Solve for Second Order

YSZ

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1 Solution

With θ' being the unlensed plane and θ being the lensed (observed plane). We have the equation

$$\theta' i \approx A_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k \tag{1}$$

We essentially wanna solve this green function-like quantity $\square$, for which

$$\theta_i = \square_{ij}\theta'_j \tag{2}$$

Ignore the D_{ijk} term, we solve for

$$\theta_{0i} = A^{-1}_{ij}\theta'_j \tag{3}$$

with $\theta = \theta_0 + \theta_1$, with θ_0 being the first order solution and θ_1 being a higher order correction. We expand the above equation

$$\theta'_i = A_{ij}(\theta_{0j} + \theta_{1j}) + \frac{1}{2}D_{ijk}(\theta_{0j} + \theta_{1j})(\theta_{0k} + \theta_{1k}) \tag{4}$$

$$= \theta'_i + A_{ij}\theta_{1j} + \frac{1}{2}D_{ijk}A^{-1}_{jl}\theta'_l A^{-1}_{km}\theta'_m + \tag{5}$$

$$\frac{1}{2}D_{ijk}A^{-1}_{jl}\theta'_l\theta_{1k} + \frac{1}{2}D_{ijk}\theta_{1j}A^{-1}_{km}\theta'_m + \mathcal{O}(\theta_1^2) \tag{6}$$

So we have solved (after relabeling dummy index)

$$-\frac{1}{2}D_{ijk}A^{-1}_{jl}\theta'_l A^{-1}_{km}\theta'_m = A_{in}\theta_{1n} + \frac{1}{2}D_{ijn}A^{-1}_{jl}\theta'_l\theta_{1n} + \frac{1}{2}D_{ink}A^{-1}_{km}\theta'_m\theta_{1n} \tag{7}$$

and hence

$$-\frac{1}{2}(A_{in} + \frac{1}{2}D_{ijn}A^{-1}_{jl}\theta'_l + \frac{1}{2}D_{ink}A^{-1}_{km}\theta'_m)^{-1}D_{ijk}A^{-1}_{jl}\theta'_l A^{-1}_{km}\theta'_m = \theta_{1n} \tag{8}$$