Abstract

These are notes on physics I learned throughout my junior year outside of normal classes.

Notes on Astroparticle Physics and Cosmology

Part I

Particles

1 Fermi's Golden Rule

Fermi's Golden Rule is used for calculating transition rates between particles. Assume there's a time-independent perterbation to the system, and thus to the Schrodinger equation

$$i\frac{d\psi}{dt} = \left[H + H'\right]\psi \tag{1}$$

Still expand $\psi = c_k \left|k\right\rangle e^{-iE_k t}$ in terms of (time dependent) energy eigenfunction will give us

$$i\sum_k \left[\frac{dc_k}{dt}\left|k\right\rangle e^{-iE_k t} - iE_k c_k \left|k\right\rangle e^{-iE_k t}\right] = \sum_k c_k H\left|k\right\rangle e^{-iE_k t} + \sum_k c_k H'\left|k\right\rangle e^{-iE_k t} \tag{2}$$

Use property of the imaginary numbers and the fact that we are operating under eigenstates, we can cancel the middle terms to get

$$i\sum_k \frac{dc_k}{dt}\left|k\right\rangle e^{-iE_k t} = \sum_k c_k H'\left|k\right\rangle e^{-iE_k t} \tag{3}$$

If we do a dot product with $\left\langle f\right|$, and assume only $c_i \approx 1$, others $c_k \approx 0$, we would get

$$i\frac{dc_f}{dt}e^{-iE_f t} = \left\langle f\middle|H'\middle|i\right\rangle e^{-iE_i t} \tag{4}$$

or

$$\frac{dc_f}{dt} = -i\left\langle f\middle|H'\middle|i\right\rangle e^{-i(E_i - E_f)t} \tag{5}$$

For simplicity, we just call $\left\langle f\middle|H'\middle|i\right\rangle := T_{fi}$

The whole coefficient is $\int_0^T -i\left\langle f\middle|H'\middle|i\right\rangle e^{-i(E_i-E_f)t}dt$, and the probability is just

$$P_{fi} = |T_{fi}|^2 \int_0^T \int_0^T e^{-i(E_i-E_f)t} e^{i(E_i-E_f)t'} dt dt'$$

Note this is essentially $c_k c_k^*$, and we can relabel the dummy variable in integration so it won't change as the other changes.

We define the transition rate as the the probability to transit within sometime, so it is

$$d\Gamma_{fi} = P_{fi}/T \tag{6}$$

Moreover, we can simpify this a bit with $\int_{\infty}^{\infty} e^{i(k-k_0)} = 2\pi\delta(k-k_0)$,

$$d\Gamma_{fi} = 2\pi \frac{|T_{fi}|^2}{T} \lim_{T\to\infty} \int_{T/2}^{T/2} e^{-i(E_i-E_f)t}\delta(E_i - E_f)dt \tag{7}$$

The dirac delta essentially specify that $E_i \approx E_f$. Another thing we can do is to assume multiple E_f are possible going from E_i. So there are dn states in $E_f \to E_f + dE_f$. If n is the total number of states, then $\frac{dn}{dE_f}$ symbolizes how many states where added going through $E_f \to E_f + dE_f$. So it is an appropriate value for density of states.

So the above becomes

$$d\Gamma_{fi} = 2\pi \int_{E_f}^{E_f+dE_f} \frac{dn}{dE_f} \frac{|T_{fi}|^2}{T} \lim_{T\to\infty} \int_{T/2}^{T/2} e^{-i(E_i-E_f)t}\delta(E_i - E_f)dtdE_f \tag{8}$$

When we evaluate the integral by plugging in $E_f = E_i$, the above just becomes, with the integral over $T/2$ to $-T/2$ cancel with T to give 1, use dE_f to cancel with 1, the change of E_f to $E_f + dE_f$ combines density of states and the dirac deltat to give $\frac{dn}{dE_f}|_{E_i}$ $\Gamma_{fi} = 2\pi|T_{fi}|^2 \frac{dn}{dE_f}|_{E_i}$

Upgrading a little bit, we can plug in $c_f = -i \langle f|H'|i\rangle \int_0^T e^{-i(E_i-E_f)t}dt$ This makes (3) then (after dot product with $\langle f|$)

$$\frac{dc_f}{dt} = -i \langle f|H'|i\rangle e^{i(E_f-E_i)t} + (-i)^2 \sum_{k\neq i} \langle f|H'|k\rangle e^{i(E_f-E_k)} \int_0^t (\langle k|H'|i\rangle)e^{i(E_k-E_i)t}dt \tag{9}$$

Notice we switched expression for coefficient from f to k. This, after doing explicity integration assuming time-independent perturbation, we have, to **second order**,

$$\frac{dc_f}{dt} = -i \langle f|H'|i\rangle e^{i(E_f-E_i)t} + (-i)^2 \sum_{k\neq i} \langle f|H'|k\rangle e^{i(E_f-E_k)} \frac{\langle k|H'|i\rangle e^{i(E_k-E_i)t}}{i(E_k - E_i)} \tag{10}$$

$$= -i \langle f|H'|i\rangle e^{i(E_f-E_i)t} + (-i) \sum_{k\neq i} \langle f|H'|k\rangle e^{i(E_f-E_i)} \frac{\langle k|H'|i\rangle}{(E_k - E_i)} \tag{11}$$

$$\tag{12}$$

Extract the common exponential factor and (-i), we get $T_{fi} = (-i)(\langle f|H'|i\rangle + \sum_{k\neq i} \langle f|H'|k\rangle \frac{\langle k|H'|i\rangle}{(E_k-E_i)})$

Part II

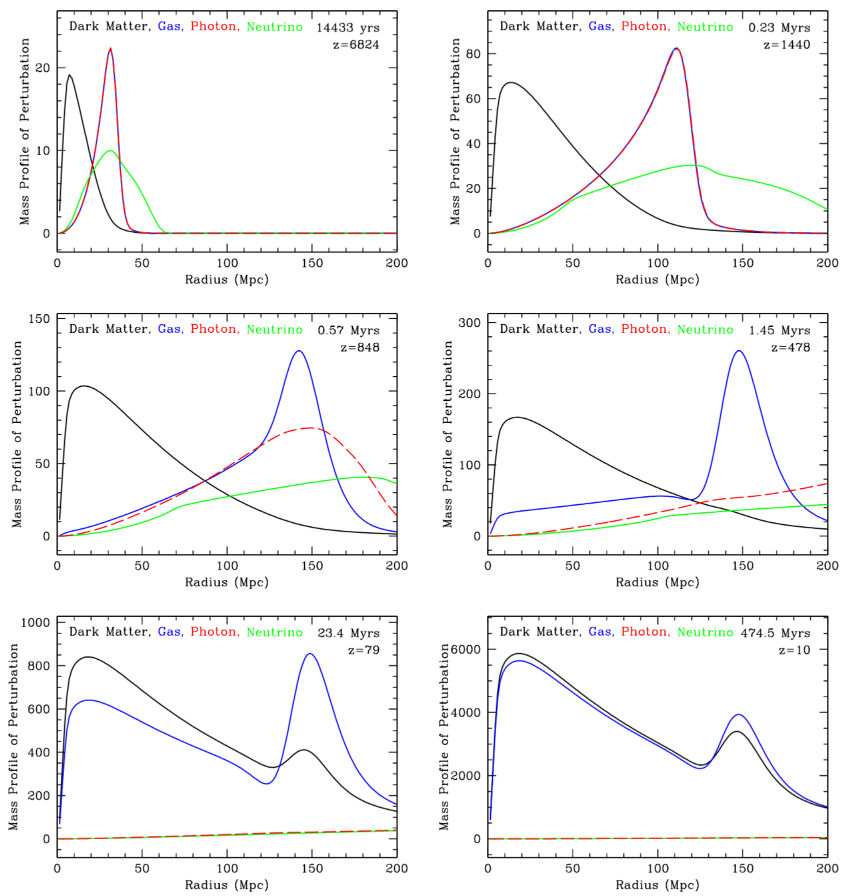
Cosmology

2 Baryon Acoustic Oscillations (BAO) [2]

One of my most intersted topics in cosmology! BAO started off from coupling between photon and baryons with compton scattering. Because everything was hot, there were actually photon-baryon plasma and it oscillates due to interplay between gravitational potential and radiaition pressure. According to these canonical plot by Eisenstein, a purterbation would propage outwards as a sounds wave , and after decoupling photons fly away, and baryons are left at around 150 Mpc position. At the bottom pictures, baryon and dark matter tend to drag the other toward themselves. The upshots is an overdensity of baryons and dark matter at the center and spherical halo of baryons on the outskirt. You can solve for momentum conservation by expanding using mean free path of photons in fourier space, with gravity as source of perturbation. As the photon decouples, it is possible to characterize growth of amplitude of different modes.

Higher order processes are generally on the scale of less than 150 Mpc, so it barely affects measurements of BAO. Larger systematic source of error (make the circle smaller or larger) are found to be canceling using technqiues including **Zel' dovich technique**. This makes BAO robust measurement of distance.

Zel' dovich approxiamtion is an intuitive approxiamtion technique that succesfully predicts filament, clusters, voids, and sheets of the universe.



BAO serves as good **stabdard ruler** because it has known size at a certain wavelength and we know how its size changes with z. It is considered as a **statistical standard ruler (SSR)**. SSR assumes there's a favorite distance for stars to spread apart: an ideal situation would be you drop a galaxy, draw a circle with radius r, drop another galaxy on the circle, use the new galaxy as the center of a circle again... This is best seen in terms of two point correlation function.

In (this review article), we use the Friedmann equation to write dimensionless hubble constant using info on dark energy density and other cosmological constants. With dark energy as a **barotropic fluid** (density only depends on pressure), and we can write equation of state of dark energy in terms of redshift dependence (w(z)). This can be contrained using distance measurements. Some distance measurements are determined with the comoving distance (r in Ryden, χ in weinberg and others) and curvature. **Angular diameter distance** d_A and **luminosity distance** d_L.

In review article, with the help of **Alcock-Paczynski test** (evaluation of the ratio of observed angular size to radial/redshift size, it uses the fact that circlular object becomes elliptical when viewed slanted), we get value of $d_A(z)$ and H together.

If the two point correlation function ξ has and only has 1 peak, then its fourier transform will be a perfect sin wave. This is the power spectrum. By measuring both on BAO, much more information can be extracted and BAO can provide better constaints on cosmological parameters.

3 First Stars

Also one of the hottest topics in cosmology/astrophysics research. It offers info on the histroy of structure formation and the chemicals in early ages of universe.

Most popular opinion is that primordial gas form in low-mass halos and then are cooled through molecular hydrogen cooling. Radiation can eitehr stop the formation process by dissociating molecular hydrogen or use X-ray to promote H_2 production.

3.1 Brief Summary Of [1]

With an $N-$body simulation with nine chemical species, electro, hydrogen (ionized or not), helium atom and gas, using the GADGET code, some cosmological parameter and cooling rate determined by other scholars, we advanced the time evolution. After simulation we then search for dark matter halos and look for promising gas clouds. We found that there's a critical line as a function of fraction of H_2 and tempterature T that determines whether the clouds were able to cool or not. Function of this line can be theoretically calculated.

Accounting for the size of the halo, we note the faster it accretes, the harder it is to cool, which came from the dynamical heating. Therefore we can find a critical growth rate of halos. Similar phenomenon occurs with too much mass. The initial angular momentum of gas may determine property of gas clouds. Especially, a fast spinning system may form a disk like structure. It turns out system with high **spin parameter** is pretty hard to find. The spin parameter does affect the shape of the gas coulds formed. We then fitted a spherical collapse model to the cooling of the gas clouds in our simulation, the agreement is great.

Accounting for uniform backgroud radiation in the LW band and self-shielding, we found that radiation does prevents cooling significantly, and self-shielding is less affective with small (optically thin objects).

In order to run bigger simulations, we intend to couple a N-body simulation with some semi-analtic model. We accounted for radiation during the formation of stars and modeled gas formation pattern. We then use some simulation just with CDM particles and got agreeing results.

4 Line Intensity Mapping (LIM)[3]

Overview

LIM is the next generation of detection method, that can probe larger areas faster, cheaper, and deeper into high redshifts, compared to normal galaxy surveys. This is because Galaxies must be bright enough in high z regions for it to be detected by telescope; but LIM can deal with unresolved galaxies, as it takes the noise as part of its analysis as well. Because dilution of aggregated signal is less sever than dilution of signal from 1 galaxies as redshift increases.

LIM takes in spatial fluctions (large scale structre) in many unresolved galaxies, and the frequency dependence of these fluctation tells us ditribution of emission. The power spectrum contains information on the clustering and shot noise (modeled by Poisson process, came from discrete sampling of galaxies), redshift bias, and average intensity. The spectrum depends on DM fluctuation and other astrophysical processes.

What LIM Does

Neutrino dynamics is very different from CDM and Baryon matters, because they are traveling fast, leaving signatures on observables. Pairing galaxy survey with hydrodynamics simulation, one may contraint the mass of neutrino well enough.

Of course, we also want to contrain the cosmological constants better. But notice, both astrophysical process and cosmological parameter affect emission lines. Therefore, we need to either pair LIM results with simulation, cross correlate different methods of observation, or use analytical methods (e.g. redshift space distortion, how stars are elongated towards us in redshift space) to get rid of density fluctuations caused by astrophysical phenomenon.

By plotting predictions of Horndeski and quintessence models for gravity, we found that their deviation from Benchmark model differ at high redshft ($z > 2$), and LIM can just probe into that. Other alternative theories LIM can probe include: heating of dark matter by annilation, collision between DM and baryon .... We need $z > 20$ probe for these information to be extracted.

Because current measurement into deep stars need the stars to be bright, there is a biased sampling in selecting targets for studying the first stars. Carbon-monoxide intensity mapping is the better method to delve into primodrial gas. This determines the star-forming-gas reservoir at high redshifts.

How LIM Does It

One can correlated LIM with overlapping galaxy surveys (not so hard to find one due to large area probed by LIM). This can be used to filter out contaimation due to say dusts and synchrotron radiation (foreground contamination).

We also need to model astrophysical/cosmological process happens in LIM pictures. This helps us predict signal to noise ratio and interpret results of the surveys. We can empirically interpolate line luminosity and halo mass, and use halo model to compute say power spectrum. The other way is to run numerical simulations and apply known physics to the results (semi-analytic).

5 Cosmic Bell Test

Using (high z) quasars to test quantum theory.

Entanglement

1930s, QM community began to study Entanglement. A. Einstein, B. Podolsky, N. Rosen made paper that behavior of particle B seems to be depend on particle A **instantaneously**, even they are arbitrarily far.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|u\rangle_A |v\rangle_B + |v\rangle_A |u\rangle_B) \tag{13}$$

This does not factorize, based on local realism.

Bell test:

1. dichotomic observables. $A(a), B(b) = \pm 1$. Say we have 2 SG devices.
2. $E(a,b) = \langle A(a)B()b\rangle$. Does the measure the same or opposite?
3. Vary question asks by
4. EPR predicts an upperbound for $S = E(a,b) + E(a',b) + E(a,b') - E(a',b')$ of 2
5. But QM says $S_{max} = 2\sqrt{2}$.

Early results at LBL by John Clauser, 1972. $|S| > 2$ by 5σ. Recently, Jian-Wei Pan, $|S| > 2$ by 4.1σ.

Loop holes

1. Locality Loophole: hidden communication between parties: information is transmitted during difference between measurements. Need spacelike spearated arrangement.
2. Detector Efficiency loophole $\rightarrow$ detector fails to respond.
3. **Freedom of choice loophole**. Whether properties of particle correlated to questions being asked. Whether the type of measurement is entirely independent and random. The assumption was choice of detector setting was correlated with particle property. Only a small such correlation can reproduce what has been observed.

Some used volunteers to play games to generated randomness.

Cosmic Bell Test

Conformal times ($\tau = \int_0^{t0} \frac{dt'}{a(t')}$) vs. comoving coordinate r. This means that the light ray is still 45 degrees in spacetime diagram. This is because $r = c\int \frac{dt'}{a(t')}$ by setting FRW metric to get the null geodesic. 2 quasers lights up, lights began to travel toward us. On earth, set up entangled photon to detectors. Before measurement of photon, we measure the light from quasers, and that determines which measurement we gonna perfom on the entangled photons. This information we extract is variantion in color (noise). The other quasers should be outside of the past light cone of detection

6 Simulations

Hydrodynamics simulations by [5] (Illustrius)

Usually, simulation on cosmology only involves DM. These simulation tend, combined with semi-analytical models tend to erroneously predict properties including distribution of neutral hydrogen, distribution and types of galaxies, metallicity in gases. This is why we need to include need to include baryonic matters in simulation, which entail us to model gas, stars, and blackholes. Notice we also want to model feedback processes and how small scale structure may couple large structures. Examples of these effect inlude how star and blackholse affect winds driven by star formation. The initial condition is usually encoded in CMB.

One success by the Illustrius is that we can provide virtual observations and extract data and mimics existing sky surveys. It also successfully predicts distribution of satellite galaxies (small galaxy circling larger ones). The **missing satellite** (how ΛCDM model predicts more satellite galaxies) and **too big to fail** (ΛCDM model predicts too large galaxies) problem both serve as challenges to the ΛCDM model. This is resolved by including dissipational processes of galaxy formation that make the stellar component more resistant to **tidal disruption** (star pulled apart by BH) close to cluster centers.

To detect large-scale characteristics of neutral hydrogen, we nned to probe through Lyman-α forest absorption lines. Observationally, we have statistically strong data to contraint the hydrogen column densities. This is done by meausring quasar spectra after being absorped. If the gas density is high, the hydrogen becomes self-shielded from the ionizing ultraviolet background and forms dense neutral cluds with absoprtion characteristic different from that of regular Ly-α forest, callled **damped Lyman-α aborbers**. This entity contains information of the initial stages of galaxy formation. In this simulation, we study the metallicity and column density distribution function of Lyman series and DLA.

To see not only how including baryonic physics soundly connect cosmological prediction and galaxy observations, we see how baryons can affect dark matter. We measure matter power spectrum $P(k)$ and use cosmological prediction to connect its current value to initial values. Observation tools that does this includes weak lensing. Several present predictions for the evolution of $P(k)$ can only be caliberated by pure DM simulation, rendering them not so effective.

Advancement is still possible: Illustrius predicts the formation of lower mass galaxies too early. This can be remedied by new stellar feedback model that maybe could include radiation pressure on dust.

Brief Note on the Algorithm

Friends of Friends This is a very common tool to identify, say halos or potential blackholes in an astrophysical simulation. You start by defining a "linking length" l, and particles within distance l are considered "friends" of that particles. However, each of these friends have friends of their own. Therefore we have created a network of linked particles which we call groups.

AREPO[4] Except the moving, unstructured mesh, we have other two algorithms. The first is called "smoothed particle hydrodynamics", which essentially treats fluid element as discrete particles which follow gravity and hydrodynamics. This is the method used by the famous GADGET. It is suitable for large dynamics ranges and can increase resolution in collapsing regions. This SPH algorithm also helps conserve total energy.

The other one is "Eulerian" algorithms, which meshes the space and account for the flow of gas between neighboring cells. Of course, we have adaptive mesh refinement which helps determine how big a mesh is needed for correct physics. AMR codes have better accuracy in resolving higher dradients and instabilities.

THE AREPO code uses the moving, unstructured mesh, with the mesh themselves moves around in space and account for the flow of gas in their vicinity. The space is first partitioned by Voronoi tessellation points paired with mesh. Then these points move around in space with mesh, gaining adptivity of SPH and Galilean-invariancy. Just like AMR with flux computation, moving mesh does great with resolving shocks and instabilities.

Detailed Physical Process Involved [6]

We have 5 physical process to take care of. These processes have to be implemented subgrid prescriptions (below ressolution) and link the physical processes actually resolved by simulation.

1. Star Formation

   By adopting an effective equation of state, we dictate that stars form stochastically above a critical density with a time scale. If we increase he resolution to include ISM processes, we do not need this eEOS treatment because we could directly model ISM physics including momentum/energy injection and radiation associated with star formation.

2. Stellar Evolution and Chemical Enrichment

   This accounts for what happens after a star dies and how will the remnant fall back into ISM. An enrichment process adopted in Illustrius is it tracks mass and metal return of stars as a function of time. For shorted lived stars, this can be achieved by lower star forming rate and increase metal content in nearby gas.

3. Cooling and heating

   Examples of colling and heating include: cooling due to primordial mixture of hydrogen and helium and heating tue to collision, ionization, photoionization. Some of the processes are complicated by additional metal elements in gas. There has been development using the the $CLOUDY$ to create a table of how elements cool down.

   Self-shielding is present at $\rho > 10^{-3}\text{cm}^{-3}$. The gas began to absorb radiation and changed the heating rate. The heating rates are suppressed as a function of number density [7].

4. Stellar Feedback

   To solve the infamous overcooling problem, we uses some subgrid model for galactic winds that blows away gas to prevent over cooling.

5. BH

   To account for presence of blackholes, we represent them as collisionless, massive sink particles that can grow in mass. We seed (identify?) them use Friends-Of-Firends algorithm to see massive groups. It abosrbs particle at Eddington-limited rate and merge with other blackholes.

   BH and AGN gives feedback to its surround by jetting how gas and release radiative energy. The three components are mechanical, thermal, and EM. (to be honest I am not so clear how the feebback process works)

References

[1] Yoshida, N., Abel, T., Hernquist, L. and Sugiyama, N., 2003. Simulations of Early Structure Formation: Primordial Gas Clouds. The Astrophysical Journal, 592(2), pp.645-663.

[2] Bassett, B. and Hlozek, R., 2022. Baryon Acoustic Oscillations. [online] arXiv.org. Available at: ¡https://arxiv.org/abs/0910.5224v1¿ [Accessed 28 January 2022].

[3] arXiv:1709.09066v1 [astro-ph.CO].

[4] arXiv:1109.4638[astro-ph.CO]

[5] Vogelsberger, M., Genel, S., Springel, V., Torrey, P., Sijacki, D., and Xu, D. et al. (2014). Properties of galaxies reproduced by a hydrodynamic simulation. Nature, 509(7499), 177-182. doi: 10.1038/nature13316

[6] arXiv:1305.2913 [astro-ph.CO]

[7] arXiv:1707.06993v2 [astro-ph. CO]