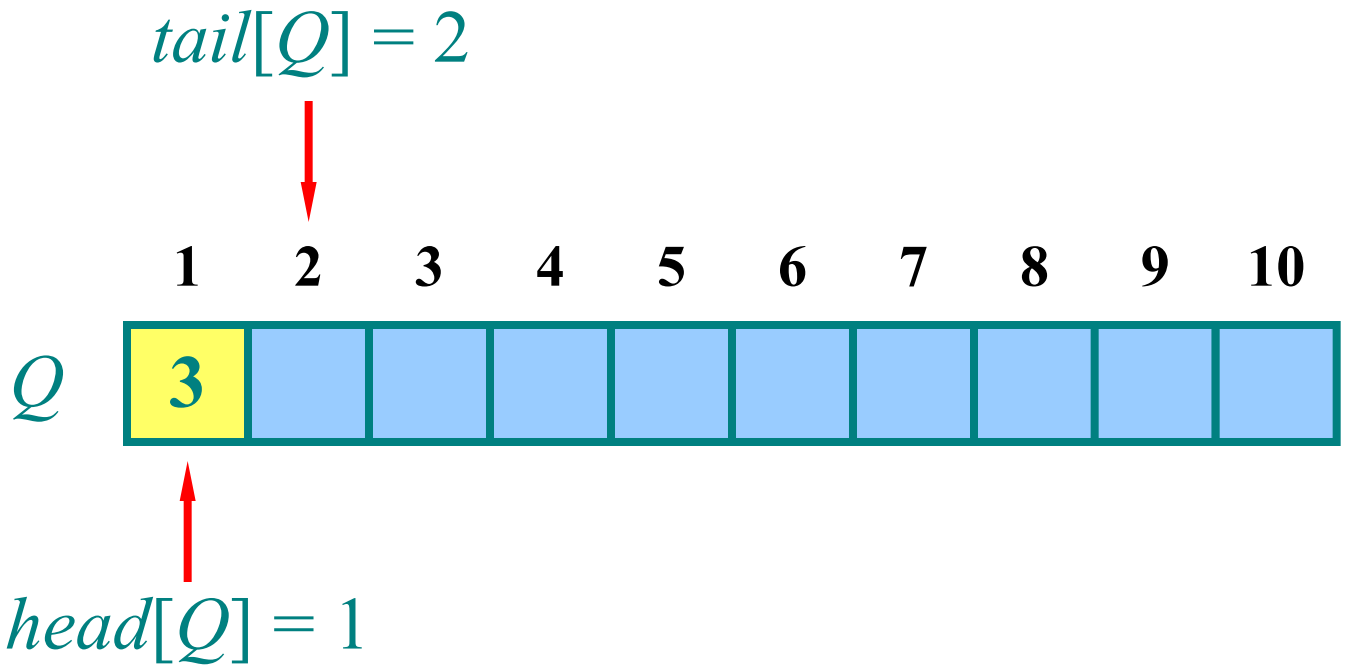
POP(*S*)
1. **if** STACK-EMPTY(*S*)
2. **then error** "underflow"
2. **else** *top*[*S*] ← *top*[*S*] − 1
4. **return** *S*[*top*[*S*] + 1]

Queues



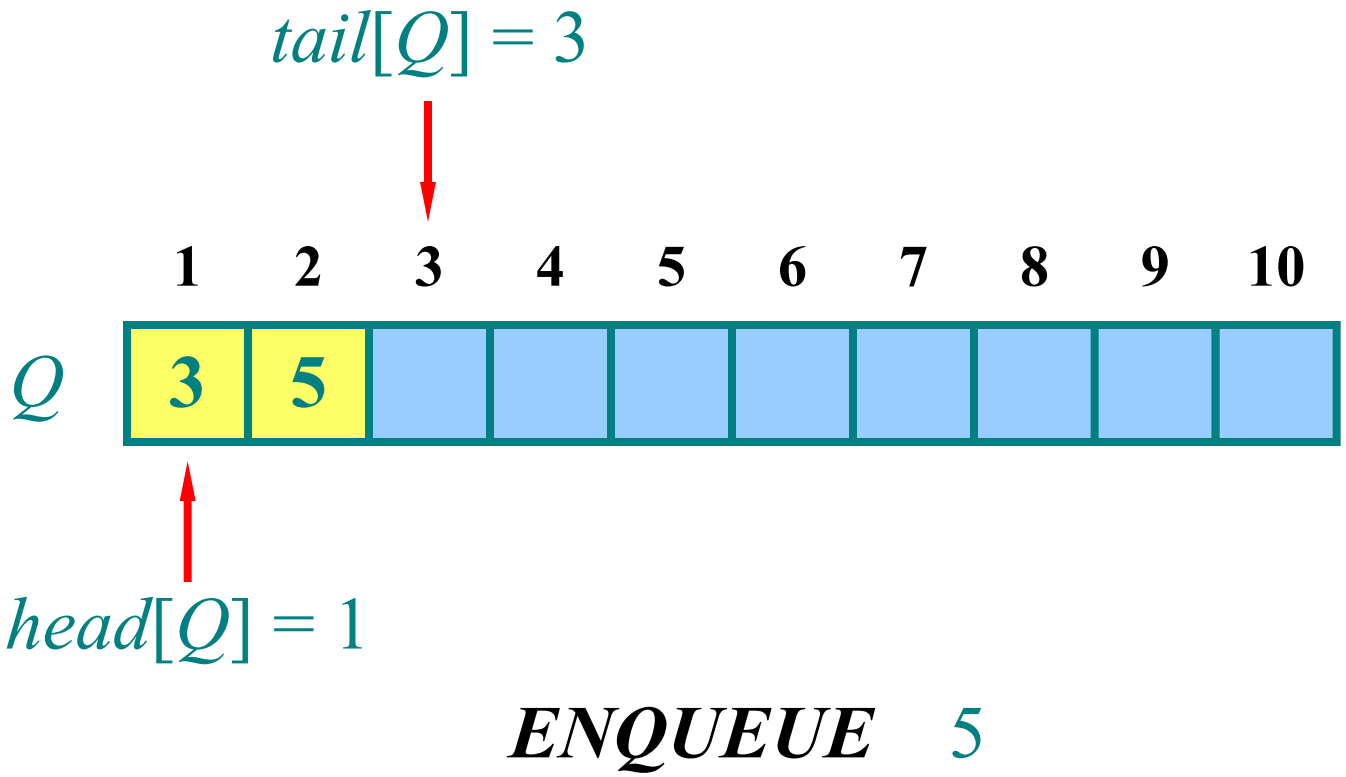
$head[Q] = tail[Q] = 1$

Queues



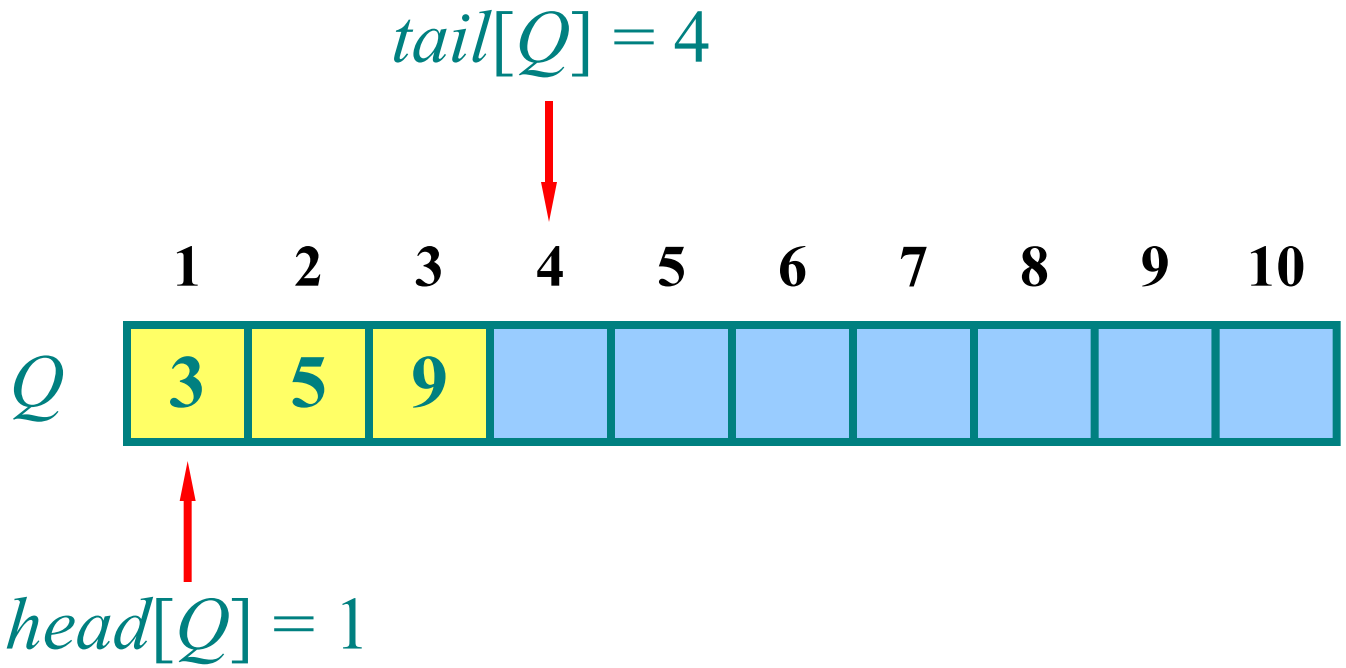
ENQUEUE 3

Queues



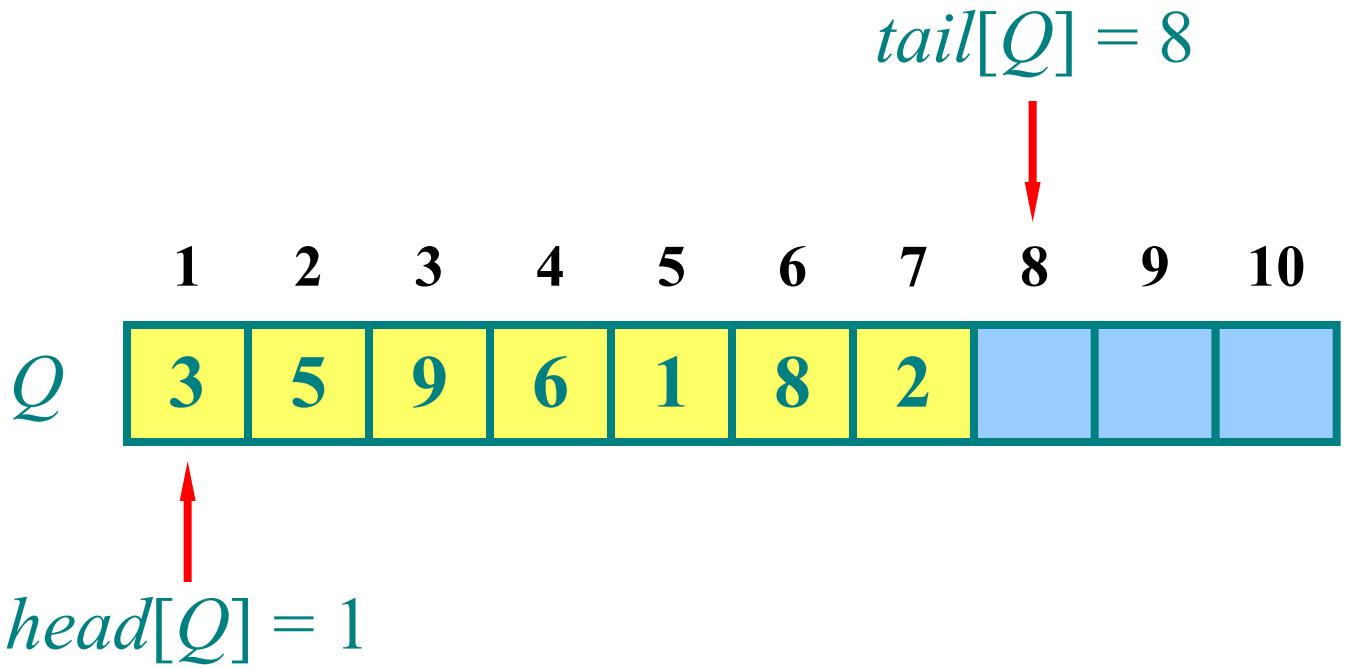
ENQUEUE 5

Queues



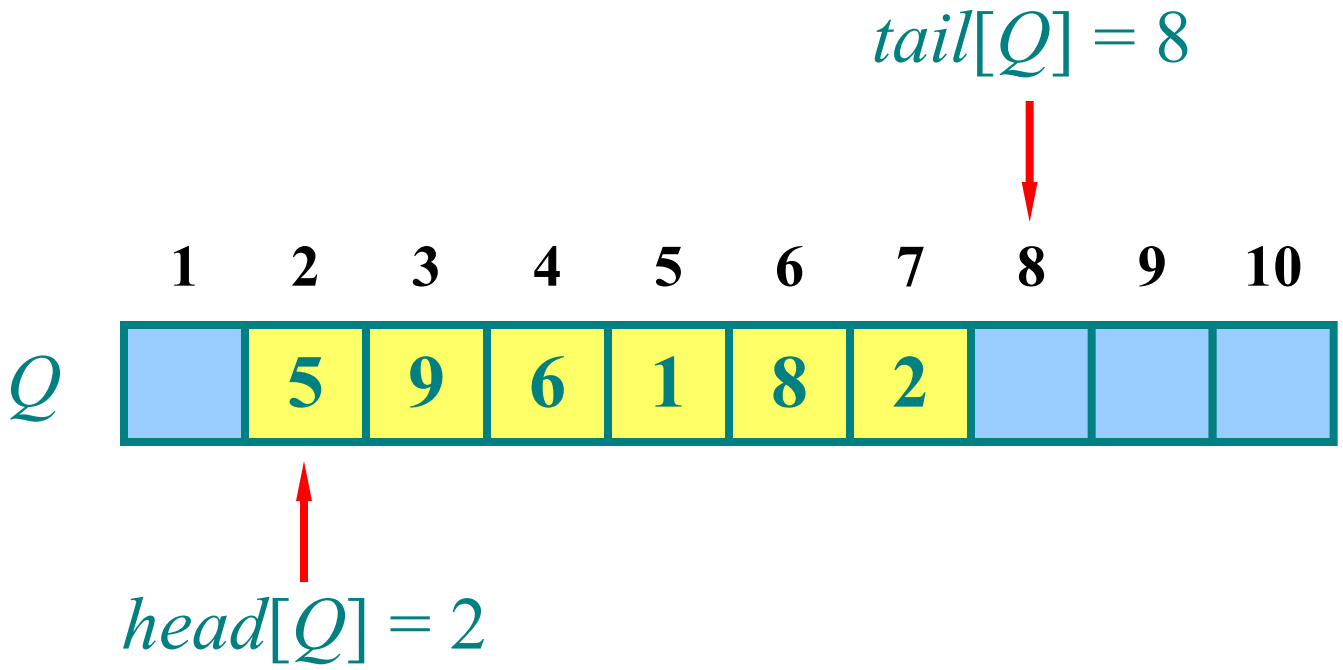
ENQUEUE 9

Queues



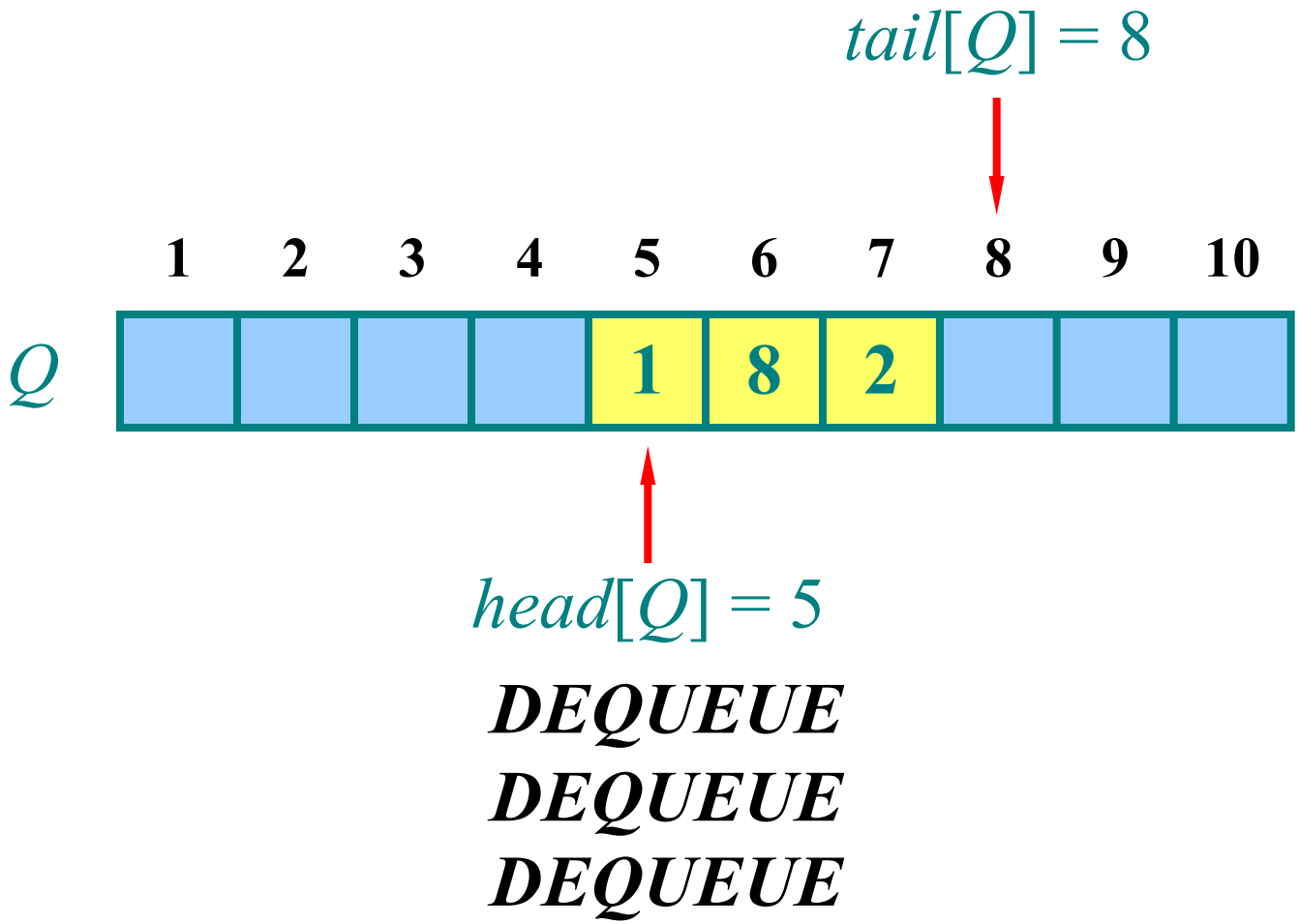
ENQUEUE 6
ENQUEUE 1
ENQUEUE 8
ENQUEUE 2

Queues

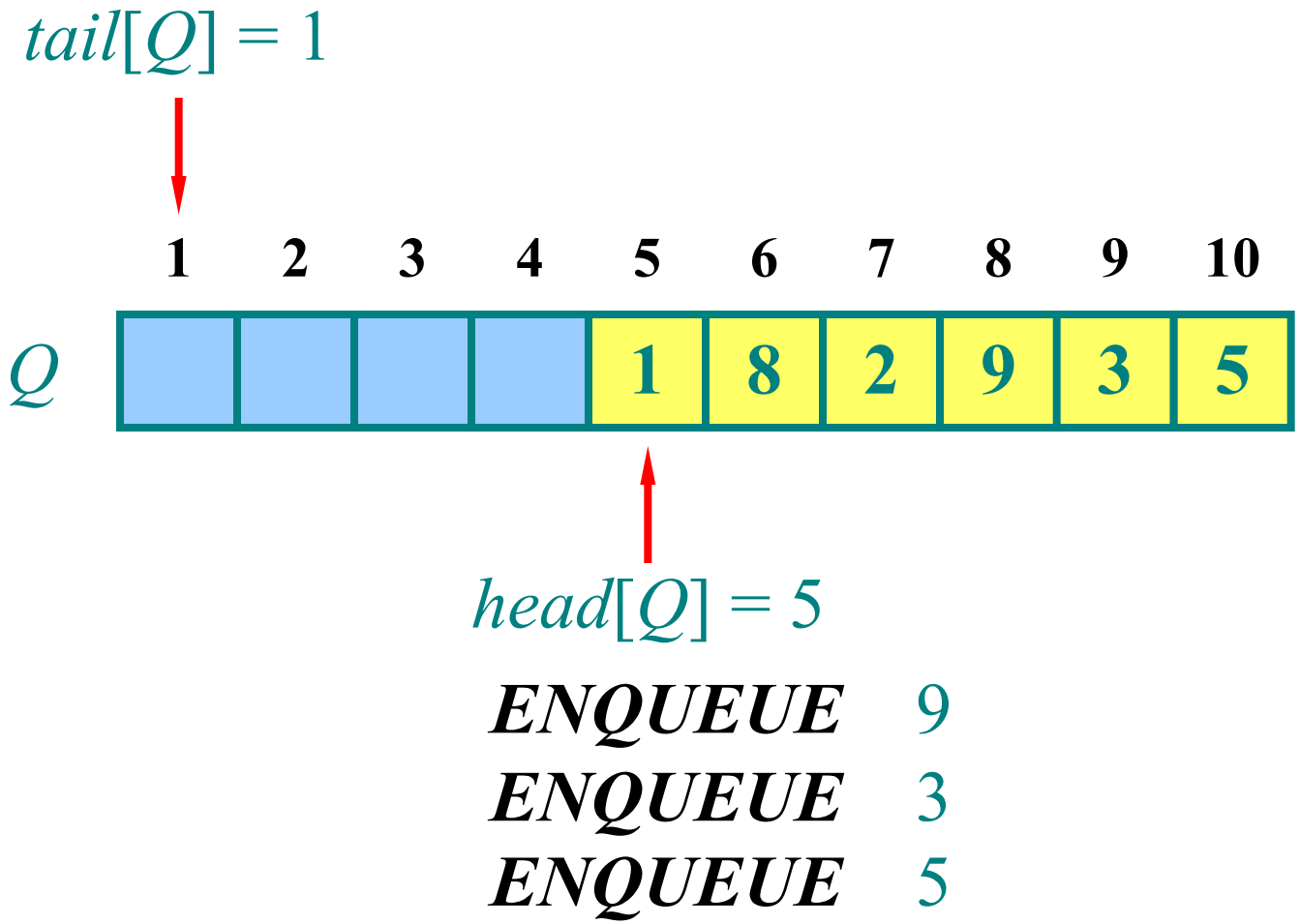


DEQUEUE

Queues

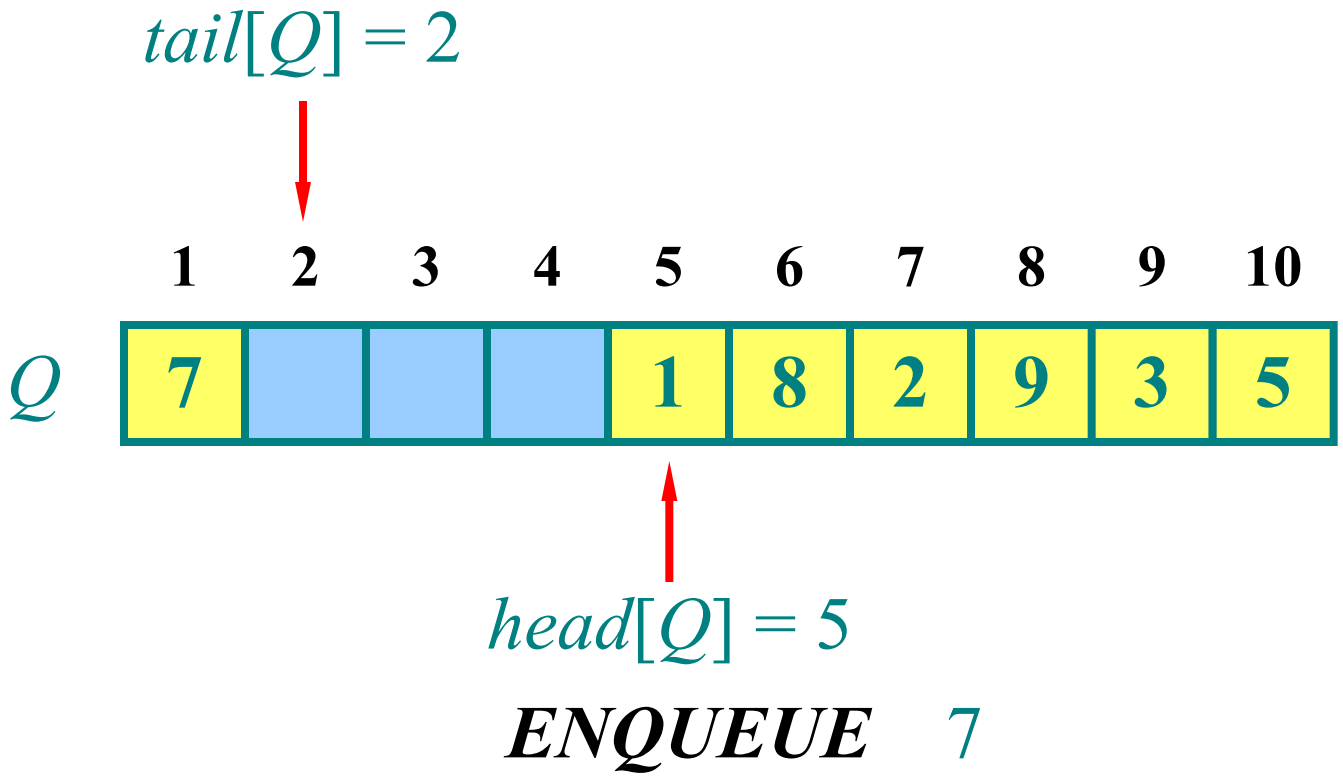


Queues

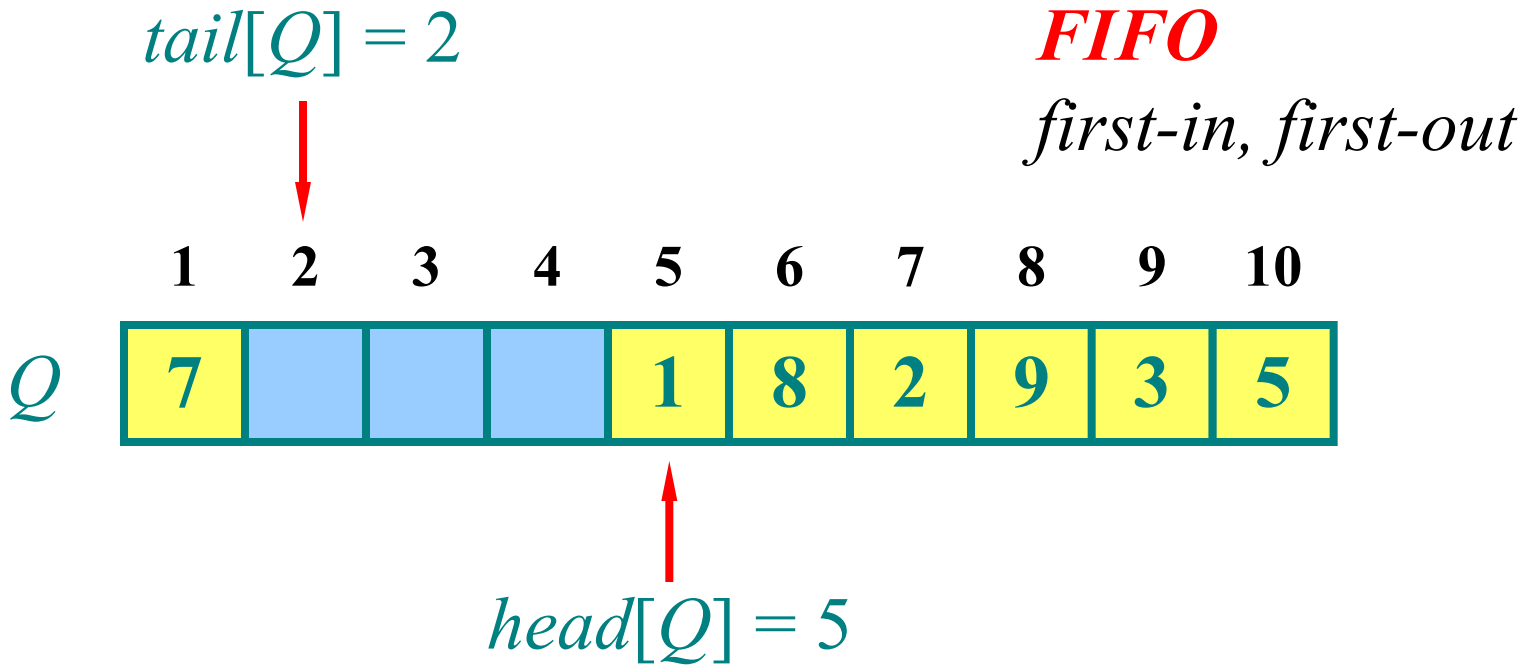


ENQUEUE 9
ENQUEUE 3
ENQUEUE 5

Queues



Queues



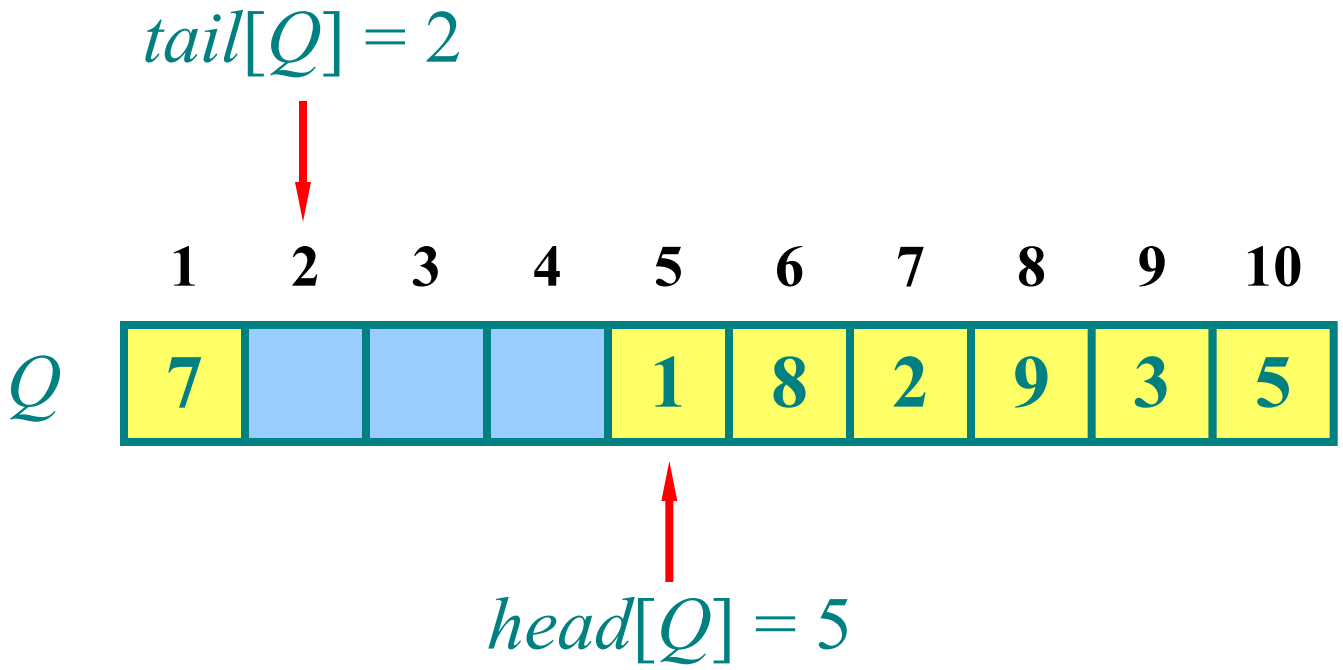
Empty

$head[Q] = tail[Q]$

Full

$head[Q] = tail[Q] + 1$

Queues



Underflow

When the queue is empty, an attempt to dequeue and element.

$head[Q] = tail[Q]$

Overflow

When the queue is full, an attempt to dequeue and element

$head[Q] = tail[Q] + 1$

Queues

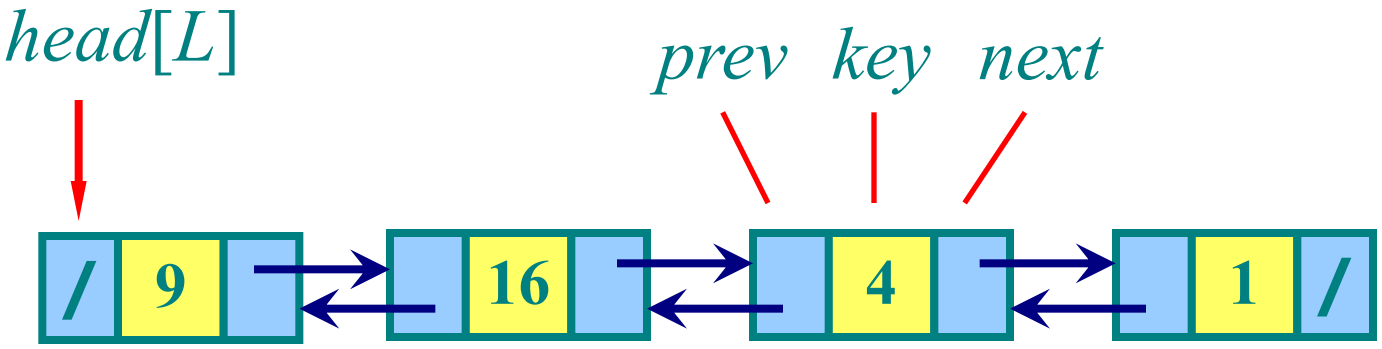
ENQUEUE(Q, x)

1. $Q[tail[Q]] \leftarrow x$
2. **if** $tail[Q] = length[Q]$
3. **then** $tail[Q] \leftarrow 1$
4. **else** $tail[Q] \leftarrow tail[Q] + 1$

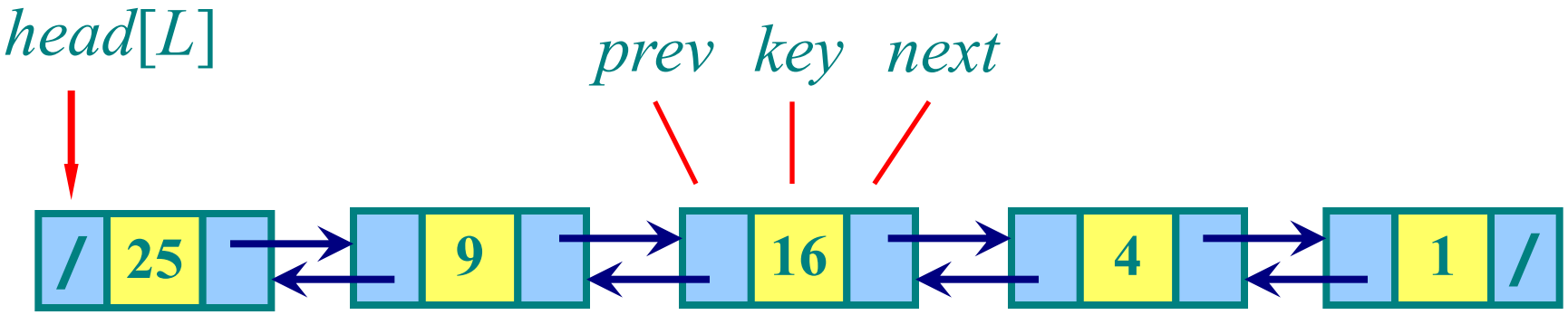
DEQUEUE(Q, x)

1. $x \leftarrow Q[head[Q]]$
2. **if** $head[Q] = length[Q]$
3. **then** $head[Q] \leftarrow 1$
4. **else** $head[Q] \leftarrow head[Q] + 1$
5. **return** x

Linked lists

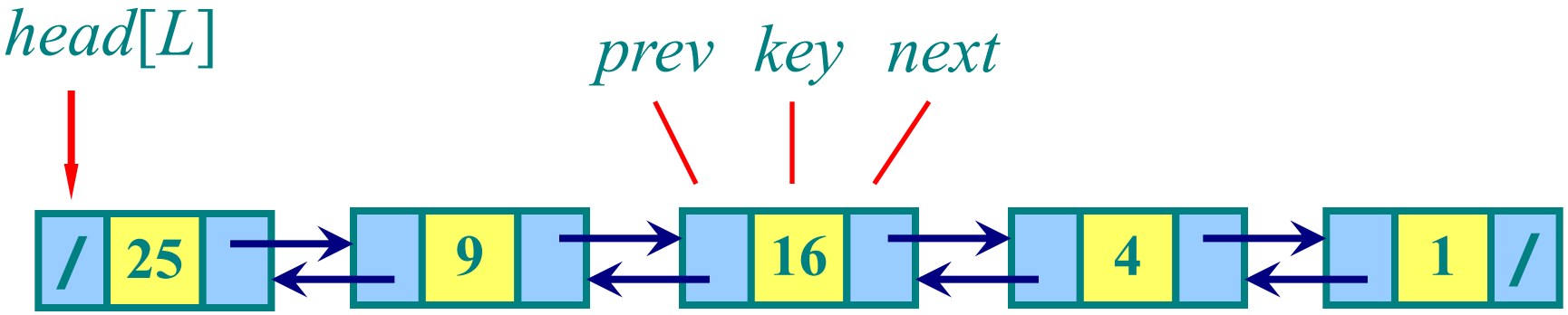


Linked lists



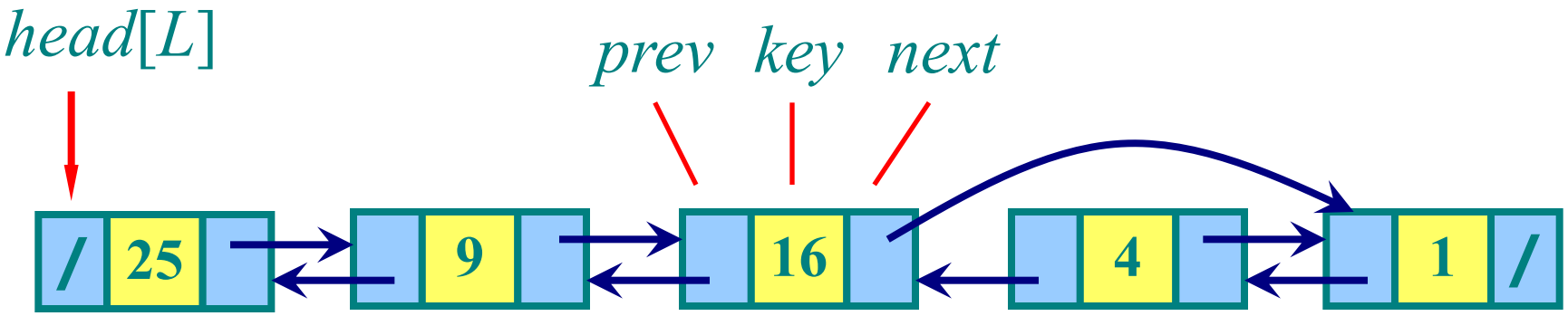
LIST-INSERT 25

Linked lists



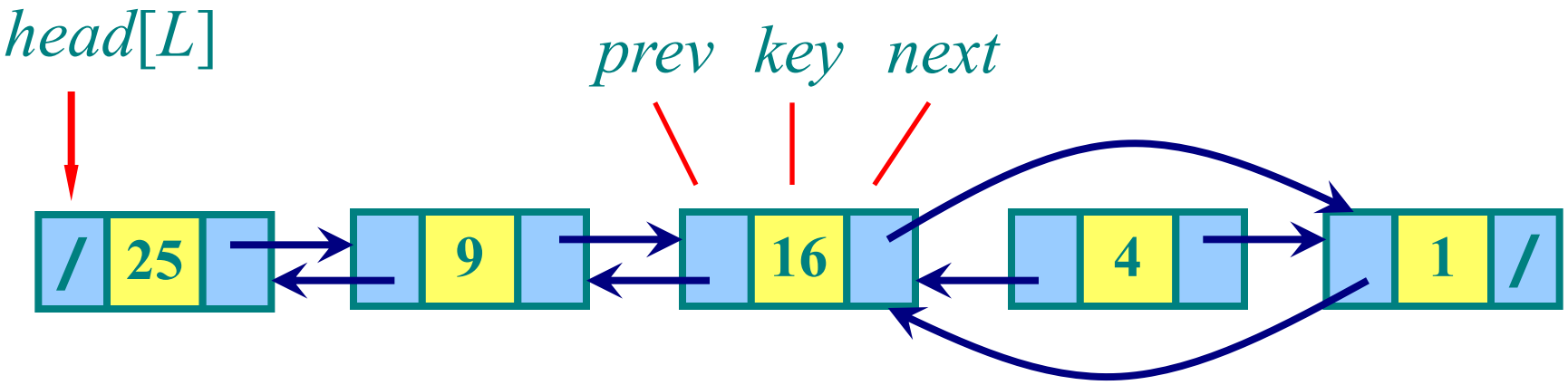
LIST-DELETE 4

Linked lists



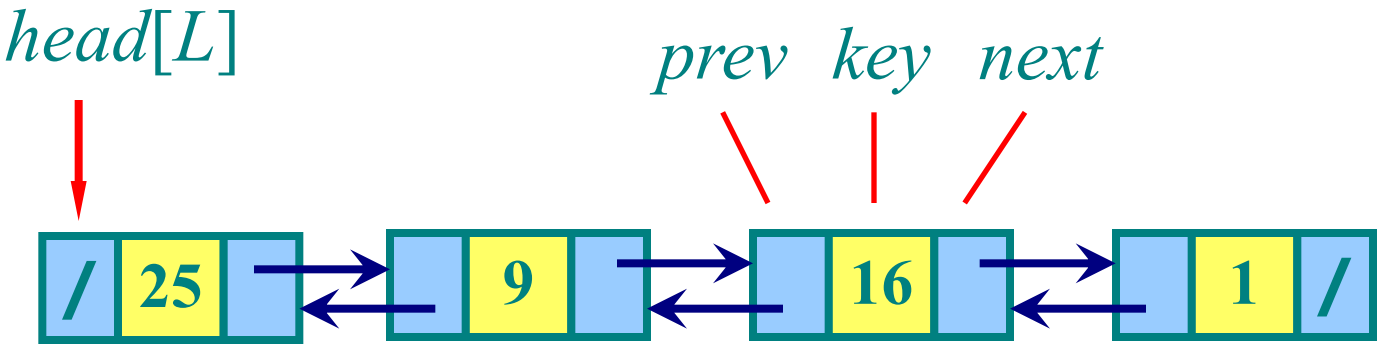
LIST-DELETE 4

Linked lists



LIST-DELETE 4

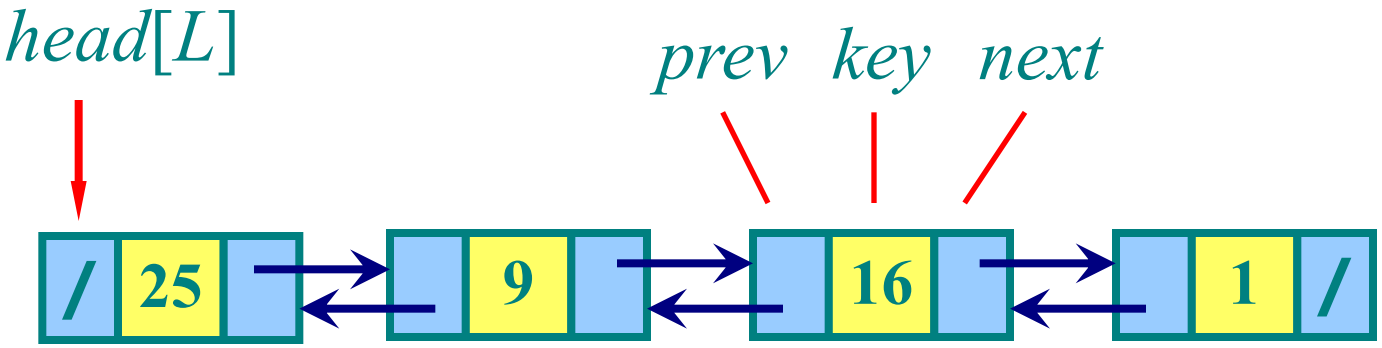
Linked lists



LIST-SEARCH(L, k)

1. $x \leftarrow head[L]$
2. **While** $x \neq$ NIL **and** $key[x] \neq k$
3. **do** $x \leftarrow next[x]$
4. **return** x

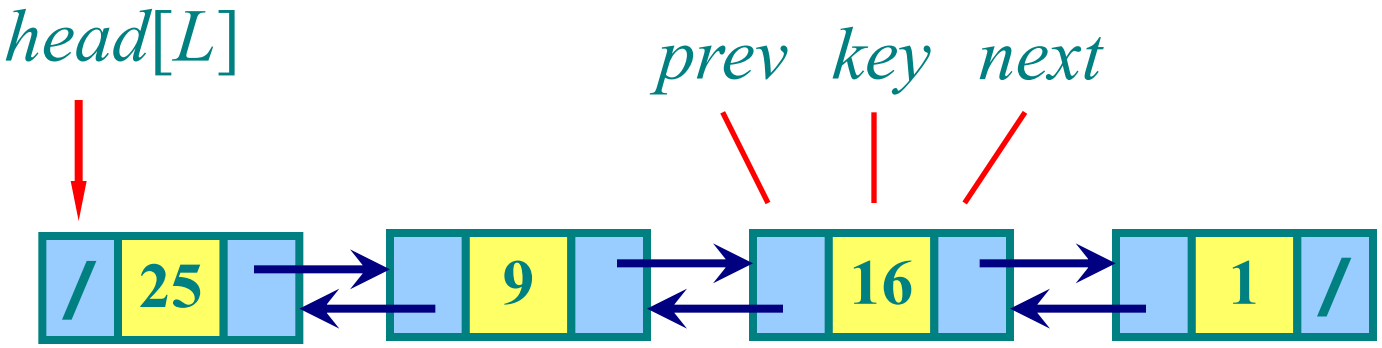
Linked lists



LIST-INSERT(L, x)

1. $next[x] \leftarrow head[L]$
2. **if** $head[L] \neq$ NIL
3. **then** $prev[head[L]] \leftarrow x$
4. $head[L] \leftarrow x$
5. $prev[x] \leftarrow$ NIL

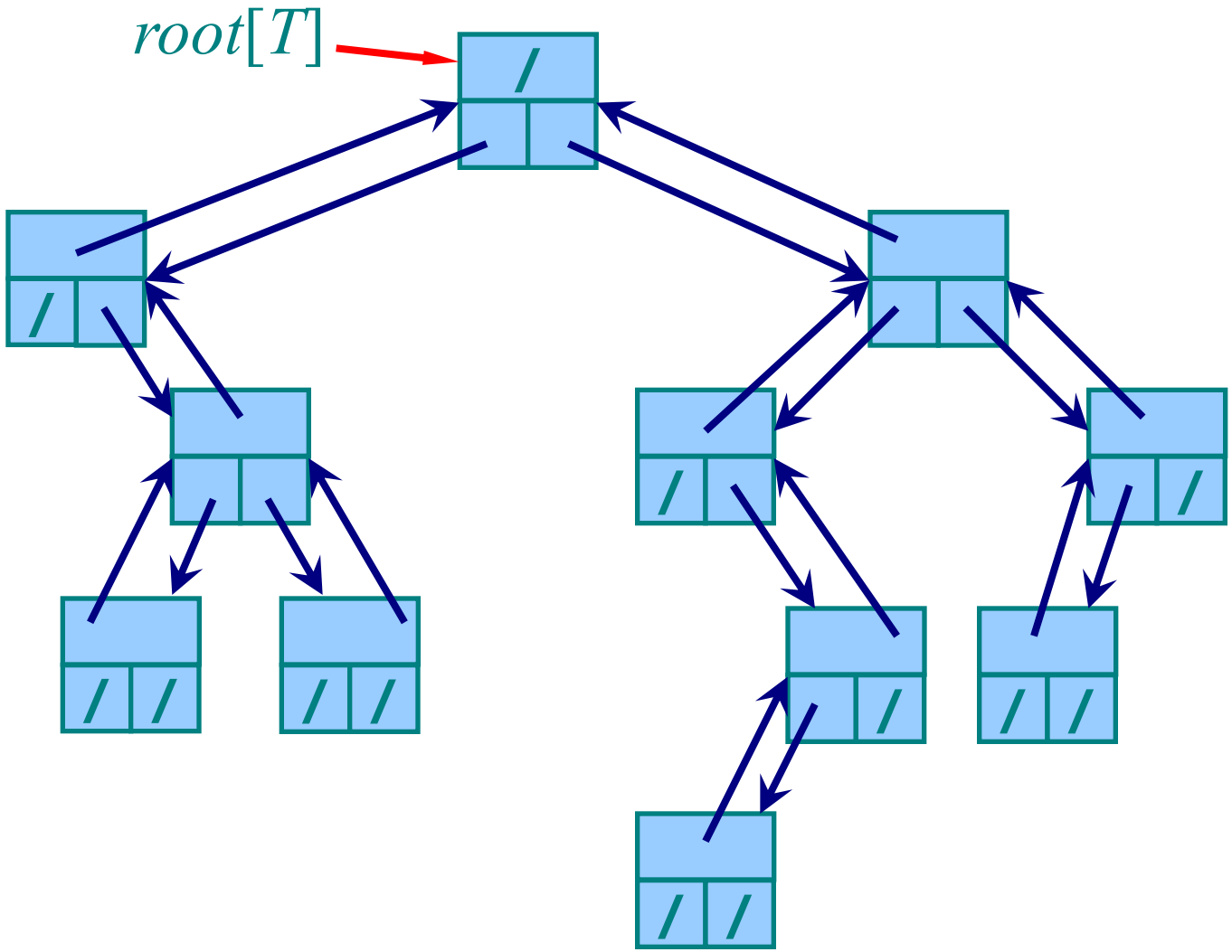
Linked lists



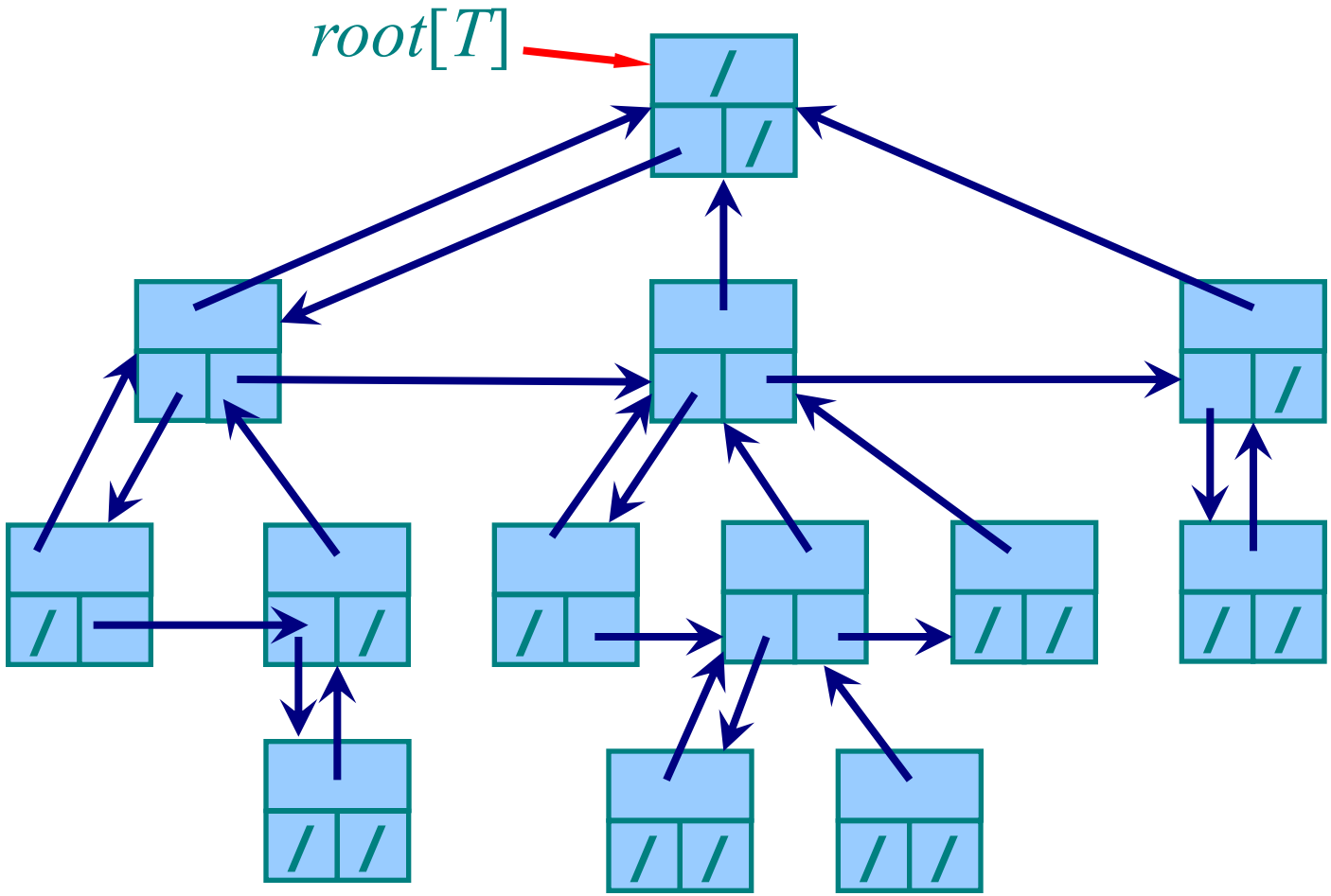
LIST-DELETE(L, x)

1. **if** $prev[x] \neq$ NIL
2. **then** $next[prev[x]] \leftarrow next[x]$
3. **else** $head[L] \leftarrow next[x]$
4. **if** $next[x] \neq$ NIL
5. **then** $prev[next[x]] \leftarrow prev[x]$

Rooted trees



Rooted trees



Postfix

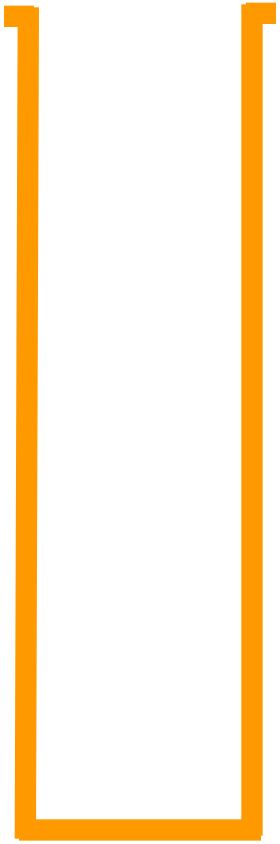
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Postfix (reverse Polish notation)

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Stack

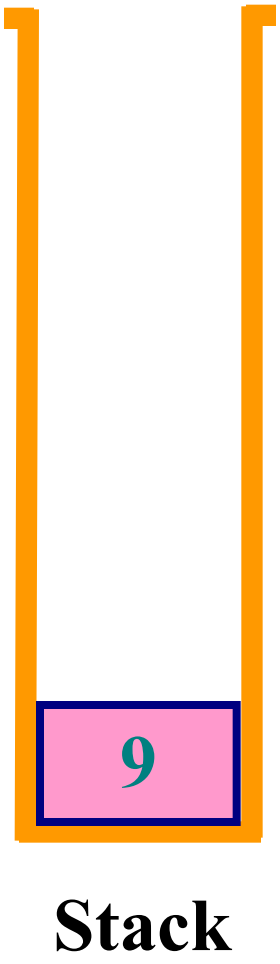
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



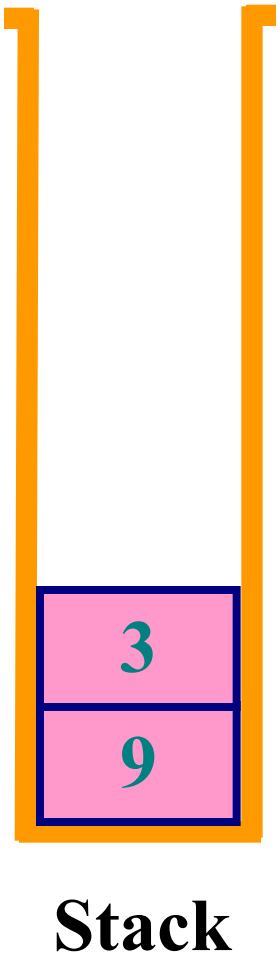
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



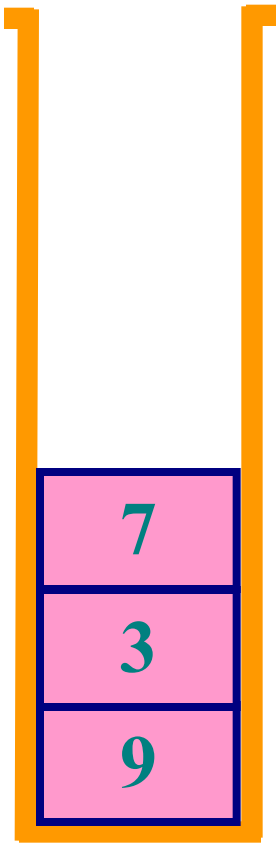
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Stack

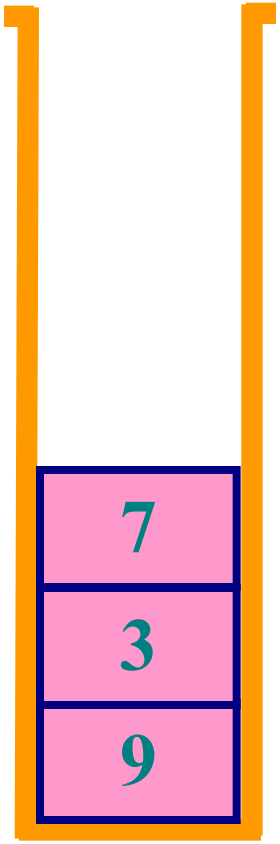
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Stack

Infix

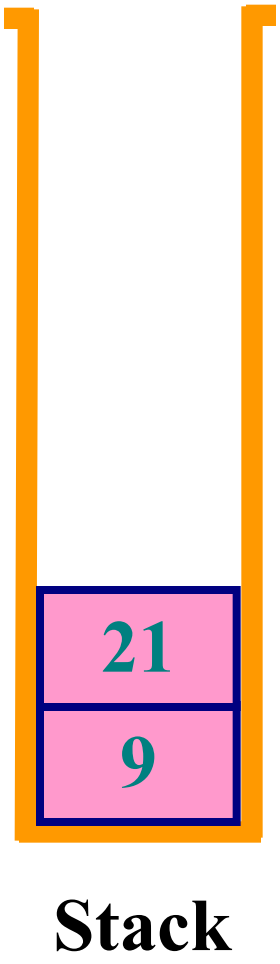
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

3 and 7 are popped and 3 * 7 = 21 is pushed

Postfix expression



Infix

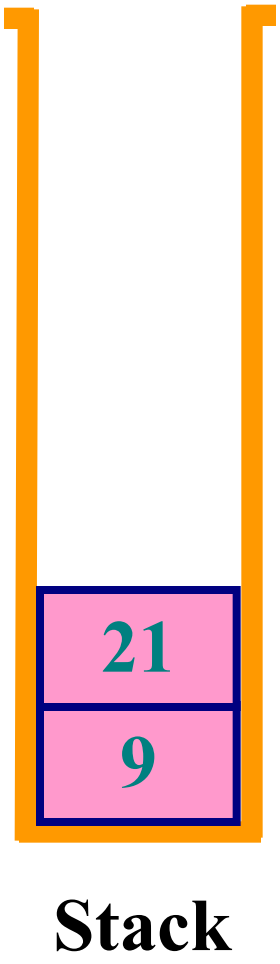
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

3 and 7 are popped and 3 * 7 = 21 is pushed

Postfix expression



Infix

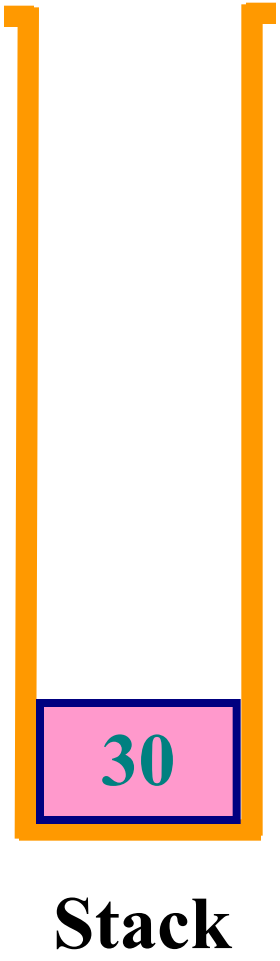
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

9 and 21 are popped and 21 + 9 = 30 is pushed

Postfix expression



Stack

Infix

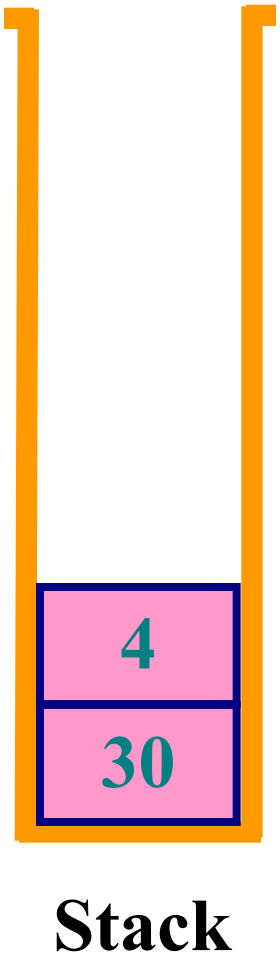
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

9 and 21 are popped and 21 + 9 = 30 is pushed

Postfix expression



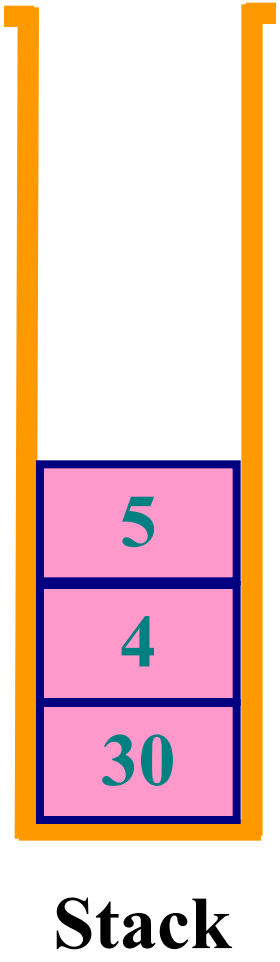
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



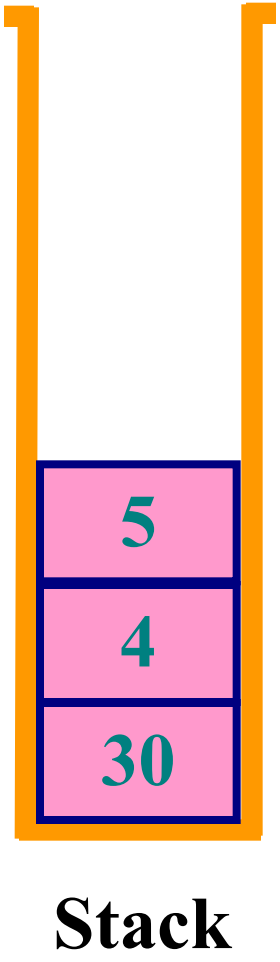
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Infix

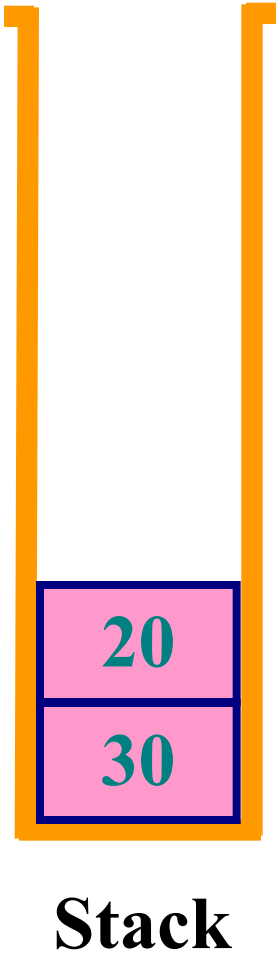
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

4 and 5 are popped and 4 * 5 = 20 is pushed

Postfix expression



Infix

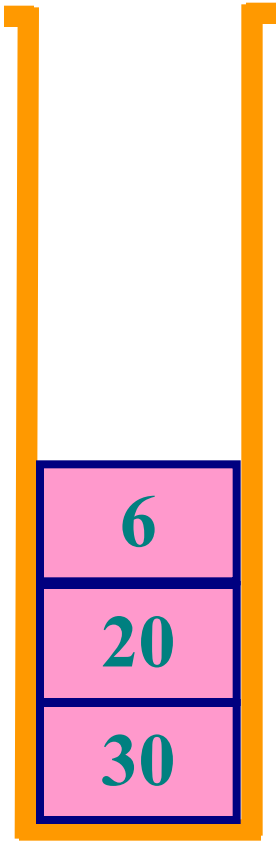
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

4 and 5 are popped and 4 * 5 = 20 is pushed

Postfix expression



Stack

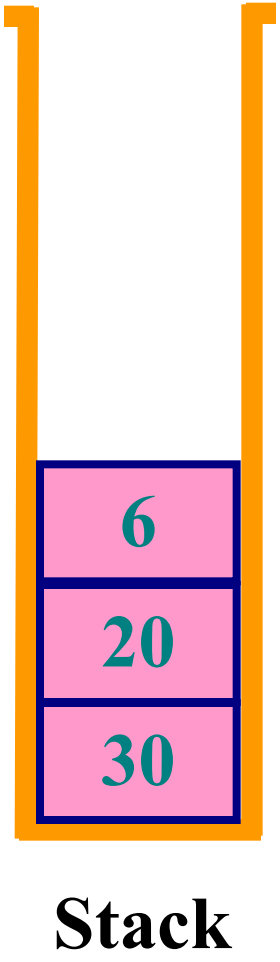
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Infix

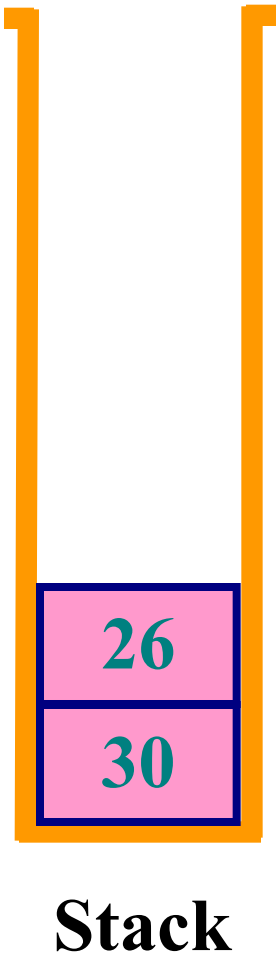
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

6 and 20 are popped and 6 + 20 = 26 is pushed

Postfix expression



Infix

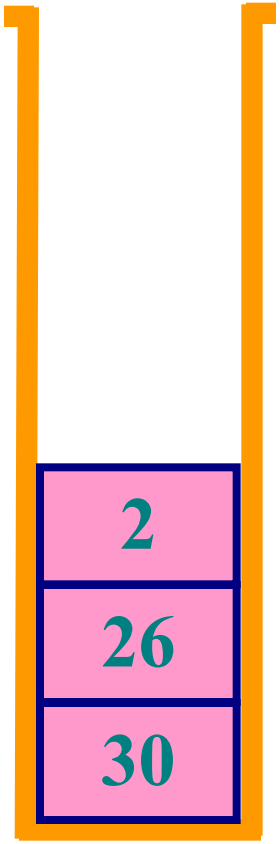
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

6 and 20 are popped and 6 + 20 = 26 is pushed

Postfix expression



Stack

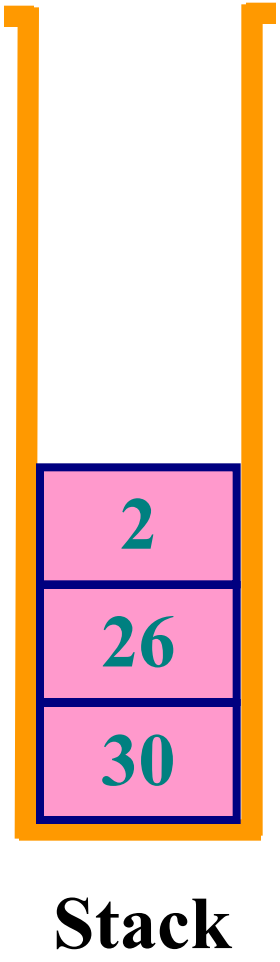
Infix

9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

Postfix expression



Infix

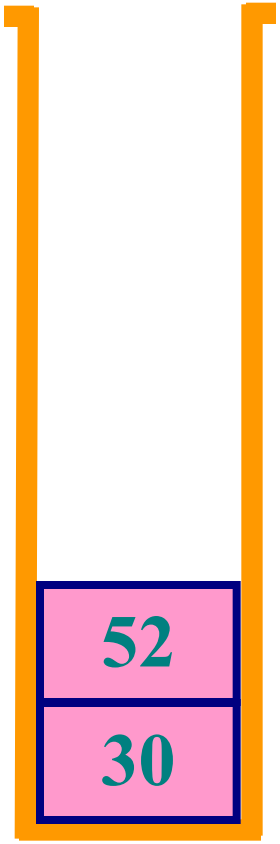
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

2 and 26 are popped and 2 * 26 = 52 is pushed

Postfix expression



Stack

Infix

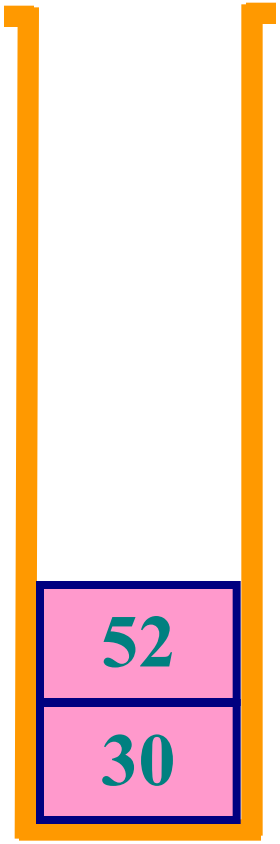
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

2 and 26 are popped and 2 * 26 = 52 is pushed

Postfix expression



Stack

Infix

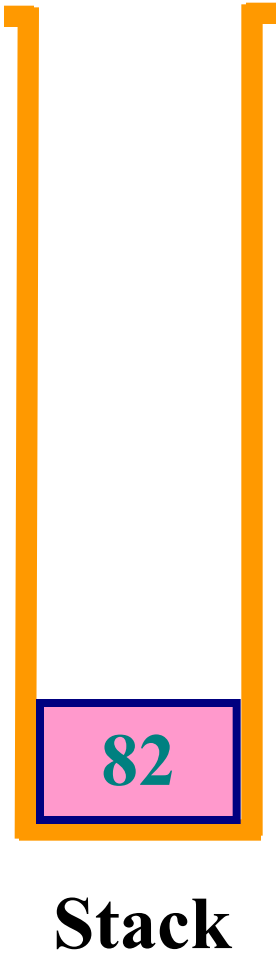
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

52 and 30 are popped and 52 + 30 = 82 is pushed

Postfix expression



Infix

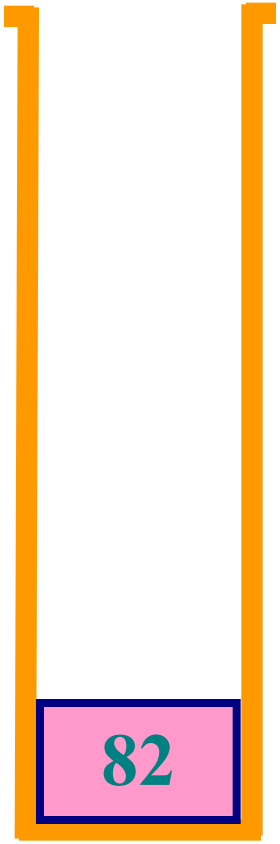
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

52 and 30 are popped and 52 + 30 = 82 is pushed

Postfix expression



Stack

Infix

done

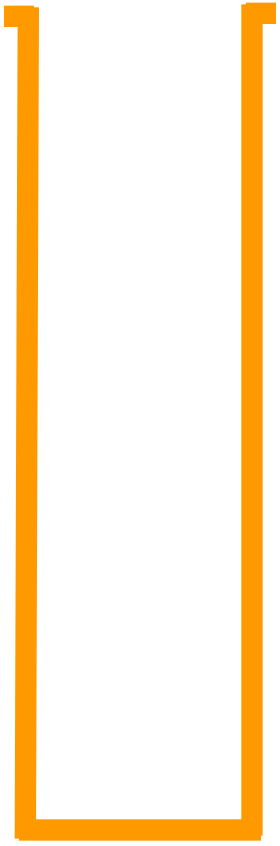
9 + 3 * 7 + (4 * 5 + 6) * 2

Input

9 3 7 * + 4 5 * 6 + 2 * +

82 are popped and we get the answer

Infix to postfix conversion



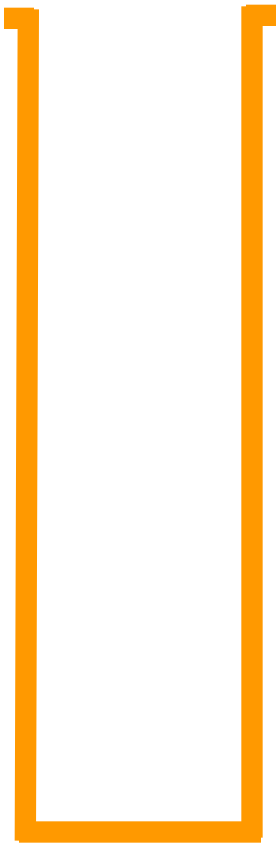
Stack

Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

Infix to postfix conversion



Stack

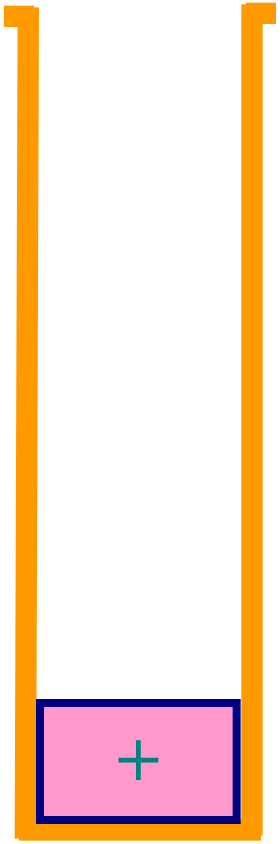
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9

Infix to postfix conversion



Stack

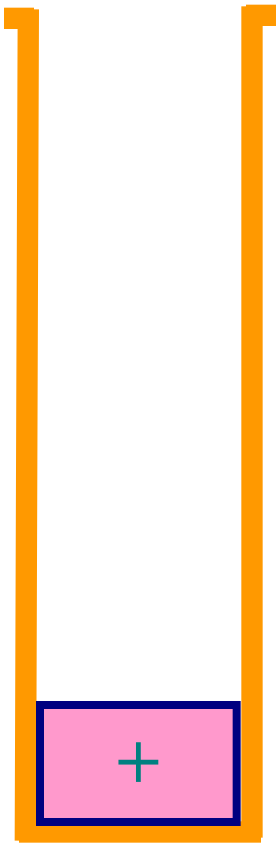
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9

Infix to postfix conversion



Stack

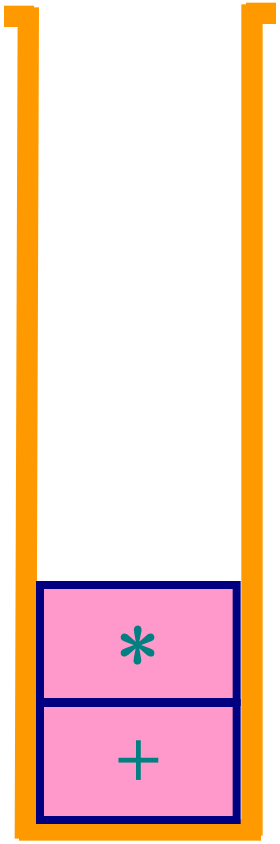
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3

Infix to postfix conversion



Stack

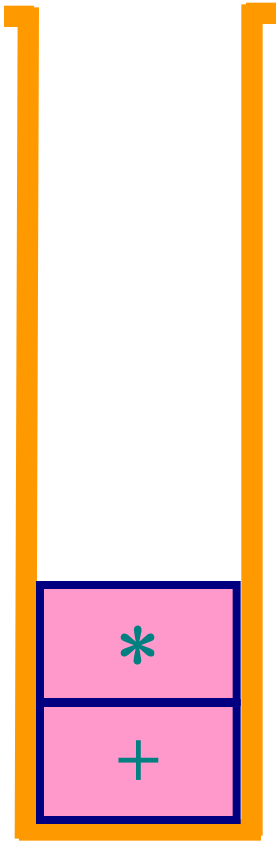
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3

Infix to postfix conversion



Stack

Input

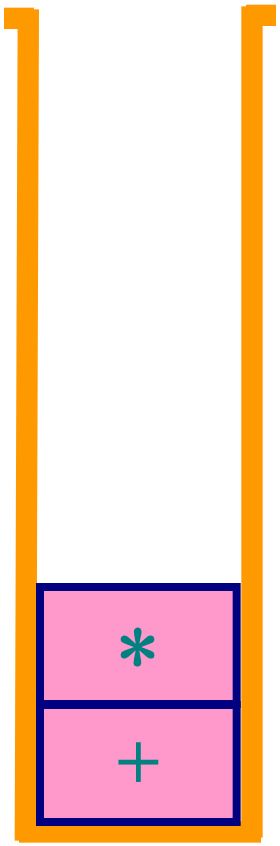
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7

Infix to postfix conversion

+ has lower priority than *



Stack

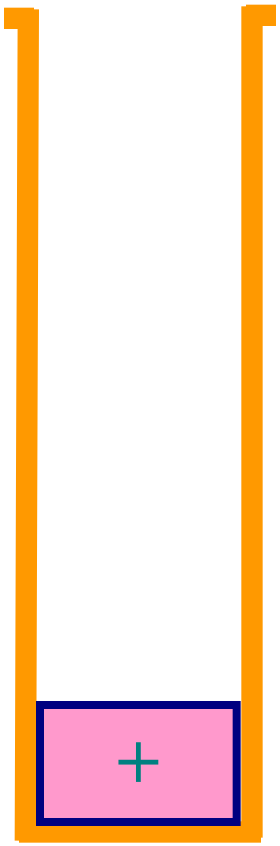
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 *

Infix to postfix conversion



Stack

Pop the other +

Input

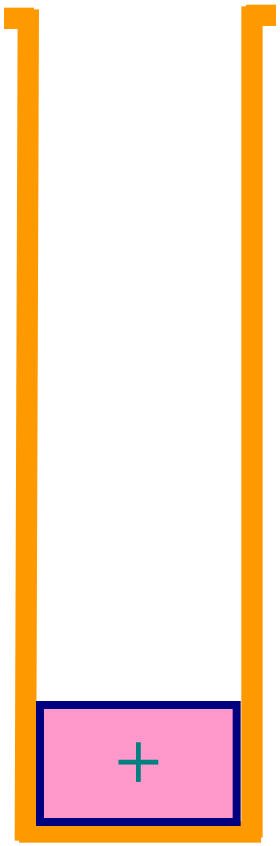
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * +

Infix to postfix conversion

Push the input + onto stack



Stack

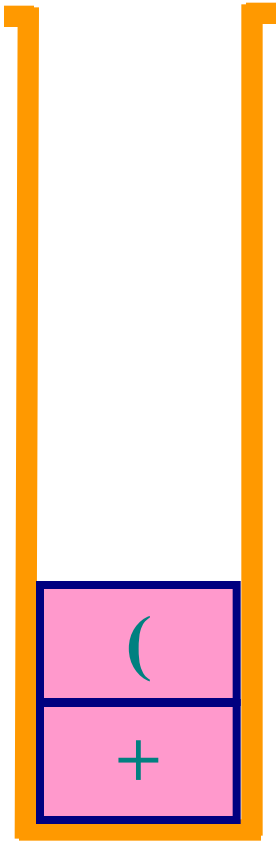
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * +

Infix to postfix conversion



Stack

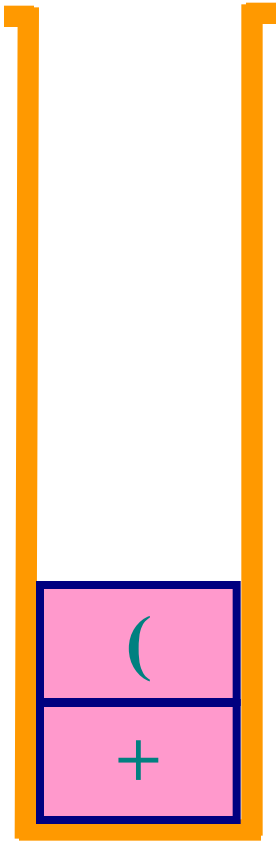
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * +

Infix to postfix conversion



Stack

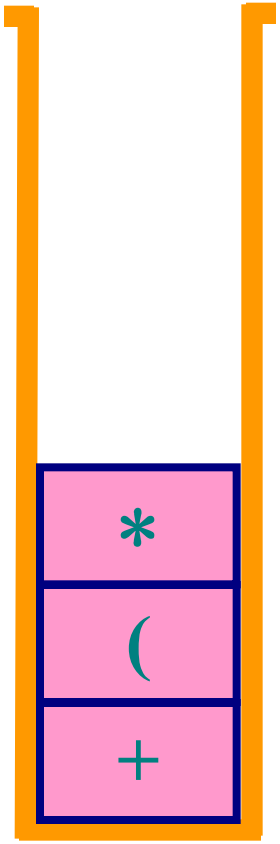
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4

Infix to postfix conversion



Stack

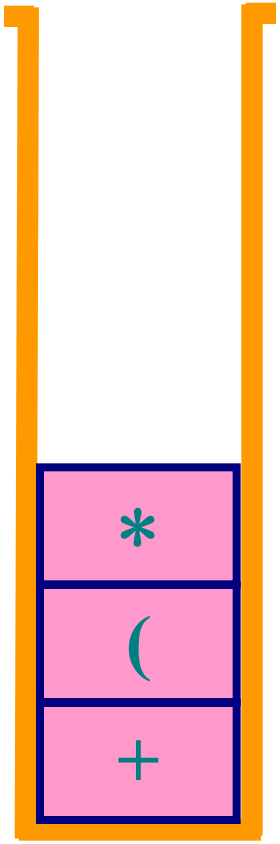
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4

Infix to postfix conversion



Stack

Input

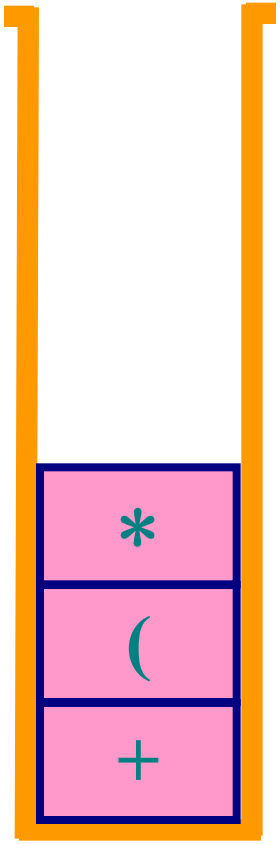
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5

Infix to postfix conversion

+ has lower priority than *



Stack

Input

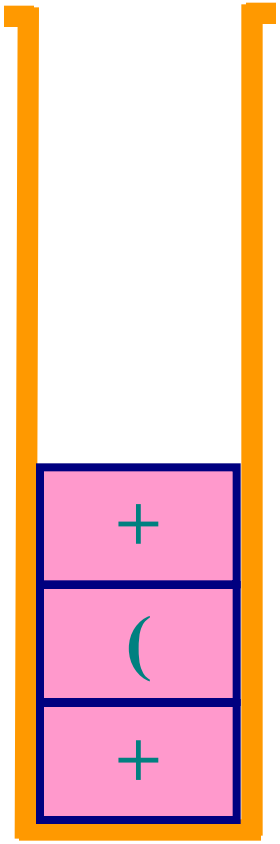
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 *

Infix to postfix conversion

Push the input + onto stack



Stack

Input

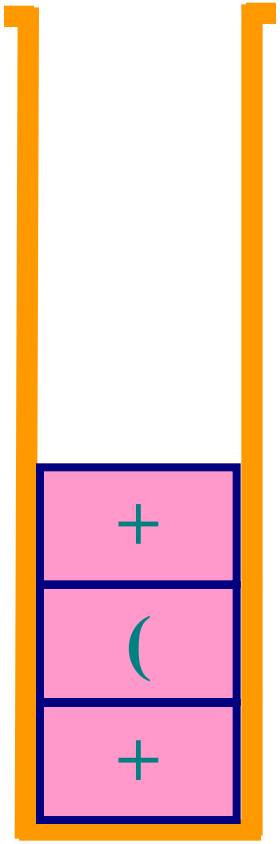
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 *

Infix to postfix conversion

Push the input + onto stack



Stack

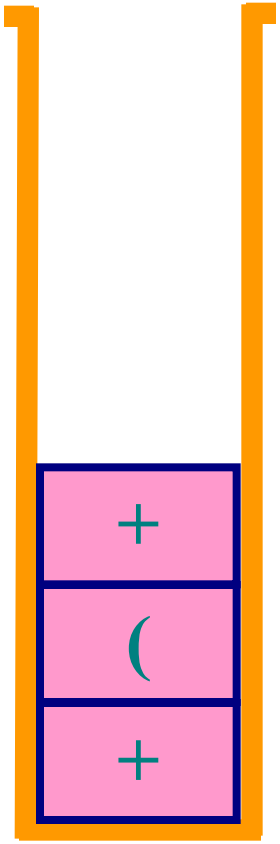
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 * 6

Infix to postfix conversion



Stack

Pop the stack until we encounter left parenthesis

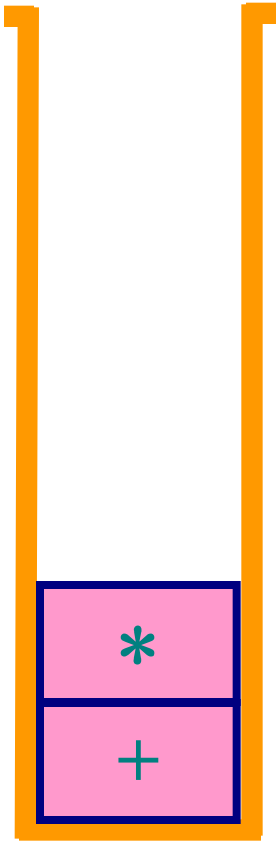
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 * 6 +

Infix to postfix conversion



Stack

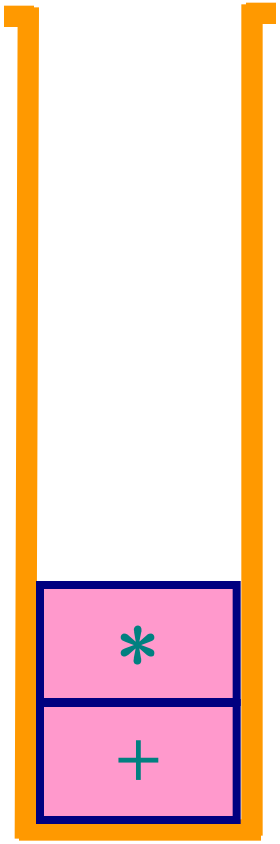
Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 * 6 +

Infix to postfix conversion



Stack

Input

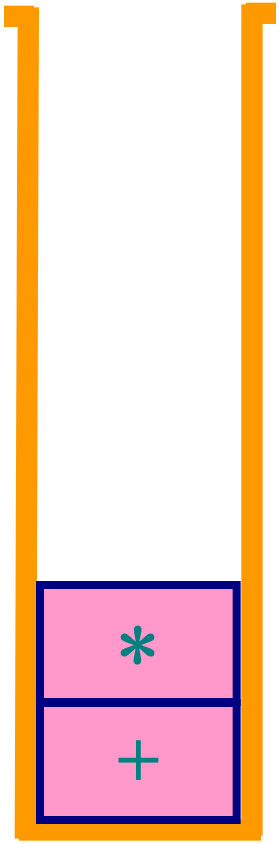
9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 * 6 + 2

Infix to postfix conversion

Pop the stack until it is empty



Stack

Input

9 + 3 * 7 + (4 * 5 + 6) * 2

Output

9 3 7 * + 4 5 * 6 + 2 * +

done

Conversion rules

- Operands are immediately placed onto the output and operators are placed onto the stack .
- When the operator on the top of the stack that has higher priority than the input operator, it is popped.
- If we see a right parenthesis, then we pop the stack, writing symbols until we encounter a left parenthesis.
- If we read the end of input, we pop the stack until it is empty.

Any questions?

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