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Spin Number in WL

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Abstract: Notes on Spin Number

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1 Derivation in 2208.10522

We are investigating the spin number of two products, say v_x and v'_x. They can be defined as

$$v_x = v\exp(im(\theta_0 - \theta))$$

$$v'_x = v'\exp\big(im'(\theta'_0 - \theta)\big)$$

We know to get the component of a complex number, we have $\operatorname{Re} z = (z + z^*)/2$ and $\operatorname{Im} z = (z - z^*)/2i$.

Then 2208.10522 then computes $v_x^* v'_x$ and $v_x v' x$:

$$v_x^* v'_x = v' v\exp\big(-i(m' - m)\theta + m'\theta_0 - m\theta_0)\big) \tag{1.1}$$

$$v_x v'_x = v' v\exp\big(-i(m' + m)\theta + m'\theta_0 + m\theta_0)\big), \tag{1.2}$$

and argues you can compute the "**products between the components**" of v_x and v'_x with these two quantities. This is not true, unless taking the real and imaginary part is also considered a linear operation (it is not, by definition of linearity however). Moreover, the components of a field does **have a definition of spin**, because it has no phase (just real number). I think the authors actually meant that they want the **components of the product**, but in that case, what we need is actually $v_x v'_x$ and $(v'_x v_x)^*$, according to the relationship in the previous section. They are, quite naively,

$$v_x^* v_x'^* = v' v\exp\big(i(m' + m)\theta + m'\theta_0 + m\theta_0)\big) \tag{1.3}$$

$$v_x v'_x = v' v\exp\big(-i(m' + m)\theta + m'\theta_0 + m\theta_0)\big), \tag{1.4}$$

This essentially says that, the product between a spin-m and a spin-m' product should just be spin-$m + m'$.

2 Other Reasons

I also think there are other evidence for my argument. First, I think the statement in 2208.10522:

γ is a spin-2 vector since $(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}, 2\frac{\partial^2}{\partial x \partial y})$ negates when coordinates of the vector space rotate by $\pi/2$. If this is true, then, because $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ negates when vector space is rotated by $\pi/2$, the κ should also be spin-2. This is not true.

This is to argue that, we need to treat the spin number of a quantity in a complex-plane way and view spin number as the coefficient before the complex phase (as just opposed to a sign).

Define $\partial = \frac{\partial}{\partial z} = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$, we may calculate the following:

$$\kappa = \frac{1}{2}\partial^* \partial \psi, \tag{2.1}$$

$$\gamma = \frac{1}{2}\partial \partial \psi = |\gamma| e^{2i\phi}, \tag{2.2}$$

We then see κ is a spin-0 quantity. If we say $z' = e^{-i\theta} z$, we have $\partial = \frac{\partial}{\partial e^{-i\theta} z} = e^{i\theta}\partial$ and $\partial^* = \frac{\partial}{\partial e^{i\theta} z^*} = e^{-i\theta}\partial^*$. Because ψ never changes phase with rotation (being a real potential).

Taking a further complex gradient, we may define two more complex fields:

$$F = \frac{1}{2}\partial^* \partial \partial \psi = |F| e^{i\phi}, \quad (10) \tag{2.3}$$

$$G = \frac{1}{2}\partial \partial \partial \psi = |G| e^{3i\phi}, \quad (11) \tag{2.4}$$

These are the ways to derive spin quantities more comfortable to me. Note that derivative and the coordinates, while having the same spin number, transform differently.

So the correct way to see the spin number of product of two γ

$$\gamma\gamma = |\gamma|^2 e^{4i\phi}$$

Acknowledgments

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Note added. This is also a good position for notes added after the paper has been written.

References

[1] Author, *Title*, *J. Abbrev.* **vol** (year) pg.

[2] Author, *Title*, arxiv:1234.5678.

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