Abstract

Notes I gathered during my summer research 2022.

Notes on Astroparticle Physics and Cosmology

Part I

Gravitaional Lensing in General

1 Weak Lensing Basics & its relationship to DM and DE

The good thing about weak lensing is that it reponds only to DM and not to visible matter. So it directly probes the dark sector. It is perceivable that N-body simulations will reach a resolution so that modeling weak lensing with N-body simulation is accurate.

To start our analysis, we can transform convergence into multipole space (χ is radial position)

$$\langle \kappa_{lm} \rangle = \int d\hat{n} \kappa(\hat{n}\chi) Y^*_{lm} \tag{1}$$

with statistical isotropy, the power spectrum of convergence can be defined as the harmonic transform (repeated fourier transform) of the two point correlation function of convergence. The convergence power spectrum is identicla to the shear power spectrum in the limit of weak distortions $P^\gamma_l \approx P^\kappa_l$.

The main thing we observe in lensing is shear – the smearing of galaxies' shapes. Because lensing is weak and galaxies have their own shapes (ellipticity), we don't measure individual shapes of galaxy but correlation of shapes. By this, we mean if galaxies near one point are elliptical in a same way, there is probably a halo in front of them. If two galaxies are close togher but does not exhibit shape correlation , then probably there is no halo in front of them. In addition, to actually measure or calculate lensing effects, we need catalog of galaxy position and galaxy shear measurements.

To do this, we can use tomography, which uses photometric redshift estimates to divide galaxies by their redshifts. After that, we can correlate redshift of galaxies within and across bins of redshift. We know that galaxies with lower redshift must have lower distortion because there isn't many mass in front of them.

We can also use galaxy and galaxy or cluster galaxy lensing. This measures correlation of shear of the background galaxies with mass of the foreground galaxies. However, what we actually observe also has contribution from the shot noise given by random galaxy shapes. We mainly measure the surface density, which can be inverted to halo density profile. Galxy-galaxy lensing is mainly used to constrain DE and modified gravity models. You can also count halo numbers and ultimately cosmological parameters.

Overall, weak lensing by LSS, mainly helps contrain $\sigma_8\Omega_M^{0.6}$ and DE equation of state w. This is possible because lensing is sensitive to growth of density perturbation.

Interestingly enough, weak lensing will magnify SN Ia and perturb these luminosities and the resulting magnifications will be correlated and may be measurable with future surveys.

$$C^{\kappa}_{ij}(l) = P^{\kappa}_{ij}(l) + \delta_{ij}\frac{\langle\gamma^2_{\text{int}}\rangle}{\hat{n}_i}, \tag{2}$$

where γ_{int} is rms of intrisic shear and $\hat{n}$ is the average number of galaxies. Therefore we see increasing number of galaxies and less ellipticity helps with shot noise.

Similarly for uncertainty, there is a term that dominates on large scale, and another term accounting for ellipticity and number of galaxies.

Note that, while two-point correlation function has been by far the most well-understood, primordial non-gaussianity and nonlinear clustering of structure can also gives signal to 3 point function. To compute bispectrum, we need halo model or perturbation theory techniques (or calibrate it with N-body simulations).

For dark energy, weak lensing mainly contribute to equation of state of the cosmological constant. For HSC, constraint is best at $z = 0.7$. For lower z, we have less sample galaxies so a smaller sample size. For higher z, there are less appropriate source.

1.1 Observation Errors

the most important systematic error here is the photometric redshft erros. This can be reduced by calibrating by spectra of a subset of galaxies and training the photometric spectra using the spectroscopic subset.

The other source of error are observed shapes of galaxies, due to PSF, atmospheric blurring, telescope. There are multiplicative and additive errors. Furue survey has to self-calibrate: use part of the survey itself to partially calibrate the systematic effects.

2 Lensing with HSC

HSC is powerful because it has a pretty deep field view, and fast, deep, and sharp at the same time. HSC is really suited for large scale survey because of this property.

For HSC SSP survey, we have wide (1400 deg^2), deep (28 deg^2), and ultradeep (3 deg^2). HSC focus on survey area with declination=0, and it overlapse with SDSS. The seeing FWHM(full width of half maximum, the smaller the better) is about 0.5-0.7 arcsec.

There has been effort to correlate HSC data with BOSS data. HSC data will add background galaxies as well as member galaxy around each BOSS galaxies. Together, we will have a better understanding of structure growth.

Analysis with HSC is done with LSST survey software pipeline. In doing so, we found out that a big source of error comes from blending. Some terms that came up:

1. LRG: luminous red galaxy. There should be a correlation between existence of LRG and underlying density distribution.
2. PSF: point spread function. Describes response of image to a point source (during image simulation). PSF = 1 means infinitely resolved. $PSF = 0$ means unresolved.

To see systematic errors, we can get two statistical measures and correlate them. If two things that are not related by phyiscs turned out to be correlated, then there might be some systematic error.

Part II

GR in General

Part III

(Weak) Lensing Research

3 Triaxial Model

Although NFW is a pretty good approximation for halos in the universe, research have found [1] that traxial halos are better approximations. It is also note in the same article that it is reasonable to adopt $\alpha = 1.5$ in

$$\rho(R) = \frac{\rho_{\mathrm{crit}(z)\delta_{\mathrm{ce}}}}{(R/R_0)^{\alpha}(1 + R/R_0)^{3-\alpha}} \tag{3}$$

With the SPLINV code written by Xiangchong Li [3], the *nfwTJ03* class contains calculation of Σ the surface density, and $\delta\Sigma$, the excess surface density. I haven't found information on excess surface density for triaxial halos. But we do have in Eqn 23. in [2], convergence κ and the fact that $\Sigma = \kappa\Sigma_{\mathrm{crit}}$.

3.1 Basics

In addition to spherically symmetric NFW halo, a triaxial halo has more parameters. In addition, I have defined a/b, a/c (ellipticity), c_e (concentration parameter). In initializing the triaxial halo object.

3.2 Angles

In FIG. 1 of [2], We have defined x coordinate system as DM halo's, with origin at its center. x' as the "observer's" frame, with z' along the line of sight and origin also at the center. We are told that z' points in θ, ϕ which are polar *coordinate of the line of sight direction in the DM halo frame*. By aligning halo's frame with the HSC telescope frame, we can say that

$$\theta = \text{Right Ascension}$$

$$\phi = \text{Declination}$$

Here's my hand-drawn illustration:

IMG_9017.png

3.3 Position

In definiing κ, we also used x' and y' which are position in the "observer's" frame. In this case, I believe that *kept the orientation, but moved the prime axis so that the origin aligns with halo center*. Because the angle difference is small between the actual halo and the area of density in question, we can essentially use right ascension and declination of the position and halo center as rectangular coordinates: **the difference between right ascension gives x' and that of delcination gives y'**. Here's my hand-drawn illustration.

IMG_9018.png

3.4 scaling

The NFW and general triaxial halos have different scaling convention. In the original NFW paper, the virial radius is seen as r_{200} as the radius whose inner mean density is $200\rho_{\text{crit}}$. So the only redshift dependence in virial radius is in the critical density of the universe at that point.

In triaxial halos, however, we have virial radius defined as in [5]:

$$\Delta_{\text{vir}} = \frac{3M_{\text{vir}}}{4\pi r_{\text{vir}}^3 \rho_{\text{crit}}} = 18\pi^2(1 + 0.4093\omega_{\text{vir}}^{0.9052}) \tag{4}$$

if we have $\Omega_{m,0} + \Omega_{\Lambda,0} = 1$ and

$$\omega_{\text{vir}} = 1/\Omega_{\text{vir}} - 1$$

and the density parameter at virialization is

$$\Omega_{\text{vir}} = \frac{\Omega_{m,0}(1+z_{\text{vir}})^3}{\Omega_{m,0}(1+z_{\text{vir}})^3 + (1-\Omega_{m,0}-\Omega_{\Lambda,0})(1+z_{\text{vir}}^2) + \Omega_{\Lambda,0}}$$

In later version of SPLINV, we reconciled the definition of virial radius. Between to form of halos. Here's what we did.

3.5 Standardizing Halo Definition

We adopt characteristic density

$$\delta_{\text{NFW}} = \frac{\Delta_{\text{vir}}\Omega_m}{3}\frac{c^3}{m(c)}$$

While noting:

$$c_e = 0.45c$$

$$R_e = 0.45 r_{\text{vir}}$$

$$R_0 = 0.45 r_{\text{vir}}/c_e = r_{\text{vir}}/c$$

We have

$$\delta_{\text{triaxial}} = \frac{\Delta_{\text{vir}}(\frac{c^2}{ab})^0.75\Omega_m}{3}\frac{c^3}{m(c)}$$

which is slight different from [2], but it gives the correct mass after integrating from $r = 0$ to $r = r_{vir}$ of density.

For both the above definition, we have

$$m\left(c\right) \equiv \frac{c^{3-\alpha}}{3-\alpha} {}_2F_1\left(3-\alpha, 3-\alpha; 4-\alpha; -c\right)$$

or

$$m\left(c\right) = \begin{cases} \ln\left(1+c\right) - \frac{c}{1+c} & (\alpha = 1) \\ 2\ln\left(\sqrt{c} + \sqrt{1+c}\right) - 2\sqrt{\frac{c}{1+c}} & (\alpha = 1.5) \end{cases}$$

3.6 results and plots

Because we took $\alpha = 1.5$, the density profile of the triaxial halo should, while staying close to NFW halo at raidus close to order of their characteristic lengths, should peak higher at $r = 0$ and also drop down faster. The units in the following plots are: (1) arbitrary because I'm just looking at how close they look (2) solar mass per Mpc^2 (surface density).

3.7 Detailed Derivation for shear and convergence.

First, note that we start with the observer's coordinate: (x', y', z') (z' points along the line of sight, observer sits at halo's center). From there, we get an expression for the convergence κ as:

$$\begin{aligned} \kappa = \frac{R_0}{\Sigma_{\text{crit}}} \int_{-\infty}^{\infty} \rho(R) dz' &= \frac{R_0}{\Sigma_{\text{crit}}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{f}} \rho\left(\sqrt{z_*'^2 + \zeta^2}\right) dz_*' \\ &\equiv \frac{b_{\text{TNFW}}}{2} f_{\text{GNFW}}(\zeta) \end{aligned}$$

where ζ is defined as

$$\zeta^2 \equiv h - \frac{g^2}{4f}$$

this is the lethal typo made by [2] in eqn 21. You can find h, g and other definitions above eqn 21.

Next, we rewrote the expression for ζ in terms of x', y' (A, B, C can be found in [2]):

$$\zeta^2 = \frac{1}{f}\left(Ax'^2 + Bx'y' + Cy'^2\right).$$

But if we make a change of coordinates, stipulating ($\psi = \frac{1}{2}\arctan\frac{B}{A-C}$):

$$x'' = x'\cos\psi + y'\sin\psi$$

$$y'' = y'\cos\psi - x'\sin\psi$$

we would have gotten

$$\zeta^2 = \frac{x''^2}{q_x^2} + \frac{y''^2}{q_y^2}$$

such that $zeta^2$ does not change value. The caveat here is that, if $A > C$, the position of q_x, q_y are switched. I assume [2] didn't care about this point? Also not that right now as $q = \frac{q_y}{q_x}$ it does not have to be smaller than 1.

We have also defined

$$\xi^2 = x''^2 + y''^2/q^2 = \zeta^2 q x^2$$

so that $\xi/qx = \zeta$. This will come up when we calculate κ.

Note that, because we, as observers, are using x', y', z' coordinates, we must use $\phi_{x'x'}$ etc. Meaning that we must take derivative with respect to the prime coordinates. We already have (subscripts means partial differentiation).

$$\begin{aligned}
I(x,y) &= \int_0^1 \frac{\xi(u)}{u} \frac{\phi_r(\xi(u))}{\left[1-(1-q^2)\,u\right]^{1/2}} du \\
J_n(x,y) &= \int_0^1 \frac{\kappa\left(\xi(u)^2\right)}{\left[1-(1-q^2)\,u\right]^{n+1/2}} du \\
K_n(x,y) &= \int_0^1 \frac{u\kappa'\left(\xi(u)^2\right)}{\left[1-(1-q^2)\,u\right]^{n+1/2}} du \\
\text{where } \xi(u)^2 &= u\left(x^2 + \frac{y^2}{1-(1-q^2)\,u}\right)
\end{aligned}$$

$$\begin{aligned}
\phi(x'',y'') &= \frac{q}{2} I(x'',y'') \\
\phi_{x''}(x'',y'') &= qxJ_0(x'',y'') \\
\phi_{y''}(x'',y'') &= qyJ_1(x'',y'') \\
\phi_{x''x''}(x'',y'') &= 2qx''^2 K_0(x'',y'') + qJ_0(x'',y'') \\
\phi_{y''y''}(x'',y'') &= 2qy''^2 K_2(x'',y'') + qJ_1(x'',y'') \\
\phi_{x''y''}(x'',y'') &= 2qx''y''K_1(x'',y'')
\end{aligned}$$

To transform $\phi_{x'',x''}$ to $\phi_{x',x'}$ etc, we need to use multivariable chain rules. Because ϕ only involves distance from center and is a potential, so differentation should not affect it: ϕ is the same in x'' and x', given you convert x' to x'' first.

$$\begin{aligned}
\phi_{x'} &= \phi_{x''}\frac{\partial x''}{\partial x'} + \phi_{y''}\frac{\partial y''}{\partial x'} \\
\phi_{y'} &= \phi_{x''}\frac{\partial x''}{\partial y'} + \phi_{y''}\frac{\partial y''}{\partial y'}
\end{aligned}$$

Note because there's a linear relationship between x'', y'' and x', y', this makes the partial differentation chain rule a little easier.

$$\phi_{x'x'} = \frac{\partial}{\partial x'}(\phi_{x''}\frac{\partial x''}{\partial x'} + \phi_{y''}\frac{\partial y''}{\partial x'}) \tag{5}$$

$$= \phi_{x''x''}(\frac{\partial x''}{\partial x'})^2 + \phi_{x''y''}(\frac{\partial y''}{\partial x'}\frac{\partial x''}{\partial x'}) + \phi_{x''y''}(\frac{\partial y''}{\partial x'}\frac{\partial x''}{\partial x'}) + \phi_{y''y''}(\frac{\partial y''}{\partial x'})^2 \tag{6}$$

$$\phi_{x'y'} = \frac{\partial}{\partial y'}(\phi_{x''}\frac{\partial x''}{\partial x'} + \phi_{y''}\frac{\partial y''}{\partial x'}) \tag{7}$$

$$= \phi_{x''x''}(\frac{\partial x''}{\partial x'}\frac{\partial x''}{\partial y'}) + \phi_{x''y''}(\frac{\partial y''}{\partial y'}\frac{\partial x''}{\partial x'}) + \phi_{x''y''}(\frac{\partial y''}{\partial x'}\frac{\partial x''}{\partial y'}) + \phi_{y''y''}(\frac{\partial y''}{\partial x'}\frac{\partial y''}{\partial y'}) \tag{8}$$

$$\phi_{y'y'} = \frac{\partial}{\partial y'}(\phi_{x''}\frac{\partial x''}{\partial y'} + \phi_{y''}\frac{\partial y''}{\partial y'}) \tag{9}$$

$$= \phi_{x''x''}(\frac{\partial x''}{\partial y'})^2 + \phi_{x''y''}(\frac{\partial y''}{\partial y'}\frac{\partial x''}{\partial y'}) + \phi_{x''y''}(\frac{\partial y''}{\partial y'}\frac{\partial x''}{\partial y'}) + \phi_{y''y''}(\frac{\partial y''}{\partial y'})^2 \tag{10}$$

density profile NWF and Triaxial.png

Sigma_vs_ra.png

density vs ra vs dec.png

References

[1] Jing, Y. P. and Suto, Y., "Triaxial Modeling of Halo Density Profiles with High-Resolution N-Body Simulations", ¡i¿The Astrophysical Journal¡/i¿, vol. 574, no. 2, pp. 538–553, 2002. doi:10.1086/341065.

[2] Oguri, M., Lee, J., and Suto, Y., "Arc Statistics in Triaxial Dark Matter Halos: Testing the Collisionless Cold Dark Matter Paradigm", ¡i¿The Astrophysical Journal¡/i¿, vol. 599, no. 1, pp. 7–23, 2003. doi:10.1086/379223.

[3] Li, X., Yoshida, N., Oguri, M., Ikeda, S., and Luo, W., "Three-dimensional Reconstruction of Weak-lensing Mass Maps with a Sparsity Prior. I. Cluster Detection", ¡i¿The Astrophysical Journal¡/i¿, vol. 916, no. 2, 2021. doi:10.3847/1538-4357/ac0625.

[4] Navarro, J. F., Frenk, C. S., and White, S. D. M., "A Universal Density Profile from Hierarchical Clustering", ¡i¿The Astrophysical Journal¡/i¿, vol. 490, no. 2, pp. 493–508, 1997. doi:10.1086/304888.

[5] Masamune Oguri et al 2001 ApJ 559 572 arXiv:1305.2913 [astro-ph.CO]

[6] arXiv:1707.06993v2 [astro-ph. CO]