of its derivation from an option pricing model. From there, **Moody's-KMV model** and the **Kamakura model** take the output from the Merton model and restate it in the form of a real-world default probability.

MODULE QUIZ 25.3

1. A junior analyst is evaluating the following questions to include in her research report:
   - Which type of default probabilities should be used in valuation?
   - What is the impact on a company defaulting when the distance to default rises?

   Regarding these questions, the analyst's report should support which of the following conclusions?

	Use in valuation	Rising distance to default
A.	Risk-neutral default probabilities	Company more likely to default
B.	Risk-neutral default probabilities	Company less likely to default
C.	Real-world default probabilities	Company more likely to default
D.	Real-world default probabilities	Company less likely to default

KEY CONCEPTS

LO 25.a

Credit ratings of larger publicly traded bond issuers are provided by rating agencies. These ratings are indicative of credit quality. Therefore, they may change based on information released to the market. Rating agencies typically do not provide credit ratings for small- and mid-sized firms. As a result, many lenders have developed their own internally developed credit rating systems for such firms. Internal credit rating systems usually look at profitability, liquidity, and solvency measures.

LO 25.b

Altman's Z-score is an application of linear discriminant analysis (LDA) that is used to predict defaults. The following five financial ratios are computed:

1. X_1: working capital / total assets
2. X_2: retained earnings / total assets
3. X_3: earnings before interest and taxes (EBIT) / total assets
4. X_4: market value of equity / book value of total liabilities
5. X_5: sales / total assets

The Z-score equation for publicly traded manufacturing firms is:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

LO 25.c

The lower (higher) the borrowing rating, the greater (lower) the probability of default. For investment-grade bonds and some non-investment-grade bonds, the (marginal) probability of default in a given year will increase with time during the initial years.

For non-investment-grade bonds, it may be the case that the marginal probabilities of default start to decrease with time because the initial years are critical in terms of survival.

LO 25.d

A rating migration matrix quantifies the average default rates of issuers based on an initial credit rating. Cumulative probabilities of default are provided by the matrix. Based on those amounts, it is possible to calculate the marginal probabilities of default.

LO 25.e

The following equation can be used to calculate the probability of default by time *t*:

$$Q(t) = 1 - e^{-\bar{\lambda}(t) \times t}$$

where:

$\bar{\lambda}(t)$ = average hazard rate between time 0 and time t

An approximate calculation of hazard rates used for a bond that trades near its par value is expressed as:

$$\bar{\lambda} = \frac{s(T)}{1 - RR}$$

where:

$\bar{\lambda}$ = average hazard rate
$s(T)$ = credit spread for maturity T
RR = recovery rate

A more precise calculation of hazard rates should be used for a bond that does not trade near its par value.

LO 25.f

A bond's recovery rate can be thought of as the trading price as a fraction of the face value approximately a month following default. More senior bonds have higher recovery rates than more junior bonds. The recovery rate on debt is negatively correlated to the default rate.

LO 25.g

A credit default swap (CDS) is a credit derivative that is similar to a typical swap in that one party makes payments to another party. The purchaser of the CDS seeks credit protection and will usually make fixed quarterly payments (in advance) to the seller of the CDS for the life of the swap, or until a credit event occurs. Credit events may include nontimely payment, debt restructuring, and bankruptcy.

The underlying reference entity in a CDS is the firm in question. If default occurs, and assuming the terms of the swap agreement dictate settlement by physical delivery, the swap will be settled with the seller of the CDS paying the buyer the face value (notional principal) of the bonds and receiving the bonds in exchange. In terms of an up-front premium, there are three possibilities:

- CDS spread = fixed coupon; no up-front premium

- CDS spread > fixed coupon; up-front premium equal to the present value of (CDS spread - fixed coupon) paid by protection buyer to protection seller
- CDS spread < fixed coupon; up-front premium equal to the present value of (CDS spread - fixed coupon) paid by protection seller to protection buyer

LO 25.h

The CDS spread is essentially the price of the CDS expressed in basis points. The bond yield spread is the excess of the corporate bond yield over a comparable risk-free bond. In theory, the CDS spread and the bond yield spread should be equal:

- If CDS spread < bond yield spread, buy the corporate bond, and buy CDS protection to earn more than the risk-free rate
- If CDS spread > bond yield spread, sell the corporate bond, and sell CDS protection to borrow at less than the risk-free rate

LO 25.i

The CDS-bond basis should be zero because the CDS spread and the bond yield spread should be equal. In practice, this may not be the case due to the following reasons:

- Bonds may sell for much higher or lower than par.
- CDSs are subject to counterparty risk.
- A CTD possibility exists with a CDS.
- CDS payoffs exclude accrued interest.
- CDS contracts may contain a restructuring clause that allows for payoffs, even without default.
- The bond yield spread is computed using a risk-free rate that is dissimilar to the one used by the market.

LO 25.j

The hazard rates computed from credit spreads are significantly greater than those computed from historical data. As credit quality improves (worsens), the differences are less (greater). A conclusion is that the return for bearing credit risk is more than sufficient because the return is greater than the expected cost of the defaults. Such excess return is even more prevalent when transacting in lower-credit-quality instruments and during periods of economic turmoil with high credit spreads.

The key explanation is that bond defaults are not independent and often depend on economic conditions; on a stand-alone basis, a good (bad) economy decreases (increases) default probabilities.

LO 25.k

Under risk neutrality, the risk-free rate is assumed to be the expected growth rate of the firm's assets—whereas in the real world, the expected growth rate is the risk-free rate plus a market risk premium.

For valuation purposes, risk-neutral estimates (e.g., estimates of default rates from credit spreads) make sense because no additional premium is required for bearing risk.

In contrast, for scenario analysis, real-world estimates (e.g., default estimates from historical data) would be more appropriate.

LO 25.l

The Merton model views firm equity value as a long call option on the firm's assets. Terminology for this model is as follows:

- V_0 = current value of the firm's assets
- V_T = value of the firm's assets at time *T*
- D = total debt to be repaid at time *T*
- E_0 = current value of the firm's equity
- E_T = value of the firm's equity at time *T* (i.e., V_T - D)
- σ_V = asset volatility
- σ_E = equity volatility

In the Merton model, the strike price of the call option is the total debt to be repaid (D). Therefore, $E_T = \max(V_T - D, 0)$ because if $V_T > D$, it makes sense to continue to service the debt and retain the positive value of E_T. From a purely quantitative perspective, if $V_T < D$, then it makes sense to default on the debt because the value of the equity is zero.

LO 25.m

To estimate default probability using equity prices, apply the Black-Scholes-Merton equation to solve for unknown variable $N(d_2)$. From there, using the relationship of $N(-d_2) = 1 - N(d_2)$, it is possible to solve for $N(-d_2)$, which is the (risk-neutral) probability of default.

The distance to default (d_2) can be calculated directly using the Black-Scholes-Merton equation. Distance to default is the number of standard deviations that the asset price must fall to lead to a default at time *T*. The higher (lower) the distance to default, the lower (higher) the probability of default.

LO 25.n

Empirical evidence supports that the Merton model, Moody's-KMV model, and the Kamakura model provide reliable and reasonably accurate rankings of default probabilities. Moody's-KMV and Kamakura take the risk-neutral output from the Merton model and restate it in the form of real-world default probabilities.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 25.1

1. **C** The five financial ratios for computing Altman's Z-score are as follows:
    1. X_1: working capital / total assets = 525,000 / 5,100,000

2. X_2: retained earnings / total assets = 1,120,000 / 5,100,000
3. X_3: earnings before interest and taxes (EBIT) / total assets = 480,000 / 5,100,000
4. X_4: market value of equity / book value of total liabilities = 4,215,000 / 1,850,000
5. X_5: sales / total assets = 1,760,000 / 5,100,000

Altman's Z-score is then as follows:

Z = 1.2 × (525,000 / 5,100,000) + 1.4 × (1,120,000 / 5,100,000) + 3.3 × (480,000 / 5,100,000) + 0.6 × (4,215,000 / 1,850,000) + 0.999 × (1,760,000 / 5,100,000)

= 0.1235 + 0.3075 + 0.3106 + 1.3670 + 0.3448 = 2.4534

The following guidelines apply for assessing credit quality:

- > 3: no default is likely
- 2.7–3: potential default
- 1.8–2.7: reasonable probability of default
- < 1.8: high likelihood of default

Therefore, with a Z-score of 2.4534, Nielsen has a reasonable probability of default. (LO 25.b)

2. **D** The probability of a B-rated bond defaulting in the fourth year is 14.89% − 11.75% = 3.14%.

   The probability that the bond will survive until the end of the third year is 100% − 11.75% = 88.25%.

   Thus, the probability that the bond will default during the fourth year conditional on no earlier default is computed as 3.14% / 88.25% = 3.56%. (LO 25.d)

Module Quiz 25.2

1. **A** The investor will sell CDS protection on the 125 companies in the index for 140 bps per company. The annual receipt by the seller is 0.0141 × $1,000,000 × 125 = $1,762,500. However, because one company defaulted before the protection payment, the annual receipt by the CDS seller will be reduced by $1,762,500 / 125 = $14,100. In addition, the seller will have to pay $1 million to the CDS protection buyer as a result of the default. The CDS seller's cash inflow for the year is computed as $1,762,500 − $14,100 − $1,000,000 = $748,400. (LO 25.g)
2. **D** If the CDS spread is greater than the bond yield spread, sell the corporate bond, and sell CDS protection to borrow at less than the risk-free rate. (LO 25.h)

Module Quiz 25.3

1. **B** Risk-neutral default probabilities should be used in valuation, and real-world default probabilities should be used in scenario analysis. As the distance to default rises (falls), the company is less (more) likely to default. (LO 25.m)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Hull, Chapter 19.

READING 26

CREDIT VALUE AT RISK

Study Session 5

EXAM FOCUS

Credit value at risk (VaR) represents the credit loss over a certain time horizon that will not be exceeded given a specific level of confidence. For the exam, understand the differences between credit VaR and market VaR calculations. Also, interpreting a rating transition matrix is important, as that is often used in credit risk modeling. In addition, be familiar with the primary methodologies associated with calculating credit VaR, which include Vasicek's Gaussian copula model, Credit Risk Plus, CreditMetrics, and the correlation model. Finally, understand the impact of credit spreads on calculating credit VaR.

MODULE 26.1: DEFINING CREDIT VaR

Market VaR vs. Credit VaR

LO 26.a: Compare market risk value at risk (VaR) with credit VaR in terms of definition, time horizon, and tools for measuring them.

Credit risk VaR is defined as the credit risk loss that will not be exceeded over a given period of time within a specific level of confidence. Models can consider losses due to defaults, downgrades, and/or credit spread changes. Credit risk VaR is often used by banks to determine the amount of economic and regulatory capital.

Although credit risk VaR is similar to market risk VaR, there are some important differences, including:

- *Time horizon*. Market risk VaR is usually calculated with a one-day time horizon while credit risk VaR often uses a one-year time horizon.
- *Calculation tools*. While historical simulation is the primary tool for calculating market risk VaR, more elaborate modeling tools are often needed for credit risk VaR calculations.

Factors for Calculating Credit VaR

LO 26.b: Define and calculate credit VaR.

The following sections will describe methodologies that can be used to calculate credit VaR. Models for computing credit VaR must account for credit correlation, which is a recognition that defaults for different companies are not independent of one another. A strong economy will generally have a positive impact on companies such that default risk is lessened. However, a poor economy will have a negative impact on companies, which suggests that defaults will be more prominent. As credit correlation increases during economic downturns, financial institution risks will also increase.

LO 26.c: Describe the use of rating transition matrices for calculating credit VaR.

Rating transition matrices are often used by financial institutions when determining credit VaR. These matrices are based on historical data and reflect the likelihood that a company will migrate among rating categories over a given period of time. Figure 26.1 illustrates an actual one-year transition matrix published by S&P based on companies rated between 1981 and 2020.

Figure 26.1: One-Year Rating Transition Matrix for 1981–2020 (in %)

Initial Rating	Year-End Rating							
	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	89.85	9.35	0.55	0.05	0.11	0.03	0.05	0.00
AA	0.50	90.76	8.08	0.49	0.05	0.06	0.02	0.02
A	0.03	1.67	92.61	5.23	0.27	0.12	0.02	0.05
BBB	0.00	0.10	3.45	91.93	3.78	0.46	0.11	0.17
BB	0.01	0.03	0.12	5.03	86.00	7.51	0.61	0.70
B	0.00	0.02	0.08	0.17	5.18	85.09	5.66	3.81
CCC/C	0.00	0.00	0.12	0.20	0.65	14.72	50.90	33.42
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

The interpretation of this matrix is fairly straightforward, assuming rating changes are independent across periods. The highest probabilities are associated with a company maintaining its rating by year-end, as evidenced by the shaded boxes in Figure 26.1. For example, a company starting the year with an AA rating has a 90.76% chance of keeping its AA rating. For that same company (reading along the AA initial rating row), there is a 0.50% chance it gets upgraded to AAA and an 8.08% chance it gets downgraded to A.

To extend the analysis beyond one year, the matrix can be multiplied by itself. In other words, a three-year transition matrix is the third power of the one-year matrix. For example, given the 89.85% probability that a AAA-rated company keeps its rating over one year, there is a 72.54% chance it keeps its rating over three years (i.e., $0.8985^3 = 0.7254$).

Alternatively, a time period less than one-year can also be considered. A three-month period requires taking the fourth root for the percentages displayed in Figure 26.1. Using the same 89.85% probability as noted earlier, there is a 97.36% chance that the company will keep its AAA rating over three months (i.e., $0.8985^{1/4} = 0.9736$).

Therefore, as the time period extends out further, default probabilities become higher and the odds of maintaining the same credit rating are lower. As time periods shorten, default probabilities will be lower and the odds of maintaining the same credit rating are higher.

Note that the independence criteria across periods may be challenging, as ratings momentum indicates that a company that has been recently downgraded is more likely to experience another downgrade.

MODULE QUIZ 26.1

1. Which of the following statements most accurately reflects the time horizons typically used for market VaR and credit VaR calculations?
   A. Both are calculated for one day.
   B. Both are calculated for one year.
   C. Market risk VaR is calculated over a longer time period than credit risk VaR.
   D. Credit risk VaR is calculated over a longer time period than market risk VaR.

2. During a period of slowing economic growth, an analyst will likely identify which of the following trends regarding credit correlation and financial institution risks?
   A. A decrease in credit correlation and defaults.
   B. An increase in credit correlation and defaults.
   C. An increase in credit correlation and a decrease in defaults.
   D. A decrease in credit correlation and an increase in defaults.

3. Historical data shows that over a one-year period, there is a 91.93% chance that a company rated BBB will keep its rating. What is the percentage chance this rating will remain unchanged over a four-year period?
   A. 36.77%.
   B. 67.72%.
   C. 71.42%.
   D. 95.97%.

MODULE 26.2: CREDIT VaR MODELS

Vasicek's Model

LO 26.d: Describe the application of the Vasicek model to estimate capital requirements under the Basel II internal-ratings-based (IRB) approach.

Vasicek's Gaussian copula model is a method used on a portfolio of loans to calculate high percentiles of the distribution of the default rate. The model outputs WCDR(T,X), which is the worst-case default rate during time period *T* at the *X*th percentile of the default rate distribution. The model relates the probability of default (PD) to the credit correlation (ρ) and time period *T*.

$$\text{WCDR}(T,X) = N\left(\frac{N^{-1}(PD) + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right)$$

The *X*th percentile of the loss distribution for an individual loan can be calculated by determining the WCDR for each loan and multiplying it by the exposure at default (EAD) and the loss given default (LGD). For a large portfolio with *n* small loans, the *X*th percentile of the loss distribution can be approximated as:

$$\sum_{i=1}^{n} \text{WCDR}_i(T,X) \times \text{EAD}_i \times \text{LGD}_i$$

The credit correlation (ρ) used in Vasicek's model should be approximately equivalent to the correlation between the two companies returns on assets (ROA) or returns on equities (ROE). Therefore, the average correlation between company ROEs could be used to determine ρ for a portfolio of exposures. Similar publicly traded companies' average correlation can be used as a proxy when the companies being valued are not publicly traded. One challenge of Vasicek's model (that can be alleviated by using alternate models) is the lack of incorporating tail correlation.

LO 26.e: Interpret the Vasicek's model, Credit Risk Plus (CreditRisk+) model, and the CreditMetrics ways of estimating the probability distribution of losses arising from defaults as well as modeling the default correlation.

Credit Risk Plus Model

Credit Suisse Financial Products developed a credit VaR calculation methodology called the **Credit Risk Plus** (a.k.a. CreditRisk+) **model**. Assuming independent defaults, a *binomial distribution* can be used to estimate the number of defaults (*m*). In this case, the probability of *m* defaults, with *n* loans and *q* probability of default of each loan, can be expressed as follows:

$$P(m \text{ defaults}) = \frac{n!}{m!(n-m)!} q^m (1-q)^{n-m}$$

An alternative calculation for when the probability of default is small, and the number of loans is large uses a *Poisson distribution* as follows:

$$P(m \text{ defaults}) = \frac{e^{-qn}(qn)^m}{m!}$$

Although the default probability (*q*) is unknown, a reasonable assumption is that the expected number of defaults (i.e., *qn*) will follow a *gamma distribution* (with mean μ and standard deviation σ). The Poisson distribution then becomes a *negative binomial distribution* as follows:

$$P(m \text{ defaults}) = p^m (1-p)^{\alpha} \frac{\Gamma(m+\alpha)}{\Gamma(m+1)\Gamma(\alpha)}$$

where:

$\alpha = \mu^2 / \sigma^2$

$p = \sigma^2 / (\mu + \sigma^2)$

$\Gamma(x)$ = the gamma function

The outcome of this modeling technique is that as σ decreases, the negative binomial distribution will follow the same probability distribution as the Poisson distribution. On the other hand, as σ increases, the likelihood of experiencing a large number of defaults increases.

Monte Carlo simulation is a way to incorporate default rate uncertainty into the modeling process. The reality is that the default rate in one year is unlikely to be independent of the default rate in the prior year. A model which ties the default rate in a given year to prior year default rates or economic elements from the prior year is desirable. As default rate uncertainty rises, the probability of large numbers of defaults rises because default correlation increases, and the loss probability distribution has a positive skew (vs. being symmetrical with low or no default correlation).

CreditMetrics Model

Vasicek's model and Credit Risk Plus only account for defaults and not downgrades. The **CreditMetrics model** (from JPMorgan) was designed to account for both defaults and downgrades. This model uses a rating transition matrix as described earlier, with ratings coming from either internal ratings (generated from historical bank data) or external credit rating agencies.

Monte Carlo simulation is needed for one-year credit VaR calculations on portfolios of transactions with multiple counterparties. Each trial involves determining counterparty credit ratings at the end of one year, with the credit loss calculated for each counterparty. If the credit rating at year-end for a specific counterparty equates to a default, the loss is equal to EAD multiplied by LGD (where LGD = 1 − the recovery rate). If the credit rating at year-end equates to no default, the credit loss calculation is the value of all transactions with that counterparty at the end of the year. The term structure of credit spreads for each rating category is needed for these calculations. This term structure is assumed to be either the same as what is observable in the market or based on a credit spread index.

Assume that a CreditMetrics simulation trial has no default during the first year. The term structure of credit spreads at the one-year point provides the probability of default in each time interval beginning in Year 1. The credit loss for each simulation trial can be calculated using the following formula:

$$\text{credit loss} = \sum_{i=j}^{n}(1 - RR)(q_i^* - q_i)v_i$$

where:

jth interval begins at Year 1

RR = recovery rate

q_i = PD for the ith interval

q_i^* = PD for the ith interval ($i \geq j$) on a particular simulation trial

v_i = present value of expected net exposure (accounting for collateral) at the interval midpoint

An improvement in the counterparty's credit rating will likely produce a credit loss calculation that is negative. If a specific simulation trial produces a Year 1 default, the loss will be determined by assessing the default timing and the EAD and then multiplying it by 1 − RR. The CreditMetrics Monte Carlo simulation produces a probability distribution for total credit losses from defaults and downgrades across all counterparties. The credit VaR is then derived using this distribution.

Correlation Model

In the **correlation model**, credit rating changes for unique counterparties are assumed to be related (not independent). A joint probability distribution of rating changes can be constructed using a Gaussian copula model, with the correlation between rating transitions for two companies equated to the correlation between their equity returns.

Using partial data from earlier Figure 26.1, if the correlation between two companies is 0.2, in random sampling two variables (x_A and x_B) with correlations of 0.2 from standard normal distributions, the probability of a rating change for an A-rated company is illustrated in Figure 26.2.

Figure 26.2: Partial Rating Transition Matrix (in %)

Initial Rating	Year-End Rating		
	AAA	AA	A
AAA	89.85	9.35	0.55
AA	0.50	90.76	8.08
A	0.03	1.67	92.61

$$N^{-1}(0.0003) = -3.3416$$
$$N^{-1}(0.0003 + 0.0167) = -2.1201$$
$$N^{-1}(0.0003 + 0.0167 + 0.9261) = 1.5813$$

So, if $x_A < -3.3416$, the A-rated company will get upgraded to AAA. If x_A is between −3.3416 and −2.1201, it will become AA-rated. The same exercise can be done for evaluating a B-rated company with a rating change, and then a comparison is done for the A-rated and B-rated companies transitioning to other ratings across the spectrum.

Note that any differences in predicted outcomes between the CreditMetrics and Credit Risk Plus models using the same assumptions are likely due to the predicted timing of losses.

Credit Spread Risk

LO 26.f: Define credit spread risk and assess its impact on calculating credit VaR.

Credit-sensitive products have values that naturally depend on credit spreads. Credit VaR calculations will therefore involve assessing potential credit spread changes. A historical simulation can be used to calculate a one-day 99% VaR, which can in turn be adjusted to calculate VaR over longer time periods. Scenarios may assume that credit spread changes stay consistent for companies. However, there are two problems with this approach: (1) the lack of daily credit spread updates and (2) the fact that currently operating companies are unlikely to have defaulted in the past, so the assumption going forward is no default.

The CreditMetrics approach can be used here, where a rating transition matrix is developed over a specific period. Historical data on rating changes provide a probability distribution for credit spread changes over that same time period. Monte Carlo simulation is then used, where each trial samples the matrix to show whether a company keeps the same rating, changes to a different rating, or defaults. The credit spread for each rating category over that same time period will also be determined, which then facilitates a portfolio value for each trail as well as a credit VaR amount.

To introduce credit correlation, either a Gaussian copula model can be used for different company rating change correlations or rating category credit spread changes can be assumed to have very high correlations such that spreads for different rated instruments move in unison.

Suppose that a company holds a 3-year zero-coupon bond with a face value of $1,000. The risk-free rate is 2.5%, and the current credit spread is 150 basis points. The current price of the bond is computed as $\$1{,}000/(1.04)^3$, or $889. The bond has a current rating of BBB and the probable ratings one month from now, along with the three equally likely credit spreads associated with the ratings, are as follows:

- *Rating increase to A:* 1.0% probability; credit spreads: 70, 90, and 110
- *No change to current rating (BBB)*: 97.6% probability; credit spreads: 120, 150, and 180
- *Rating decrease to BB:* 1.3% probability; credit spreads: 300, 350, and 400
- *Default:* 0.1% (assume the bond would be worth $300 if it defaults)

The outcomes of these scenarios are shown in Figure 26.3, assuming equal probabilities associated with the three potential credit spreads assigned to each rating. For a 3-year bond, one month from now equates to 2.917 years remaining to maturity.

Figure 26.3: Computing Credit VaR With Credit Spreads

Rating	Spread	Probability	Bond Value ($)	Loss ($)
Default		0.100%	300.00	589.00
BB	400	0.433%	832.19	56.81
BB	350	0.433%	843.69	45.31
BB	300	0.433%	855.41	33.59
BBB	180	32.533%	884.43	4.57
BBB	150	32.533%	891.90	−2.90
BBB	120	32.533%	899.44	−10.44
A	110	0.333%	901.98	−12.98
A	90	0.333%	907.08	−18.08
A	70	0.333%	912.21	−23.21

The output shows that the credit VaR is $589.00 for a confidence level higher than 99.9%. For a confidence level between 99.9% and 99.467%, the credit VaR is $56.81.

For credit VaR calculations, a constant level of risk is often assumed such that risk is brought back to its initial levels through periodic portfolio rebalancing. A *constant level of risk strategy* is where a company holding bonds with a specific rating (like AA) would then sell those bonds if they no longer hold that rating and replace them with AA-rated bonds. A *buy-and-hold strategy* involves holding the original bond for a period of time (say, one year) and then selling.

The buy-and-hold strategy will generally produce larger losses from big downgrades and defaults than the constant level of risk strategy. However, small rating changes naturally produce lower probabilities of losses. Credit VaR tends to be smaller with the constant level of risk strategy.

MODULE QUIZ 26.2

1. Company A has an ROE of 8%, while Company B has an ROE of 6%. The average correlation between the two is 0.24, and both companies are publicly traded. The credit correlation (ρ) most likely used in Vasicek's Gaussian copula model will be closest to:
   A. 0.12.
   B. 0.24.
   C. 0.48.
   D. 0.76.

2. If an analyst wants a credit risk model that accounts for both defaults and downgrades, she will most likely use which of the following models?
   A. CreditMetrics.
   B. Credit Risk Plus.
   C. Vasicek's model.
   D. Monte Carlo simulation.

3. When accounting for credit spreads and potential bond losses for a bond currently rated A, an analyst will likely assign the:
   A. biggest spreads to situations where the bond rating increases.
   B. lowest probability to situations where the bond rating increases.
   C. lowest spreads to situations where the bond maintains its rating.
   D. highest probability to situations where the bond maintains its rating.

KEY CONCEPTS

LO 26.a

Credit risk VaR is defined as the credit risk loss that will not be exceeded over a given period of time within a specific confidence level. Key differences between credit risk VaR and market risk VaR include:

- Market risk VaR is usually calculated with a one-day time horizon. For credit risk VaR, a one-year time horizon is often used.
- While historical simulation is the primary tool for calculating market risk VaR, more elaborate modeling tools are often needed for credit risk VaR calculations.

LO 26.b

Credit VaR models must account for credit correlation, which is a recognition that defaults for different companies are not independent of one another. A strong economy will generally have a positive impact on companies such that default risk is lessened, while a poor economy will have a negative impact on companies such that defaults will be more prominent. As credit correlation increases during economic downturns, financial institution risks increase.

LO 26.c

Rating transition matrices can be used to calculate credit VaR. These matrices are based on historical data and will reflect the likelihood that a company will migrate among rating categories over a given period of time. Naturally, the highest probabilities are associated with a company maintaining its rating by year-end.

To extend the analysis beyond one year, the matrix can be multiplied by itself. For example, a three-year transition matrix is the third power of the one-year matrix. Alternatively, a time period less than one year can also be considered. As the time period extends out further, default probabilities are higher and the odds of maintaining the same credit rating are lower. As time periods shorten, default probabilities are lower and the odds of maintaining the same credit rating are higher.

LO 26.d

Vasicek's Gaussian copula model is a method used on a portfolio of loans to calculate high percentiles of the distribution of the default rate. The formula computes WCDR(T,X), which is the worst-case default rate during time period *T* at the *X*th percentile of the default rate distribution. The model relates the probability of default (PD) to the credit correlation (ρ) and time period *T*.

$$\mathrm{WCDR(T,X)} = \mathrm{N}\left(\frac{\mathrm{N}^{-1}(\mathrm{PD}) + \sqrt{\rho}\,\mathrm{N}^{-1}(\mathrm{X})}{\sqrt{1-\rho}}\right)$$

The *X*th percentile of the loss distribution for an individual loan can be calculated by determining the WCDR for each loan and multiplying it by the exposure at default (EAD) and the loss given default (LGD). For a large portfolio with *n* small loans, the *X*th percentile of the loss distribution can be approximated as:

$$\sum_{i=1}^{n} \mathrm{WCDR}_i(\mathrm{T,X}) \times \mathrm{EAD}_i \times \mathrm{LGD}_i$$

The credit correlation (ρ) used in Vasicek's model should be approximately equivalent to the correlation between the two companies returns on assets or returns on equities.

LO 26.e

Credit Suisse developed a credit VaR calculation methodology known as the Credit Risk Plus model. Assuming independent defaults, the binomial distribution can be used to estimate the number of defaults. An outcome of this modeling is that as volatility (σ) decreases, the negative binomial distribution will follow the same probability distribution as the Poisson distribution. As σ increases, the likelihood of experiencing a large number of defaults increases.

Monte Carlo simulation is a way to incorporate default rate uncertainty into the modeling process. A model that ties the default rate in a given year to prior year default rates or economic elements from the prior year is desirable. As default rate uncertainty rises, the probability of large numbers of defaults rises because default correlation increases, and the loss probability distribution has a positive skew (versus being symmetrical with low or no default correlation).

While Vasicek's model and Credit Risk Plus only account for defaults and not downgrades, the CreditMetrics model (from JPMorgan) accounts for both defaults and downgrades. The model uses a rating transition matrix, with ratings either from bank internal ratings or from external credit rating agencies.

Monte Carlo simulation is needed for one-year credit VaR calculations on portfolios of transactions with multiple counterparties. Each trial involves determining counterparty credit ratings at the end of one year, with the credit loss calculated for each counterparty. The term structure of credit spreads for each rating category is needed for these calculations and is assumed to be either the same as what is observable in the market or based on a credit spread index. The model produces a probability distribution for total credit losses from defaults and downgrades across all counterparties.

In the correlation model, credit rating changes for unique counterparties are assumed to be related (not independent). A joint probability distribution of rating changes can be constructed using a Gaussian copula model, with the correlation between rating transitions for two companies equated to the correlation between their equity returns.

LO 26.f

Credit-sensitive products have values that naturally depend on credit spreads. Credit VaR calculations will therefore involve assessing potential credit spread changes. A CreditMetrics approach can be used, where a rating transition matrix is developed over a specific period. Historical data on rating changes provide a probability distribution for credit spread changes over that same time period. Monte Carlo simulation is then used, where each trial samples the matrix to show whether a company keeps the same rating, changes to a different rating, or defaults.

To introduce credit correlation, either a Gaussian copula model can be used for different company rating change correlations or rating category credit spread changes can be assumed to have very high correlations such that spreads for different rated instruments move in unison.

A constant level of risk strategy is where a company holding bonds with a specific rating (like AA) would then sell those bonds if they no longer hold that rating and replace them with AA-rated bonds. A buy-and-hold strategy involves holding the original bond for a period of time (say, one year) and then selling. The buy-and-hold strategy will generally produce larger losses from big downgrades and defaults than the constant level of risk strategy. Credit VaR tends to be smaller with the constant level of risk strategy.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 26.1

1. **D** Market risk VaR is usually calculated over a one-day time horizon, while credit risk VaR will often use a one-year time horizon. (LO 26.a)
2. **B** When the economy is slowing, companies are negatively impacted, and defaults will become more prominent. At the same time, credit correlation (which captures the lack of independence between defaults for different companies) increases as well. (LO 26.b)
3. **C** If 91.93% is the likelihood that the company keeps its rating over a one-year period, then there is a 71.42% chance it keeps that rating over a four-year period. $0.9193^4 = 0.7142 = 71.42\%$. (LO 26.c)

Module Quiz 26.2

1. **B** The average correlation between company ROEs can be used to determine ρ. Because the average correlation is given as 0.24, that is the most likely correlation used in Vasicek's model. (LO 26.d)
2. **A** The CreditMetrics model is used to account for both defaults and downgrades, whereas Vasicek's model and the Credit Risk Plus model do not. Monte Carlo simulation is an underlying technique applied to various models. (LO 26.e)
3. **D** An analyst will likely assign the highest probability to situations where the bond maintains its rating. The biggest spreads will be for situations where the bond rating decreases, and the lowest spreads will be for situations where the bond rating increases. The lowest probability will likely be for a bond default. (LO 26.f)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Malz, Chapter 8.

READING 27

PORTFOLIO CREDIT RISK

Study Session 5

EXAM FOCUS

In this reading, we discuss the role that default correlation plays in measuring the default risk for a credit portfolio. For the exam, be prepared to list drawbacks of using default correlation and explain the single-factor model approach under the assumption that defaults are independent and returns are normally distributed. Know how to calculate the mean and standard deviation of the default distribution under the single-factor model conditional approach for correlations of 0 and 1 and the unconditional approach for correlations between 0 and 1. Lastly, be able to explain how VaR is determined using the single-factor model and copula methodology based on simulated terminal values.

MODULE 27.1: CREDIT PORTFOLIOS AND CREDIT VaR

Default Correlation for Credit Portfolios

LO 27.a: Define and calculate default correlation for credit portfolios.

Risks to consider when analyzing credit portfolios include default probability, loss given default (LGD), probability of deteriorating credit ratings, spread risk, and risk of loss through restructuring in bankruptcy. **Default correlation** measures the probability of multiple defaults for a credit portfolio issued by multiple obligors.

Suppose there are two firms whose probabilities of default over the next time horizon t are π_1 and π_2 for each firm, respectively. In addition, there is a joint probability that both firms will default over time horizon t equal to π_{12}.

The default correlation for this simple two-firm credit portfolio can be framed around the concept of Bernoulli-distributed random variables x_i, that have four possible outcomes over a specific time horizon t. Figure 27.1 illustrates the four possible random outcomes where 0 denotes the event of no default and 1 denotes default. The random variables for Firm 1 and 2 are x_1 and x_2. The probabilities of the four random events (Firm 1 defaults, Firm 2 defaults, both firms default, and neither firm defaults) are illustrated in Figure 27.1.

Figure 27.1: Default Probabilities for Two Firms

Event	x_1	x_2	$(x_1 x_2)$	Default Probability
Firm 1 defaults	1	0	0	$\pi_1 - \pi_{12}$
Firm 2 defaults	0	1	0	$\pi_2 - \pi_{12}$
Both default	1	1	1	π_{12}
No default	0	0	0	$1 - \pi_1 - \pi_2 + \pi_{12}$

Thus, the probability that one of the firms defaults or both firms default equals: $\pi_1 + \pi_2 - \pi_{12}$. Since the probabilities of all four events must equal 1, the probability that no firm defaults is $1 - \pi_1 - \pi_2 + \pi_{12}$. The means of the two Bernoulli-distributed default processes are: $E[x_i] = \pi_i$, where i equals 1 or 2. The expected value of joint default is simply the product of the two denoted as: $E[x_1x_2] = \pi_{12}$. The variances are computed as: $E[x_i]^2 - (E[x_i])^2 = \pi_i(1 - \pi_i)$ and the covariance is computed as: $E[x_1x_2] - E[x_1]E[x_2] = \pi_{12} - \pi_1 \pi_2$.

Equation 1 defines the default correlation for a two-firm credit portfolio as the covariance of Firm 1 and 2 divided by the standard deviations of Firm 1 and 2.

Equation 1

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

EXAMPLE: Calculating default correlation

Assume a portfolio of two credits, one rated BBB+ and one rated BBB, whose probabilities of default over the next time horizon t are 0.002 and 0.003, respectively. In addition, assume there is a joint probability that both credits will default over time horizon t equal to 0.00015. **Calculate** the default correlation for this credit portfolio.

Answer:

Default correlation can be calculated using the following formula for a two-credit portfolio:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}} = \frac{0.00015 - (0.002 \times 0.003)}{\sqrt{0.002(1-0.002)}\sqrt{0.003(1-0.003)}}$$

$$= \frac{0.000144}{\sqrt{0.001996}\sqrt{0.002991}} = \frac{0.000144}{(0.04468 \times 0.05469)}$$

$$= 0.0589 \text{ or } 5.89\%$$

Credit Portfolio Framework

LO 27.b: Identify drawbacks in using the correlation-based credit portfolio framework.

A major drawback of using the default correlation-based credit portfolio framework is the number of required calculations. For example, to specify all possible outcome events in a three firm framework requires three individual firm default outcome probabilities, three two-default outcome probabilities, the three-default outcome probability, and the no default outcome probability. Thus, there are 2^n event outcomes with only $(n + 1) + [n(n - 1) / 2]$ conditions. If we have ten firms, there will be 1,024 event outcomes with 56 conditions. The number of pairwise correlations is equal to $n(n - 1)$. In modeling credit risk, the pairwise correlations are often set to a single, nonnegative parameter.

In addition, certain characteristics of credit positions do not fit well in the default correlation credit portfolio model. Guarantees, revolving credit agreements, and other contingent liabilities have features similar to options that are not reflective of this simplistic framework. For example, credit default swap (CDS) basis trades may not be modeled simply by credit or market risk. Rather technical factors may play an important role as was evident in the subprime mortgage crisis where there was a lack of liquidity. Furthermore, convertible bonds have characteristics of credit and equity portfolios driven by market and credit risks.

Additional drawbacks in using the default correlation-based credit portfolio framework are related to the limited data for estimating defaults. Firm defaults are relatively rare events. Therefore, estimated correlations vary greatly depending on the data time horizon and industry. Most studies use an estimated correlation of 0.05. Thus, default correlations are small in magnitude, and the joint probability of two firms defaulting is even smaller.

Credit VaR

LO 27.c: Assess the impact of correlation on a credit portfolio and its Credit VaR.

LO 27.h: Assess the effect of granularity on Credit VaR.

The effects of default, default correlation, and loss given default are important determinants in measuring credit portfolio risk. A portfolio's **credit value at risk** (credit VaR) is defined as the quantile of the credit loss less the expected loss of the portfolio. Default correlation impacts the volatility and extreme quantiles of loss rather than the expected loss. Thus, default correlation affects a portfolio's credit VaR.

If default correlation is 1, then there are no credit diversification benefits, and the portfolio behaves as if there were just one credit position. A default correlation equal to 0 implies the portfolio is a binomial-distributed random variable because there is no correlation with other firms/credits.

EXAMPLE: Computing credit VaR (default correlation = 1, number of credits = n)

Suppose there is a portfolio with a value of $1,000,000 that has *n* credits. Each of the credits has a default probability of π percent and a recovery rate of zero. This implies that in the event of default, the position has no value and is a total loss.

What is the extreme loss given default and credit VaR at the 95% confidence level if π is 2% and the default correlation is equal to 1?

Answer:

With the default correlation equal to 1, the portfolio will act as if there is only one credit. Viewing the portfolio as a binomial-distributed random variable, there are only two possible outcomes for a portfolio acting as one credit. Regardless of whether the number of credits in the portfolio, *n*, is 1, 20, or 1,000, it will still act as one credit when the correlation is 1.

The portfolio has a π percent probability of total loss and a (1 − π) percent probability of zero loss. Therefore, with a recovery rate of zero, the extreme loss given default is $1,000,000. The expected loss is equal to the portfolio value times π and is $20,000 in this example (= 0.02 × $1,000,000). There is a 98% probability that the loss will be 0, given the fact that π equals 2%. The credit VaR is defined as the quantile of the credit loss minus the expected loss of the portfolio. Therefore, at the 95% confidence level, the credit VaR is equal to −$20,000 (= 0 minus the expected loss of $20,000).

Note that if π was greater than (1 − confidence level), the credit VaR would have been calculated as $1,000,000 − $20,000 = $980,000.

EXAMPLE: Computing credit VaR (default correlation = 0, number of credits = 50)

Again suppose there is a $1,000,000 portfolio with *n* credits that each have a default probability of π percent and a zero recovery rate. However, in this example the default correlation is 0, n = 50, and π = 0.02. In addition, each credit is equally weighted and has a terminal value of $20,000 if there is no default. The number of defaults is binomially distributed with parameters of n = 50 and π = 0.02. The 95th percentile of the number of defaults based on this distribution is 3. What is the credit VaR at the 95% confidence level based on these parameters?

Answer:

The expected loss in this case is also $20,000 (= $1,000,000 × 0.02). If there are three defaults, the credit loss is $60,000 (= 3 × $20,000). The credit VaR at the 95% confidence level is $40,000 (calculated by taking the credit loss of $60,000 and subtracting the expected loss of $20,000).

The term "granular" refers to reducing the weight of each credit as a proportion of the total portfolio by increasing the number of credits. As a credit portfolio becomes more granular, the credit VaR decreases. However, when the default probability is low, the credit VaR is not impacted as much when the portfolio becomes more granular.

EXAMPLE: Computing credit VaR (default correlation = 0, number of credits = 1,000)

Suppose there is a $1,000,000 portfolio with *n* credits that each have a default probability, π, equal to 2% and a zero recovery rate. The default correlation is 0 and n = 1,000. There is a probability of 28 defaults at the 95th percentile based on the binomial distribution with the parameters of n = 1,000 and π = 0.02. What is the credit VaR at the 95% confidence level based on these parameters?

Answer:

The 95th percentile of the credit loss distribution is $28,000 [= 28 × ($1,000,000 / 1,000)] The expected loss in this case is $20,000 (= $1,000,000 × 0.02). The credit VaR is then $8,000 (= $28,000 – expected loss of $20,000).

Thus, as the credit portfolio becomes more granular, the credit VaR decreases. For very large credit portfolios with a large number of independent credit positions, the probability that the credit loss equals the expected loss eventually converges to 100%.

MODULE QUIZ 27.1

1. Which of the following equations best defines the default correlation for a two-firm credit portfolio?

   A. $\rho_{12} = \dfrac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$

   B. $\rho_{12} = \dfrac{\pi_{12}}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$

   C. $\rho_{12} = \dfrac{\pi_{12}}{\sqrt{(1-\pi_1)}\sqrt{(1-\pi_2)}}$

   D. $\rho_{12} = \dfrac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1}\sqrt{\pi_2}}$

2. Suppose a portfolio manager is using a default correlation framework for measuring credit portfolio risk. How many unique event outcomes are there for a credit portfolio with eight different firms?
   A. 10.
   B. 56.
   C. 256.
   D. 517.

3. Suppose a portfolio has a notional value of $1,000,000 with 20 credit positions. Each of the credits has a default probability of 2% and a recovery rate of zero. Each credit position in the portfolio is an obligation from the same obligor, and therefore, the credit portfolio has a default correlation equal to 1. What is the credit value at risk at the 99% confidence level for this credit portfolio?
   A. $0.
   B. $1,000.

C. $20,000.
D. $980,000.

MODULE 27.2: CONDITIONAL DEFAULT PROBABILITIES AND CREDIT VaR WITH COPULAS

Conditional Default Probabilities

LO 27.d: Describe the use of a single factor model to measure portfolio credit risk, including the impact of correlation.

LO 27.e: Define beta and calculate the asset return correlation of any pair of firms using the single factor model.

The **single-factor model** is used to examine the impact of varying default correlations based on a credit position's beta. Each individual firm or credit, *i*, has a beta correlation, β_i, with the market, *m*. Firm *i*'s individual asset return is defined as:

Equation 2

$$a_i = \beta_i m + \sqrt{1-\beta_i^2}\,\varepsilon_i$$

where:

$\sqrt{1-\beta_i^2}$ = firm's standard deviation of idiosyncratic risk
ε_i = firm's idiosyncratic shock

Assuming that each ε_i is not correlated with other credits, each return on asset, a_i, is a standard normal variate. The correlation between pairs of individual asset returns between two firm's *i* and *j* is $\beta_i\beta_j$. The model assumes that Firm *i* defaults if $a_i \le k_i$, the logarithmic distance to the defaulted asset value that is measured by standard deviations.

An important property of the single-factor model is conditional independence. Conditional independence states that once asset returns for the market are realized, default risks are independent of each other. This is due to the assumption for the single-factor model that return and risk of assets are correlated only with the market factor. The property of conditional independence makes the single-factor model useful in estimating portfolio credit risk.

So, how can the single-factor model be used to measure default probabilities that are conditional on market movements or economic health? Suppose that the market factor, *m*, has a specific value of $\bar{m}$. Substituting this value $\bar{m}$ into Equation 2 and subtracting $\beta_i \bar{m}$ from both sides results in Equation 3. Default risk is measured by the distance to default, $a_i - \beta_i\bar{m}$. This distance to default either increases or decreases, and the only random parameter is the idiosyncratic shock, ε_i.

Equation 3

$$a_i - \beta_i\overline{m} = \sqrt{1-\beta_i^2}\,\varepsilon_i$$

As a result of this conditioning, the default distribution's mean shifts based on the specific market value for any beta, β_i, that is greater than zero. The default threshold, k_i, does not change, but the standard deviation of the default distribution is reduced from 1 to $\sqrt{1-\beta_i^2}$.

The unconditional default distribution is a standard normal distribution. However, the conditional distribution is a normal distribution with a mean of $\beta_i\overline{m}$ and a standard deviation of $\sqrt{1-\beta_i^2}$. Specifying a specific value $\overline{m}$ for the market parameter, m, in the single-factor model results in the following implications:

1. The conditional probability of default will be greater or smaller than the unconditional probability of default as long as $\overline{m}$ or β_i are not equal to zero. This reduces the default triggers or number of idiosyncratic shocks, ε_i, so that it is less than or equal to $k_i - \beta_i\overline{m}$. As the market factor goes from strong to weak economies, a smaller idiosyncratic shock will trigger default.
2. The conditional standard deviation $\sqrt{1-\beta_i^2}$ is less than the unconditional standard deviation of 1.
3. Individual asset returns, a_i, and idiosyncratic shocks, ε_i, are independent from other firms' shocks and returns.

Conditional Default Distribution Variance

LO 27.f: Using the single factor model, estimate the probability of a joint default of any pair of credits and the default correlation between any pair of credits.

Suppose a firm has a beta, β_i, equal to 0.5 and a default threshold, k_i, equal to −2.33. The unconditional probability of default $\Phi(-2.33) = 0.01$. If the market return is −0.5, what is the conditional variance of the default distribution using the single-factor model?

PROFESSOR'S NOTE
Recall that the symbol Φ represents a standard normal distribution function.

The conditional distribution is a normal distribution with a mean of $\beta_i\overline{m}$ and a conditional variance of $1-\beta_i^2$. For this example, the mean is $\beta_i\overline{m} = 0.5(-0.5) = -0.25$, and the conditional variance is $1 - 0.5^2 = 0.75$. The conditional standard deviation is then 0.866 (the square root of the variance of 0.75).

The conditional cumulative default probability function is stated as a function of m as follows:

$$p(m) = \Phi\left(\frac{k_i - \beta_i\overline{m}}{\sqrt{1-\beta_i^2}}\right)$$

The mean is the new distance to default based on the realized market factor, $\beta_i \overline{m}$, and the standard deviation assumes conditional independence and is equivalent to $\sqrt{1-\beta_i^2}$. Thus, given a realized market factor, $\overline{m}$, the probability of default is based on the distance of the new default trigger of idiosyncratic shocks, ε_i, measured in standard deviations below its mean of zero.

PROFESSOR'S NOTE
For the exam, focus on how to calculate the parameters of the distribution (e.g., the mean and the standard deviation).

If we assume that distribution parameters (β, k, and π) are equal for all firms, then the probability of a joint default for two firms can be defined as:

$$\Phi\binom{k}{k} = P\left[-\infty \leq a \leq k, -\infty \leq a \leq k\right]$$

This assumption also allows us to define the default correlation for any pair of firms as follows:

Equation 4

$$\rho = \frac{\Phi\binom{k}{k} - \pi^2}{\pi(1-\pi)}$$

Although the derivation of this default correlation equation is not required for the exam, you may wish to understand how Equation 1 (from LO 27.a) is used to derive Equation 4.

The single-factor model assumes the cumulative return distribution of any pair of credit positions *i* and *j* is distributed as a bivariate standard normal distribution with a correlation coefficient equal to $\beta_i\beta_j$. The cumulative distribution function for this pair, *i* and *j*, is $\Phi\binom{a_i}{a_j}$. We are interested in the probability of a joint default that will occur in the extreme tail of the distributions. Thus, the probability that the realized value for credit *i*, a_i, is less than the default threshold, or critical value, k_i, and is denoted for the pair of credits *i* and *j* as:

$$\Phi\binom{k_i}{k_j} = P\left[-\infty \leq a_i \leq k_i, -\infty \leq a_j \leq k_j\right]$$

In LO 27.a, Equation 1 defined the default correlation as the covariance of Firm 1 and 2 divided by the standard deviations of Firm 1 and 2 as follows:

$$\rho_{ij} = \frac{\pi_{ij} - \pi_i\pi_j}{\sqrt{\pi_i(1-\pi_i)}\sqrt{\pi_j(1-\pi_j)}}$$

Substituting $\Phi\binom{k_i}{k_j}$ for π_{ij} results in:

$$\rho_{ij} = \frac{\Phi\binom{k_i}{k_j} - \pi_i\pi_j}{\sqrt{\pi_i(1-\pi_i)}\sqrt{\pi_j(1-\pi_j)}}$$

If we assume that parameters (β, k, and π) are equal for all firms, then the pairwise asset return correlation for any two firms must equal β^2 and the previous equation simplifies to Equation 4.

Credit VaR With a Single-Factor Model

Previously, the loss distribution was estimated when the default correlation was either 0 or 1. In order to define the distribution of loss severity for values between 0 and 1, we need to determine the unconditional probability of default loss. Using the single-factor model framework, the unconditional probability of a default loss level is equal to the probability that the realized market return results in a default loss. In other words, the individual credit asset returns, a_i, are strictly a function of the market return and the asset return's correlation, or β_i, with the market. The unconditional distribution used to calculate credit VaR is determined by the following steps:

1. The default loss level is assumed to be a random variable X with realized values of x. Under this framework, x is not simulated.
2. Given a loss level of x, the value for the market factor, m, is determined at the probability of the stated loss level. The relationship between the loss level and market factor return is equal to:

   $$x(m) = p(m) = \Phi\left(\frac{k - \beta\overline{m}}{\sqrt{1-\beta^2}}\right)$$

   The market factor return, $\overline{m}$, for a given loss level, $\overline{x}$, is determined based on the following relationship:

   $$\Phi^{-1}(\overline{x}) = \left(\frac{k - \beta\overline{m}}{\sqrt{1-\beta^2}}\right)$$

3. The market factor is assumed to be standard normal, and therefore, a loss level of 0.01 (99% confidence level) is equal to a value of −2.33 based on the standard normal distribution.
4. These steps are repeated for each individual credit to determine the loss probability distribution.

EXAMPLE: Realized market value

Suppose a credit position has a correlation to the market factor of 0.25. What is the realized market value used to compute the probability of reaching a default threshold at the 99% confidence level?

Answer:

At the 99% confidence level, the default loss level has a default probability, π, of 0.01. A default loss level of 0.01 corresponds to −2.33 on the standard normal distribution. The relationship between the default loss level and the given market return, $\overline{m}$, is defined by:

$$p(\overline{m}) = 0.01 = \Phi\left(\frac{k - \beta\overline{m}}{\sqrt{1 - \beta^2}}\right)$$

This is approximately equal to the probability of obtaining a realized market return of −2.33 as follows:

$$\Phi^{-1}(0.01) \approx -2.33 = \left(\frac{k - \beta\overline{m}}{\sqrt{1 - \beta^2}}\right)$$

The realized market value is computed as follows:

$$-2.33 = \frac{-2.33 - (0.25)\overline{m}}{\sqrt{1 - 0.25^2}}$$
$$-2.33(0.9682) = -2.33 - (0.25)\overline{m}$$
$$-2.256 + 2.33 = -(0.25)\overline{m}$$
$$0.074 = -(0.25)\overline{m}$$
$$-0.296 = \overline{m}$$

The probability that the default threshold is reached is the same probability that the realized market return is −0.296 or lower.

The parameters play an important role in determining the unconditional loss distribution. The probability of default, π, determines the unconditional expected default value for the credit portfolio. The credit position's correlation to the market, β, determines the dispersion of the defaults based on the range of the market factor.

Credit VaR With Simulation

LO 27.g: Describe how Credit VaR can be calculated using a simulation of joint defaults.

Copulas provide a mathematical approach for determining how defaults are correlated with one another using simulated results. The following four steps are used to compute a credit VaR under the copula methodology:

1. Define the copula function.
2. Simulate default times.
3. Obtain market values and profit and loss data for each scenario using the simulated default times.
4. Compute portfolio distribution statistics by adding the simulated terminal value results.

EXAMPLE: Computing credit VaR with a copula

Suppose there is a credit portfolio with two loans (rated CCC and BB) that each has a notional value of $1,000,000. Figure 27.2 illustrates four possible event outcomes over a default time horizon of one year for this credit portfolio. The four event outcomes are only the BB rated loan defaults, only the CCC rated loan defaults, both loans default, or no loans default.

Figure 27.2: Event Outcomes for a Two-Credit Portfolio

Event	Default Time
BB default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} > 1)$
CCC default	$(\tau_{BB,i} > 1, \tau_{CCC,i} \leq 1)$
Both default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} \leq 1)$
No default	$(\tau_{BB,i} > 1, \tau_{CCC,i} > 1)$

How can credit VaR be estimated for this portfolio assuming a correlation of 0.25?

Answer:

The copula approach to estimating credit VaR is applied using the following steps:

1. The first step is to simulate 1,000 values using a copula function. The most common copula used to calculate credit VaR is the normal copula.
2. The 2,000 simulated values (1,000 pair simulations results in 2,000 values) are then mapped to their standard univariate normal quantile which results in 1,000 pairs of probability values.
3. The first and second elements of each probability pair are mapped to the BB and CCC default times, respectively.
4. A terminal value is assigned to each loan for each simulation. The values are added up for the two loans, and the sum of the no-default event value is subtracted to determine the loss. Figure 27.3 summarizes the sum of the terminal values and losses for 1,000 simulations.

Figure 27.3: Event Outcomes for a Two-Credit Portfolio

Event	Default Time	Terminal Value	Loss
BB default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} > 1)$	1,480,000	710,000
CCC default	$(\tau_{BB,i} > 1, \tau_{CCC,i} \leq 1)$	1,410,000	780,000
Both default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} \leq 1)$	700,000	1,490,000
No default	$(\tau_{BB,i} > 1, \tau_{CCC,i} > 1)$	2,190,000	0

The loss level sums from the simulation are then used to determine the credit VaR based on the simulated distribution. In this simulation, the 99% confidence level corresponds to the $1,490,000 loss where both loans default. The 95% confidence level corresponds to the $780,000 value because the lower 5% of the simulated values resulted in defaults with a total loss of $780,000.

MODULE QUIZ 27.2

1. A portfolio manager uses the single-factor model to estimate default risk. What is the mean and standard deviation for the conditional distribution when a specific realized market value $\overline{m}$ is used?

A. The mean and standard deviation are equivalent in the standard normal distribution.
B. The mean is $\beta_i \overline{m}$ and the standard deviation is $\sqrt{1-\beta_i^2}$.
C. The mean is $\overline{m}$ and the standard deviation is β_i.
D. The mean is $\overline{m}$ and the standard deviation is 1.

2. Suppose a credit position has a correlation to the market factor of 0.5. What is the realized market value that is used to compute the probability of reaching a default threshold at the 99% confidence level?
   A. −0.2500.
   B. −0.4356.
   C. −0.5825.
   D. −0.6243.

KEY CONCEPTS

LO 27.a

The default correlation for a two credit portfolio assuming the outcomes are Bernoulli-distributed random variables is:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

LO 27.b

Drawbacks of using the default correlation-based credit portfolio framework are the number of required calculations (2^n event outcomes with $(n + 1) + [n(n - 1) / 2]$ conditions), certain characteristics of credit positions do not fit well in the default correlation credit portfolio model, and the limited data for estimating defaults due to the fact that firm defaults are relatively rare events.

LO 27.c

Default correlation affects a portfolio's credit value at risk (credit VaR). A default correlation equal to 0 implies the portfolio is a binomially distributed random variable. As a credit portfolio becomes more granular, the credit VaR decreases.

A portfolio's credit VaR can be defined as the quantile of the credit loss less the expected loss of the portfolio.

LO 27.d

In the single-factor model, Firm *i*'s individual asset return is defined as:

$a_i = \beta_i m + \sqrt{1-\beta_i^2}\,\varepsilon_i$ where $\sqrt{1-\beta_i^2}$ is the firm's standard deviation of idiosyncratic risk, and ε_i is the firm's idiosyncratic shock. The model assumes that Firm *i* defaults if $a_i \leq k_i$.

The single-factor model framework states that the unconditional probability of a default loss level is equal to the probability that the realized market return results in a default loss. The market factor is assumed to be standard normal. The credit position's correlation to the market, β, determines the dispersion of the defaults based on the range of the market factor.

LO 27.e

Each individual firm or credit, *i*, has a beta correlation, β_i, with the market, *m*. The correlation between pairs of individual asset returns between two firm's *i* and *j* is $\beta_i\beta_j$.

LO 27.f

If we assume that distribution parameters (β, *k*, and π) are equal for all firms, then the probability of a joint default for two firms can be defined as:

$$\Phi\left(\begin{smallmatrix}k\\k\end{smallmatrix}\right) = P\left[-\infty \leq a \leq k, -\infty \leq a \leq k\right]$$

This assumption also allows us to define the default correlation for any pair of firms as follows:

$$\rho = \frac{\Phi\left(\begin{smallmatrix}k\\k\end{smallmatrix}\right) - \pi^2}{\pi(1-\pi)}$$

LO 27.g

A credit VaR under the copula methodology is computed by: defining the copula function, simulating default times, obtaining market values and profit and loss data for each scenario using the simulated default times, and computing the portfolio distribution statistics by adding the simulated terminal value results.

LO 27.h

As a credit portfolio becomes more granular, credit VaR decreases. However, when the default probability is low, credit VaR is not impacted as much when the portfolio becomes more granular.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 27.1

1. **A** The default correlation for a two-firm credit portfolio is defined as:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

(LO 27.a)

2. **C** There are 256 event outcomes for a credit portfolio with eight different firms calculated as: $2^8 = 256$. (LO 27.b)

3. **D** With the default correlation equal to 1, the portfolio will act as if there is only one credit. Viewing the portfolio as a binomial distributed random variable, there are only two possible outcomes for a portfolio acting as one credit. The portfolio has a 2% probability of total loss and a 98% probability of zero loss. Therefore, with a recovery rate of zero, the extreme loss given default is $1,000,000. The

expected loss is equal to the portfolio value times π and is $20,000 in this example (0.02 × $1,000,000). The credit VaR is defined as the quantile of the credit loss less the expected loss of the portfolio. At the 99% confidence level, the credit VaR is equal to $980,000 ($1,000,000 minus the expected loss of $20,000). (LO 27.c)

Module Quiz 27.2

1. **B** The conditional distribution is a normal distribution with a mean of $\beta_i\overline{m}$ and a standard deviation of $\sqrt{1-\beta_i^2}$. (LO 27.d)

2. **D** A default loss level of 0.01 corresponds to −2.33 on the standard normal distribution. The realized market value is computed as follows:

$$-2.33 = \frac{-2.33 - (0.5)\overline{m}}{\sqrt{1-0.5^2}}$$
$$-2.33(0.86603) = -2.33 - (0.5)\overline{m}$$
$$-2.01785 + 2.33 = -(0.5)\overline{m}$$
$$0.31215 = -(0.5)$$
$$-0.62430 = \overline{m}$$

(LO 27.d)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Malz, Chapter 9.

READING 28

STRUCTURED CREDIT RISK

Study Session 5

EXAM FOCUS

In this reading, we discuss common structured products, capital structure in securitization, structured product participants, a basic waterfall structure, and the impact of correlation. For the exam, understand the qualitative impacts of changing default probability and default correlation for all tranches for mean (average) and risk (credit VaR). Default sensitivity (similar to DV01) is introduced. Understand the process to compute implied correlation extracted from observable market prices.

MODULE 28.1: STRUCTURED PRODUCTS

Types of Structured Products

LO 28.a: Describe common types of structured products.

Securitization and structured products are two of the most important financial innovations in recent memory. Securitization is basically the pooling of credit-sensitive assets and the associated creation of new securities (structured products or portfolio credit products) whose cash flows are based on underlying loans or credit claims. Each product has its own risk and return characteristics, which can vary dramatically from the original assets. For this section, a partial list of structured products and factors that affect their valuation are discussed.

Covered Bonds

Covered bonds are on-balance sheet securitizations. A pool of mortgages, which secure a bond issue, is separated from other loans into a covered pool on the originator's balance sheet. Investors have higher priority than general creditors if a bank defaults. Principal and interest is paid and guaranteed by the originator and is not based on the performance of the underlying assets themselves. Thus, covered bonds are not true securitizations since the assets are not part of a bankruptcy-remote structure and the investors have recourse against the originator.

Mortgage Pass-Through Securities

In contrast to covered bonds, mortgage pass-through securities are true off-balance sheet securitizations. Investors receive cash flows based entirely on the performance of the pool less associated fees paid to the servicer. Most pass-throughs are agency mortgage-backed securities (MBSs) that carry implicit or explicit government guarantee of performance. Thus, default risk is not a serious concern. The primary risk is due to prepayment of principal by the homeowner, most likely from refinancing after interest rate declines or home sales.

Collateralized Mortgage Obligations (CMOs)

CMOs are MBSs that tranche (i.e., divide) cash flows into different securities based on predetermined conditions. The resulting tranches can have long or short maturities, fixed or floating cash flows, or other varieties and conditions. The most basic structure is the *waterfall* or *sequential pay structure* where Tranche 1 receives all principal and its portion of interest in each period until it is paid off. The remaining tranches will receive interest only until Tranche 1 is retired and then principal will flow down to Tranche 2, and so on. Not surprisingly, Tranche 1 will have a very low prepayment risk as it expects to receive all principal payments before other bondholders.

Structured Credit Products

Like other structured products, this pool of assets is backed by risky debt instruments. The difference is that structured credit products create tranches that have different amounts of credit risk. The most junior (i.e., equity) tranches bear the first losses and are most likely to be written down from defaulted assets. If the equity tranche is completely wiped out, the next most junior tranche will bear the credit risk of subsequent defaults. The most senior tranche will have the highest credit rating and the lowest probability of write-downs.

Asset-Backed Securities

This is the most general class of securitizations where cash-flow generating assets are pooled and subsequently tranched. Under this definition, MBS is a special case of the more general ABS. Other varieties include collateralized bond obligations (CBOs), collateralized debt obligations (CDOs), collateralized loan obligations (CLOs), and collateralized mortgage obligations (CMOs). There exist even more complex securities that pool other securitizations together such as CDO-squared (CDO of CDOs).

Structured credit products can also vary across other dimensions. First, the underlying collateral of the pool can consist of loans, bonds, credit card receivables, auto loans, and even non-debt instruments that generate cash flows, such as toll collections. Second, the size and number of tranches is specific to each transaction. Third, the pool can be passive or actively managed. In a passive pool, the existing assets, such as mortgages and auto loans, will eventually pay themselves down. On the other hand, actively managed pools will selectively add or shed assets from the pool. Managers with key insight should be able to enhance the performance of the pool by identifying overvalued and undervalued loan products. Revolving pools have a period of time where loan proceeds are reinvested in new assets. Once the revolving period ends, the asset balances are fixed (e.g., credit card balances) and will spend themselves down.

Capital Structure in Securitization

LO 28.b: Describe tranching and the distribution of credit losses in a securitization.

The capital structure of a securitization refers to the priority assigned to the different tranches. In general, the most **senior tranches** at the top of the capital structure will have the highest priority to receive principal and interest. Since these securities are perceived to be the safest, they also receive the lowest coupon.

The **equity tranche** is the slice of the cash flow distribution with the lowest priority and will absorb the first losses up to a prespecified level. These securities typically do not carry a fixed coupon but receive the residual cash flows only after the other security claims are satisfied. Therefore, the return is variable and, hence, the term "equity." Typically, the equity tranche is the smallest part of the capital structure.

Between the senior and equity tranches is the **mezzanine tranche** (i.e., the junior tranche). The mezzanine tranches will absorb losses only after the equity tranche is completely written down. Thus, the senior tranches are protected by both the equity and mezzanine debt (termed subordination or credit enhancement). Terminology-wise, the mezzanine debt attaches to the equity tranche from above and detaches from the senior tranche from below. These junior debt claims offer a relatively high coupon (if the claim is fixed) or high spread (if the claim is floating). To keep the securitization viable, the mezzanine tranches will be purposefully thin.

There are many creative ways to provide credit protection to various security classes, but this must come at the expense of shifting risk to other parts of the capital structure. In general, credit enhancement can be divided into internal and external credit enhancement mechanisms. The term **external credit enhancement** means that the credit protection takes the form of insurance or wraps purchased from a third party, typically a monoline insurer.

Two examples of **internal credit enhancements** are overcollateralization and excess spread. **Overcollateralization** is when the pool offers claims for less than the amount of the collateral. For example, consider a collateralized MBS with 101 mortgages in the collateral pool, but the face value of the bonds across all tranches only totals 100 mortgages. Overcollateralization is a *hard credit enhancement* because the protection is available at the origination of the pool.

The **excess spread** is the difference between the cash flows collected and the payments made to all bondholders. For example, if the weighted average of the collateral is 8% (net of fees) and the weighted average of the payments promised to the senior, junior, and equity tranches is 7%, then the residual 1% accumulates in a separate trust account. The excess spread will be invested and is available to make up future shortfalls. Since the excess spread is zero at origination, it is considered a *soft credit enhancement.*

Waterfall Structure

LO 28.c: Describe a waterfall structure in a securitization.

A waterfall structure outlines the rules and conditions that govern the distribution of collateral cash flows to different tranches. In the simplest example of a securitization, the senior and junior bonds will receive their promised coupons conditional on a sufficient amount of cash inflows from the underlying loans. The residual cash flow, if any, is called the excess spread. The overcollateralization triggers will decide how the excess is divided between the equity investors and the accumulating trust. Intuitively, the underlying cash flows will be largest in the earlier periods so the trust will build up a reserve against future shortfalls.

In practice, this process can be quite complex as there may be a dozen tranches or more with different coupons, maturities, and overcollateralization triggers. The waterfall is further complicated by loan defaults. A simplifying assumption would incorporate a constant default rate which can be built into the waterfall distribution. As the loans mature, the actual incidence of the loan defaults will increase or decrease the value of the respective tranches. For example, suppose that fewer loans default than previously assumed, then collateral cash flows are larger than expected and will benefit all bondholders, in particular, the equity tranche.

Let's analyze the cash flows in a waterfall structure by considering the following examples. Assume there are 1,000 identical loans with a value of $1 million each. The interest rate on the loans is floating with a rate equal to the market reference rate + 300 bps, reset annually. The senior, junior, and equity tranches are 80%, 15%, and 5% of the pool, respectively. The spreads on the senior and mezzanine tranches are 1% and 5%. There is one overcollateralization trigger where the equity holders are entitled to a maximum of $15 million and any excess is diverted to the excess trust account. To begin, assume the default rate is 0%. The cash flows for the waterfall structure are detailed in Figure 28.1.

Note that the senior tranche has a principal value of $800 million while the junior tranche has an initial principal of $150 million. Using a current market reference rate of 5%, their respective coupons are 6% (= 5% + 1% spread) and 10% (= 5% + 5% spread). The total cash flows flowing into the pool are $1 billion × 8% = $80 million, which is sufficient to pay the senior and junior claims. The residual cash flow is $80 million – ($48 million + $15 million) = $17 million.

Next, the overcollateralization test must be applied. Since the maximum the equity tranche can receive is $15 million, the equity investors will receive the full $15 million and the excess of $2 million will flow into the trust account. This is shown in the last row of Figure 28.1.

Now assume that the expected default rate is 4% each year. The first difference from the 0% default rate example is that the total loan proceeds is reduced by defaulted loans: $1 billion × 8% × (1 – 0.04) = $76.8 million. There is still sufficient cash flow to pay the senior and junior bondholders in full. However, when the overcollateralization test is applied, the equity holders will not reached their maximum. Therefore, the equity tranche receives only $13.8 million and there is no diversion to the trust account as shown in Figure 28.2.

Figure 28.1: Waterfall Structure (Default Rate = 0%)

Loan Information	
# loans	1,000
Value of identical loan	$1,000,000
Principal amount	$1,000,000,000
Reference rate	5.00%
Spread	3.00%
Coupon	8.00%
Default rate	0.00%
OC trigger	$15,000,000

Tranche Information		
Senior % of pool		80%
	Reference rate	5.00%
	Spread	1.00%
	Coupon	6.00%
Mezzanine % of pool		15%
	Reference rate	5.00%
	Spread	5.00%
	Coupon	10.00%
Equity % of pool		5%

Period	Loan Proceeds	
1	$80,000,000	
Senior Principal	**Senior Coupon**	**Interest**
$800,000,000	6.00%	$48,000,000
Mezzanine Principal	**Mezzanine Coupon**	**Interest**
$150,000,000	10.00%	$15,000,000
Excess CF	**CF to Equity**	**CF to Trust**
$17,000,000	$15,000,000	$2,000,000

Figure 28.2: Waterfall Structure (Default Rate = 4%)

Loan Information	
# loans	1,000
Value of identical loan	$1,000,000
Principal amount	$1,000,000,000
Reference rate	5.00%
Spread	3.00%
Coupon	8.00%
Default rate	4.00%
OC trigger	$15,000,000

Tranche Information		
Senior % of pool		80%
	Reference rate	5.00%
	Spread	1.00%
	Coupon	6.00%
Mezzanine % of pool		15%
	Reference rate	5.00%
	Spread	5.00%
	Coupon	10.00%
Equity % of pool		5%

Period	Loan Proceeds	
1	$76,800,000	
Senior Principal	**Senior Coupon**	**Interest**
$800,000,000	6.00%	$48,000,000
Mezzanine Principal	**Mezzanine Coupon**	**Interest**
$150,000,000	10.00%	$15,000,000
Excess CF	**CF to Equity**	**CF to Trust**
$13,800,000	$13,800,000	$0

MODULE QUIZ 28.1

1. How many of the following statements concerning the capital structure in a securitization are most likely correct?
   I. The mezzanine tranche is typically the smallest tranche size.
   II. The mezzanine and equity tranches typically offer fixed coupons.
   III. The senior tranche typically receives the lowest coupon.

   A. No statements are correct.
   B. One statement is correct.
   C. Two statements are correct.
   D. Three statements are correct.

2. Assume there are 100 identical loans with a principal balance of $500,000 each. Based on a credit analysis, a 300 basis point spread is applied to the borrowers. The market reference rate is currently 4% and the coupon rate will reset annually. The senior,

junior, and equity tranches are 75%, 20%, and 5% of the pool, respectively. The spreads on the senior and mezzanine tranches are 2% and 6%. Excess cash flow is diverted above $1,000,000. Assume the default rate is 2%. What are the cash flows to the mezzanine and excess trust account in the first period?

	Mezzanine	Trust account
A.	$1,000,000	$0
B.	$1,000,000	$180,000
C.	$2,250,000	$200,000
D.	$2,250,000	$250,000

MODULE 28.2: SECURITIZATION

Securitization Participants

LO 28.d: Identify the key participants in the securitization process and describe conflicts of interest that can arise in the process.

The nature of the securitization process from original loan to tranche issuance necessarily involves many different participants. The first step begins with the **originator** who funds the loan. The originator may be a bank, mortgage lender, or other financial intermediary. The term "sponsor" may be used if the originator supplies most of the collateral for the issue.

The **underwriter** performs a function similar to the issuance of traditional debt and equity. The underwriter structures the issue (i.e., engineers the tranche size, coupon, and triggers, and sells the bonds to investors). The underwriter warehouses the collateral and faces the risks that the issue will not be marketed or that the collateral value will drop.

The **credit rating agencies (CRAs)** are an important part of the securitization process. Without their explicit approval via credit ratings, investors would be at a severe disadvantage to assess the riskiness of the issue. The credit rating agencies can influence the size of the tranches by selecting the attachment points and thus are active participants in the process. In addition, the CRAs may influence the issue by requiring enhancements. There is a natural conflict of interest because the CRAs want to generate profit and grow their business, but it may come at the expense of allocating larger portions of the capital structure to lower interest paying senior notes. Investors can alleviate this concern by performing their own (costly) analysis or purchasing a wrap or insurance against the issue.

The role of the servicer is multifaceted and possibly understated. The servicer must collect and distribute the collateral cash flows and the associated fees. In addition, the servicer may need to provide liquidity if payments are late and resolve default situations. It is not hard to envision the conflict of interest in foreclosure: the servicer would, all else equal, like to delay foreclosure to increase their fees, while investors want as quick of a resolution as possible to minimize the damage and/or lack of maintenance from the homeowners who have no economic incentive to maintain the property.

When the pool is actively managed, another source of conflict arises. The manager naturally would like to minimize their effort to continually monitor the credit quality of the collateral unless there is a clear incentive to do so. A common feature of securitized pools is for the originator and/or manager to bear the first loss in the capital structure.

Custodians and trustees play an administrative role verifying documents, disbursing funds, and transferring funds between accounts.

Three-Tiered Securitization Structure

LO 28.e: Compute and evaluate one or two iterations of interim cashflows in a three-tiered securitization structure.

LO 28.f: Describe the treatment of excess spread in a securitization structure and estimate the value of the overcollateralization account at the end of each year.

LO 28.g: Explain the tests on the excess spread that a custodian must go through at the end of each year to determine the cash flow to the overcollateralization account and to the equity noteholders.

The cash flows in a three-tiered securitization (senior, mezzanine, and equity) can be broken out into the inflows from the collateral and the outflows to the investors. The inflows prior to maturity are the interest on the collateral (L_t) plus the recovery from the sale of any defaulted assets in the current period (R_t). Assume the collateral pool has N identical loans with coupons = market reference rate + spread. The terminal cash flows in the final year are the last interest payment plus principal and recovery of defaulted assets. As an additional consideration, the recovered funds from defaults would earn interest over the remaining life of the pool at r.

The outflows are the coupon payments paid to senior and mezzanine note holders, collectively denoted B (assumed constant). The equity holder position is a bit more complicated because the excess spread trust has first priority on the cash flows to provide soft credit enhancement to the more senior tranches. Specifically, the equity holders' cash flows are dependent on the amount of inflows to the pool less any funds diverted to the excess spread account. Denote the amount diverted to the spread trust in year t as (OC_t) with maximum allowable diversion K. To determine the cash flow to equity, the following steps must be performed:

1. Is the current period interest sufficient to cover the promised coupons: $L_t - B \geq 0$? If *yes*, then the following overcollateralization test must be performed to see how much flows to trust: Is $L_t - B \geq K$? If *yes*, then K is diverted to trust, and $L_t - B - K$ flows to equity holders: $OC_t = K$. If *no*, then $L_t - B$ is diverted to trust, and nothing flows to equity holders: $OC_t = L_t - B$.
2. Is the current period interest sufficient to cover the promised coupons: $L_t - B \geq 0$? If *no*, then the interest is not sufficient to pay bondholders and all L_t flows to

bondholders. Therefore, the shortfall is $B - L_t$. The next step is to check if the accumulated funds in the spread trust can cover the shortfall. If the trust account has enough funds, the bondholders can be paid in full. If the trust account does not have enough funds, then the bondholders suffer a write-down.

The previous steps outlined the basic procedure for tranche cash flow distribution; however, a few more factors need to be considered. First, for each period there are possible defaults. For simplicity, assume the number of defaults (d_t) is constant for each period. Second, the amount recovered in year t (assuming a 40% recovery rate) equals:

$$R_t = 0.4d_t \times \text{loan amount}$$

Therefore, the total amount deposited into the trust account in year t is:

$$R_t + OC_t$$

It follows that the total amount accumulated in the trust account in year t is:

$$R_t + OC_t + \sum_{\tau=1}^{t-1} (1 + r)^{t-\tau} OC_\tau$$

Now, if excess spread is negative ($L_t - B < 0$), the custodian must check if the trust account can cover the shortfall. Formally, the test for the custodian is:

$$R_t + \sum_{\tau=1}^{t-1} (1 + r)^{t-\tau} OC_\tau > B - L_t$$

Note that there is no OC_t term to add to R_t since there is no excess spread this period. If the test is true, then the trust account can make the bondholders whole.

If it is not true, then the fund is reduced to zero and bondholders receive $R_t + \sum_{\tau=1}^{t-1} (1 + r)^{t-\tau} OC_\tau$ from the trust account.

Using the previous exposition, the amount diverted to the overcollateralization account can be calculated as:

$$OC_t = \left\{ \begin{array}{c} \min(L_t - B, K) \\ \max\left[L_t - B, -\left[\sum_{\tau=1}^{t-1}(1 + r)^{t-\tau} OC_\tau + R_t\right]\right] \end{array} \right\} \text{for} \left\{ \begin{array}{l} L_t \geq B \\ L_t < B \end{array} \right\}$$

Note that the upper condition represents inflows to the trust account while the lower condition represents outflows from the trust account.

Finally, the equity cash flows can be expressed as:

$$\max(L_t - B - OC_t, 0) \text{ for } t = 1,\dots,T-1$$

The cash flows in the final year must be examined separately for several reasons. First, the surviving loans reach maturity and principal is returned. Second, there is no diversion to the trust account because the structure ends and all proceeds follow the waterfall. Third, since there is no diversion to the trust, there is no need to test overcollateralization triggers.

The terminal cash flows are summarized as follows:

1. Loan interest $= \left(N - \sum_{t=1}^{T} d_t\right) \times$ (reference rate + spread) $\times$ par
2. Proceeds (par) from redemption of surviving loans $= \left(N - \sum_{t=1}^{T} d_t\right) \times$ par
3. Recovery in final year: $R_T = 0.4d_T \times$ par
4. Residual in trust account: $\sum_{t=1}^{T} (1 + r)^{t-\tau} OC_t$

The sum of these terminal cash flows is compared to the amount due to the senior tranche. If the sum is large enough, the senior tranche is paid off and the remainder is available for the rest of the capital structure. If the remainder is large enough to cover the junior tranche, then the residual flows to equity. If the remainder cannot meet junior claims, the junior bonds receive the excess and equity holders receive nothing.

As an example, determine the terminal cash flows to senior, junior, and equity tranches given the following information. The original loan pool included 100 loans with $1 million par value and a fixed coupon of 8%. The number of surviving loans is 90. The par for the senior and junior tranches is 75% and 20%, respectively. The equity investors contributed the remaining 5%. There were two defaults with recovery rate of 40% recovered at the end of the period. The value of the trust account at the beginning of the period is $16 million earning 4% per annum.

1. Total size of collateral pool at origination: 100 × $1,000,000 = $100,000,000
2. Senior tranche = $75,000,000
   Junior tranche = $20,000,000
   Equity tranche = $5,000,000
3. Interest from loans: 90 × 8% × $1,000,000 = $7,200,000
4. Redemption at par: 90 × $1,000,000 = $90,000,000
5. Recovery in final year: 2 × 40% × $1,000,000 = $800,000
6. Value of OC at end of final year: $16,000,000 × 1.04 = $16,640,000
7. Total available to satisfy all claims = $114,640,000
8. Senior claim = $75,000,000 < $114,640,000. Senior claim is satisfied without impairment
9. Junior claim = $20,000,000 < $114,640,000 – $75,000,000 so junior claim is satisfied
10. Equity claim = $114,640,000 – $75,000,000 – $20,000,000 = $19,640,000

Now, continue with the same example, but change the interest rate to 5% and the beginning OC value to $3 million. The first two steps will be the same as before.

3. Interest from loans: 90 × 5% × $1,000,000 = $4,500,000
4. Redemption at par: 90 × $1,000,000 = $90,000,000
5. Recovery in final year: 2 × 40% × $1,000,000 = $800,000
6. Value of OC at end of final year: $3,000,000. × 1.04 = $3,120,000
7. Total available to satisfy all claims = $98,420,000
8. Senior claim = $75,000,000 < $98,420,000. Senior claim is satisfied without impairment
9. Junior claim = $20,000,000 < $98,420,000 – $75,000,000 so junior claim is satisfied
10. Equity claim = $98,420,000 – $75,000,000 – $20,000,000 = $3,420,000

Finally, continue with the same example, but change the interest rate to 4% and the beginning OC value to $1 million. Assume a recovery rate of zero. Again, the first two steps are the same as before.

3. Interest from loans: 90 × 4% × $1,000,000 = $3,600,000
4. Redemption at par: 90 × $1,000,000 = $90,000,000
5. Recovery in final year: 2 × 0% × $1,000,000 = 0
6. Value of OC at end of final year: $1,000,000 × 1.04 = $1,040,000
7. Total available to satisfy all claims = $94,640,000
8. Senior claim = $75,000,000 < $94,640,000. Senior claim is satisfied without impairment
9. Junior claim = $20,000,000 > $94,640,000 – $75,000,000 so junior claim is impaired

   Junior tranche receives $19,640,000
10. Equity claim = $94,640,000 – $75,000,000 – $20,000,000 < 0

    Equity tranche receives $0

MODULE QUIZ 28.2

1. Which of the following participants in the securitization process is least likely to face a conflict of interest?
   A. Credit rating agency and servicer.
   B. Servicer and underwriter.
   C. Custodian and trustee.
   D. Trustee and manager.

MODULE 28.3: SIMULATION, PROBABILITY OF DEFAULT, AND DEFAULT CORRELATION; DEFAULT

SENSITIVITIES; AND STRUCTURED PRODUCTS

Simulation Approach

LO 28.h: Describe a simulation approach to calculating credit losses for different tranches in a securitization.

The prior analysis made a few very important simplifying assumptions. In particular, the analysis assumed that the default rate was constant year over year, each loan exhibited the same default probability, and the correlation between loans was ignored. In practice, these assumptions need to be brought into the analysis and the only tractable way to do so is via simulation.

Although the technical details are well beyond the scope of the exam, we can identify the basic steps and intuition for the simulation approach to calculating credit losses.

Step 1: Estimate the parameters.

Step 2: Generate default time simulations.

Step 3: Compute portfolio credit losses.

The first step is to estimate the critical parameters, default intensity, and pairwise correlations. The default intensity can be estimated using market spread data to infer the hazard rate across various maturities. Estimating the correlation coefficients is more challenging because of a lack of usable market data. The copula correlation could be useful in theory but suffers empirical precision in practice. Instead, a sensitivity analysis is performed for various default and correlation pairs.

The second step identifies if and when the security defaults. Simulation provides information on the timing for each hypothetical outcome. The third step uses the simulation output to determine the frequency and timing of credit losses. The credit losses can be "lined up" to assess the impact on the capital structure losses. The tail of the distribution will identify the credit VaR for each tranche in the securitization.

Impact of Probability of Default and Default Correlation

LO 28.i: Explain how the default probabilities and default correlations affect the credit risk in a securitization.

There are several important comparative statistics associated with a generic securitization. The following results represent the effect of the average tranche values and write-downs. The implications of extreme tail events will be discussed shortly using VaR. The first factor to consider is the probability of default. It is straightforward to see that, for a given correlation, increasing the probability of default will negatively impact the cash flows and, thus, the values of all tranches.

The effect of changing the correlation is more subtle. Consider the stylized case where the correlation is very low, say zero, so loan performance is independent. Therefore, in a large portfolio, it is virtually impossible for none of the loans to default and it is equally unlikely that there will be a large number of defaults. Rather, the number of defaults should be very close to the probability of default times the number of loans. So, the pool would experience a level of defaults very close to its mathematical expectation and is unlikely to impair the senior tranches. The analogous situation is flipping a coin 1,000 times—the number of heads would be very close to 500. It would be virtually impossible for the number of heads to be less than 400 or greater than 600. Now, if the correlation increases, the default of one credit increases the likelihood of another default. Thus, increasing correlation decreases the value of senior tranches as the pool is now more likely to suffer extreme losses. This effect is exacerbated with a higher default probability.

Now consider the equity tranche. Recall that the equity tranche suffers the first write-downs in the pool. Therefore, a low correlation implies a predictable, but positive, number of defaults. In turn, the equity tranche will assuredly suffer write-downs. On the other hand, if the correlation increases, the behavior of the pool is more extreme, and there may be high levels of related losses or there may be very few loan losses. In sum, the equity tranche increases in value from increasing correlation as the possibility of zero (or few) credit losses increases from the high correlation.

The correlation effect on the mezzanine tranche is more complex. When default rates are low, increasing the correlation increases the likelihood of losses to the junior bonds (similar to senior bonds). However, when default rates are relatively high, increasing the correlation actually decreases the expected losses to mezzanine bonds as the possibility of few defaults is now more likely. Accordingly, the mezzanine bond mimics the return pattern of the equity tranche. In short, increasing correlation at low default rates decreases mezzanine bond values, but at high default rates it will increase mezzanine bond values.

Convexity is also an issue for default rates. For equity investors, as default rates increase from low levels, the equity tranche values decrease rapidly then moderately (a characteristic of positive convexity). Since the equity tranche is thin, small changes in default rates will disproportionately impact bond prices at first. Similarly, senior tranches exhibit negative convexity. As defaults increase, the decline in bond prices increases. As usual, the mezzanine impact is somewhere in between: negative convexity at low default rates, positive convexity at high default rates.

The previous section focused on the average (mean) value of the tranches while this section examines the distribution of possible tranche values (risk). Specifically, the goal is to analyze the impact of default probability and default correlation under extreme conditions (far into the tail). The metric used is credit VaR for various ranges of default probability and default correlation for the senior, junior, and equity tranches. The main result is that increasing default probability, while holding correlation constant, generally decreases the VaR for the equity tranches (less variation in returns) and increases the VaR for the senior tranches (more variation in returns). As usual, the mezzanine effect is mixed: VaR increases at low correlation levels (like senior bonds) then decreases at high correlation levels (like equity). These results are summarized in Figure 28.3.

Figure 28.3: Increasing Default Probability (Correlation Constant)

	Mean Value	Credit VaR
Equity tranche	↓	↓
Mezzanine tranche	↓	↑ then ↓
Senior tranche	↓	↑

The next effect to consider is the impact of a rising correlation. As a reminder, increasing correlation increases the clustering of events, either high frequency of defaults or very low frequency of defaults. Increasing correlation decreases senior bond prices as the subordination is more likely to be breached if defaults do indeed cluster. In contrast, equity returns increase as the low default scenario is more probable relative to low correlation where defaults are almost certain.

As the default correlation approaches one, the equity VaR increases steadily. The interpretation is that although the mean return is increasing so is the risk as the returns are more variable (large losses or very small losses).

All else equal, the senior VaR also increases consistently with correlation. However, we note an interesting effect: the incremental difference between high correlations (0.6 versus 0.9) is relatively small. In addition, two pairwise results are worth highlighting. If correlation is low and default frequency is relatively high, then senior bonds are well insulated. In fact, at the 10% subordination level, the senior bonds would be unaffected even at a high default rate. At the other extreme, when correlations are high (0.6 or above), then the VaRs are quite similar regardless of the default probability. Hence, generally speaking, correlation is a more important risk factor than default probability which may not be entirely intuitive.

The implications for the mezzanine tranche are, again, mixed. When default rates and correlations are lower, the mezzanine tranche behaves more like senior notes with low VaRs. However, when the default probabilities are higher and/or pairwise correlation is high, the risk profile more closely resembles the equity tranche. These results are summarized in Figure 28.4.

Figure 28.4: Increasing Correlations (Default Probability Constant)

	Mean Value	Credit VaR
Equity tranche	↑	↑
Mezzanine tranche	↓(at low default rates) ↑(at high default rates)	↑
Senior tranche	↓	↑

Measuring Default Sensitivities

LO 28.j: Explain how default sensitivities for tranches are measured.

The previous discussion highlighted the effect of increasing the probability of default, which decreases tranche values. However, this effect is not necessarily linear and also depends on the interaction with the default correlation. To analyze the marginal effects in more detail, the definition of DV01 is extended to default probabilities and is called "default '01." The default probability will be shocked up and down by the same amount (by convention 10 basis points) and each tranche will be revalued through the VaR simulations. The formulation for default '01 of each tranche is as follows:

$$1/20\ [(\text{mean value / loss based on } \pi + 0.001) \\ - (\text{mean value / loss based on } \pi - 0.001)]$$

From this equation, there are several qualitative impacts to note. First, the default sensitivities are always positive for any default probability-correlation combination. This follows from the previous observation that all tranches are negatively affected from increasing default probabilities. Second, the default '01 will approach zero as default rates become sufficiently high as the marginal impact of increasing the default rate has minimal effect. The third result follows from the second. There will be more variation in the default sensitivities when the default rate generates losses close to the tranche's attachment point. This result is similar to the high gamma (high sensitivity in delta) for options at-the-money.

Risks for Structured Products

LO 28.k: Describe risk factors that impact structured products.

Aside from the credit portfolio modeling issues discussed before, there are at least three other risks that deserve discussion: systematic risk, tranche thinness, and loan granularity.

Similar to a well-diversified equity portfolio that cannot eliminate systematic risk, the same holds true for credit portfolios. Unfortunately, even when the collateral pool is well-diversified among lenders, terms, geography, and other factors, high systematic risk expressed in high correlations can still severely damage a portfolio. As previously discussed, with increases in pairwise correlations, the likelihood of senior tranche write-downs increases as well.

The equity and mezzanine tranches are relatively thin. This also manifests itself in the relative closeness of the 95% and 99% credit VaR. The implication is that given that the tranche has been breached, the loss is likely very large.

Loan granularity references the loan level diversification. For example, in a collateralized MBS pool, the portfolio composition is a few loans but the loans are of substantial size. This reduction in sample size increases the probability of tail events in relation to an equal sized portfolio constructed with more loans of smaller amounts.

Implied Correlation

LO 28.l: Define implied correlation and describe how it can be measured.

The implied correlation is a very similar concept to the implied volatility of an equity option. For options, the Black-Scholes-Merton model is a widely accepted valuation model and so the observable market price is associated with a unique unobserved volatility. For securitized tranches, the process is exactly the same. Starting with observed market prices and a pricing function for the tranches, it is possible to back out the unique implied correlation to calibrate the model price with the market price.

The mechanical part of the process involves several intermediate steps. First, the observable credit default swap (CDS) term structure is used to extract risk-neutral default probabilities and possibly recovery rates. Assuming constant pairwise correlation and market prices for the respective tranches, the default estimates and correlation estimates can be fed into a copula. The output is the risk-neutral implied correlation (i.e., base correlation) per tranche. The correlation estimates will vary between the tranches and are not likely to be constant giving rise to correlation skew. As an example, suppose the observed market price of the equity tranche increases from $3 million to $3.2 million, but the estimates of the risk-neutral probability of default remain the same. It can be inferred that the market's estimate of the implied correlation must have increased. The precise value must be extracted from the pricing model but qualitatively the direction is correct; increasing correlations benefit equity holders.

Motivations for Using Structured Products

LO 28.m: Identify the motivations for using structured credit products.

Identifying the motivations of loan originators and investors can provide a better understanding for why securitizations are established.

Loan originators, who help create securitizations by selling loans into a trust, are attracted to borrowing via securitization given its ability to provide a lower cost of funding. Without securitization, loans would either be retained or sold in the secondary market. These alternatives would likely be more costly than securing funding via securitization. A lower cost of funding can be obtained given the diversification of the loan pool and the reputation of the originator for underwriting high-quality loans. However, some loan pools, such as commercial mortgage pools, can be difficult to diversify. Thus, an element of systematic risk may still exist, which could lead to an underestimation of overall risk. An additional benefit of securitization for loan originators is the collection of servicing fees.

Investors, who purchase the assets in a securitization, are attracted to investing in diversified loan pools that they would not otherwise have access to without securitization, such as mortgage loans and auto loans. In addition, the ability to select a desired risk-return level via tranching offers another advantage for investors. Equity tranches will offer higher risk-return levels, while senior tranches will offer lower risk-return levels. However, it is important for investors to conduct the proper due diligence when analyzing potential tranche investments in order to understand the actual level of risk involved.

MODULE QUIZ 28.3

1. Which of the following statements about portfolio losses and default correlation are most likely correct?
   I. Increasing default correlation decreases senior tranche values but increases equity tranche values.
   II. At high default rates, increasing default correlation decreases mezzanine bond prices.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following statements best describes the calculation of implied correlation?
   A. The implied correlation for the mezzanine tranche assumes non-constant pairwise correlation.
   B. Observable market prices of credit default swaps are used to infer the tranche values.
   C. The tranche pricing function is calibrated to match the model price with the market price.
   D. The risk-adjusted default probabilities are used in model calibration.

KEY CONCEPTS

LO 28.a

Securitization is the process of pooling cash flow generating assets and reapportioning the cash flows into bonds. These structured products generate a wide range of risk-return profiles that vary in maturity, credit subordination (equity, mezzanine, and senior), type of collateral (mortgages, auto loans, and credit card balances), active or passive management, and static or revolving assets. A true securitization removes the assets from the originator's balance sheet.

LO 28.b

The capital structure of a securitization refers to the different size and priority of the tranches. In general, the senior tranches are the largest, safest, and lowest yielding bonds in the capital structure. The mezzanine tranche has lower priority than the senior tranche and is promised a higher coupon. The lowest priority tranche that bears the first loss is the equity tranche. The size of the equity and mezzanine tranches provides subordination for the senior tranche. Internal credit enhancement, such as overcollateralization and excess spread, buffers the senior tranches from losses. Likewise, external wraps and insurance also protect the senior bondholders.

LO 28.c

A waterfall structure details the distribution of collateral cash flows to the different classes of bondholders. The equity tranche typically receives the residual cash flows once the senior and mezzanine investor claims are satisfied. If the cash flows to equity holders exceed the overcollateralization trigger, the excess is diverted to a trust account. Fees and defaults will reduce the net cash flows available for distribution.

LO 28.d

Securitization is a complicated process and typically involves an originator, underwriter, credit rating agency, servicer, and manager. The originator creates the initial liability; the underwriter pools and structures the terms of the deal as well as markets the issue; the credit rating agency is an active participant suggesting/requiring sufficient subordination and enhancements to justify the ratings; the servicer collects and distributes the cash flows to investors and manages distress resolution; managers, both static and active, usually bear the first loss to mitigate conflict of interest in asset selection and credit monitoring.

LO 28.e

The three-tiered waterfall will have scheduled payments to senior and mezzanine tranches. The equity tranche receives cash flows only if excess spread > 0 (i.e., interest collected > interest owed to senior + mezzanine). The overcollateralization account increases from recovery of defaulted assets and diversion of spread (usually a maximum is predetermined) and earns the money market rate.

LO 28.f and LO 28.g

If excess spread is negative (i.e., interest collected < interest owed to senior + mezzanine), the OC account will use all of its available funds until depleted. The terminal cash flows are more complicated: redemptions at par + interest from surviving loans + recovery in final period + terminal OC account. No funds are diverted in the final year as it all is aggregated and disbursed. Senior claims are paid first; if senior is paid in full, mezzanine claims are paid; if mezzanine is paid in full, the residual accrues to equity holders.

LO 28.h

Simulation is a useful technique to provide more insight into the performance of the collateral and, hence, cash flows to the tranches. In particular, the default intensity can be time-varying and estimated using a hazard distribution. The correlation between loans is critical to the performance of the pool, so various default probability/correlation pairs are used. Copulas could be used to simulate the timing of the defaults. Finally, simulations allow computation of VaRs for each tranche.

LO 28.i

Increasing default probability will decrease all tranches unconditionally. In contrast, increasing correlation will impact each tranche differently. In general, increasing default correlation increases the likelihood of extreme portfolio behavior (very few or many defaults).

Credit VaR can be used to measure the value of the tranches in the left tail. Increasing the correlation increases the VaR of all tranches. In contrast, increasing the probability of default decreases equity VaR and increases senior VaR.

LO 28.j

Default sensitivities are measured analogously to DV01 and spread '01 by shocking the default probability up and down by 10 basis points. Default sensitivities are always positive and are largest when the resulting loss is close to the attachment point.

LO 28.k

Similar to equity portfolios, systematic risk is present in credit portfolios. Extreme loss events are captured by high default correlations. The thinness of the equity and mezzanine tranches implies that conditional losses are likely to be large. A less granular pool (fewer but larger loans) is more likely to experience a tail event, all else equal.

LO 28.l

Implied default correlations for each tranche can be backed out of the tranche pricing model similar to how the implied volatility is calculated for the Black-Scholes-Merton model.

LO 28.m

Loan originators help create securitizations by selling loans into a trust. They are attracted to secured borrowing via securitization because it provides a lower cost of funding than alternatives such as retaining loans. Investors purchase the bonds and equity in a securitization. They are attracted to securitization because it allows them to invest in diversified loan pools that are typically reserved for banks.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 28.1

1. **B** Senior tranches are perceived to be the safest, so they receive the lowest coupon. The equity tranche receives residual cash flows and no explicit coupon. Although the mezzanine tranche is often thin, the equity tranche is typically the thinnest slice. (LO 28.b)

2. **A** The interest rate on the loans = 4% (market reference rate) + 3% (spread) = 7%. Therefore, the total collateral cash flows in the first period = 100 × $500,000 × 7% × (1 − 0.02) = $3,430,000. The senior tranche receives $50 million × 0.75 × (4% + 2%) = $2,250,000. Similarly, the mezzanine tranche receives $50 million × 0.20 × (4% + 6%) = $1,000,000. Next, the residual cash flows are calculated: $3,430,000 − $2,250,000 − $1,000,000 = $180,000. Since $180,000 < $1,000,000, all cash flows are claimed by the equity investors and there is no diversion to the trust account. (LO 28.c)

Module Quiz 28.2

1. **C** The custodian and trustee play the least important roles in the securitization process. The servicer, originator, underwriter, credit rating agency, and manager all face conflicts of interest to varying degrees. (LO 28.d)

Module Quiz 28.3

1. **A** Statement I is true. Increasing default correlation increases the likelihood of more extreme portfolio returns (very high or very low number of defaults). The increased likelihood of high defaults negatively impacts the senior tranche. On the other hand, the increased likelihood of few defaults benefits the equity tranche as it bears first loss. Statement II is false. At high default rates, increasing the correlation increases the likelihood of more extreme portfolio returns which benefits equity investors and mezzanine investors. (LO 28.i)

2. **C** Starting with observed market prices and a pricing function for the tranches, it is possible to back out the implied correlation to calibrate the model price with the market price. The computation of implied correlation assumes constant pairwise correlation. Both credit default swap and tranche values are observed. Observed tranche values are used in conjunction with risk-neutral default probabilities to compute implied correlation. (LO 28.l)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Hull, Chapter 24.

READING 29

CREDIT RISK

Study Session 5

EXAM FOCUS

This topic is focused on credit risk, which is the risk that the other side of a transaction (in a derivatives contract) or a borrower will default on their obligation. Credit ratings, default probabilities, and recovery rates all play a significant role in determining credit risk. For the exam, be able to calculate default probabilities using credit spreads, and understand default probabilities in both risk-neutral and real-world scenarios. Also, be aware of the credit risk for derivatives and the tools that are used to mitigate this risk. In addition, from the bank's perspective, know the differences between credit and debt (debit) valuation adjustments. Finally, be familiar with the Gaussian copula model, which is used for default probability calculations and estimating credit VaR.

MODULE 29.1: DEFAULT PROBABILITIES

LO 29.c: Calculate the probability of default using credit spreads.

Credit risk is the exposure faced by an individual or entity due to the default potential of both counterparties in derivatives contracts and individuals/entities that borrow money. Three credit ratings agencies that provide corporate bond ratings include Moody's, S&P, and Fitch. The highest rated bonds are considered investment grade, as they pose the lowest risk to bondholders. Credit risk becomes a greater concern with lower rated bonds. The Moody's data in Figure 29.1 reflects average cumulative default rates from 1970 to 2015.

Figure 29.1: Moody's Average Cumulative Default Rates, 1970–2015 (%)

Rating	Term (In Years)								
	1	2	3	4	5	7	10	15	20
Aaa	0.000	0.011	0.011	0.031	0.087	0.198	0.396	0.725	0.849
Aa	0.022	0.061	0.112	0.196	0.305	0.540	0.807	1.394	2.266
A	0.056	0.170	0.357	0.555	0.794	1.345	2.313	4.050	6.087
Baa	0.185	0.480	0.831	1.252	1.668	2.525	4.033	7.273	10.734
Ba	0.959	2.587	4.501	6.538	8.442	11.788	16.455	23.930	30.164
B	3.632	8.529	13.515	17.999	22.071	29.028	36.298	43.368	48.071
Caa-C	10.671	18.857	25.639	31.075	35.638	41.812	47.843	50.601	51.319

As shown by the shaded area in Figure 29.1, bonds rated Baa and above are considered **investment-grade bonds**. The probabilities of default for investment-grade bonds are significantly lower than they are for **non-investment-grade bonds**.

An interesting result from this data is how default probabilities relate over time. The probability that a bond rated Baa will default in Year 3 is 0.831% − 0.480% = 0.351%. In Year 4, it is 1.252% − 0.831% = 0.421% (which is *higher* than in Year 3). For a bond rated B, the probability of a Year 3 default is: 13.515% − 8.529% = 4.986%. For Year 4, it is 17.999% − 13.515% = 4.484% (which is *lower* than in Year 3). The conclusion is that investment-grade bonds may see financial health deteriorate over time, while non-investment-grade bonds may see financial health recover after making it past the critical early years.

The unconditional default probability (i.e., the probability of a default in a given year, as seen today) can be illustrated using data from Figure 29.1. The following is for a B-rated bond:

- Unconditional default probability during Year 4: 17.999% − 13.515% = 4.484%
- Probability of survival through Year 3: 100% − 13.515% = 86.485%
- Probability of a default during Year 4, conditional on no earlier default: 0.04484/0.86485 = 5.185%

The **hazard rate** (i.e., default intensity) is a measure of the probability of default, given a condition of no earlier default. Using the hazard rate, the **probability of default**, *Q(t)*, can be defined as:

$$Q(t) = 1 - e^{-\bar{\lambda}(t)\times t}$$

where:

$\bar{\lambda}(t)$ = average hazard rate between time 0 and time t

A bond's **recovery rate** can be calculated by dividing its market value post-default by its face value. Recovery rates are naturally higher for first and second lien bonds versus those for subordinated bonds. Corporate bond recovery rates reflect a negative dependence on default rates such that strong economic conditions produce lower default numbers and higher recovery rates, whereas weaker economic conditions produce higher default numbers and lower recovery rates.

Default Probabilities Using Credit Spreads

The yield spread of a bond is the difference between the promised yield on the bond and the risk-free rate. Default probabilities can be estimated using bond yield spreads. If *s(T)* is the average loss rate on a bond per year, the average hazard rate can be approximated as:

$$\bar{\lambda}(T) = \frac{s(T)}{1 - RR}$$

For example, a one-year bond yielding 125 basis points above the risk-free rate with a recovery rate of 55% would produce an average hazard rate of: 0.0125/(1 – 0.55) = 0.0278, or 2.78% per year. Greater precision can be found by aligning hazard rates with bond prices such that the hazard rate for the first period is calculated using the shortest maturity bond, and the hazard rate between the first and second period is calculated using the next shortest maturity bond, etc.

The risk-free rate, which is a critical factor in default probability calculations, is typically a Treasury rate. However, there is concern that these rates are too low to truly proxy risk-free rates. A better measure may be credit default swap (CDS) spreads, which do not depend on the risk-free rate. An asset swap spread may also be used, as the present value of the spread is the difference between the price of the risk-free bond and the price of the corporate bond.

Default Probability Estimate Comparisons

Bond yield spreads tend to produce higher estimated default probabilities than those estimated from historical data, with that difference further exacerbated during times of crisis (like the credit crisis of 2007–2009). Figure 29.2, which is derived from Merrill Lynch published data on 7-year bond yields between 1996 and 2007, illustrates the expected excess return on bonds (in basis points) for spreads over Treasuries at various rating categories.

Figure 29.2: Expected Excess Return on Bonds (in bps)

Rating	Bond Yield Spread Over Treasuries	Spread of Risk-Free Rate Over Treasuries	Spread for Historical Defaults	Excess Return
Aaa	78	42	2	34
Aa	86	42	5	39
A	111	42	12	57
Baa	169	42	22	105
Ba	322	42	108	172
B	523	42	294	187
Caa	1146	42	464	640

Evidence has shown that for investment-grade companies, the ratio of the hazard rate derived from bond prices to the hazard rate derived from historical data is very high. However, this ratio subsequently declines as ratings decline such that the spread between the hazard rates increases. Interestingly, a small expected excess return on the bond comes from larger percentage differences between hazard rates. As credit quality declines, this expected excess return increases.

The hazard rates (default probabilities), which are implied from credit spreads, are considered "risk neutral," which means they can be used to calculate forecasted cash flows when there is credit risk in a risk-neutral world. Expected cash flows are discounted at a risk-free rate. When historical data is used, hazard rates are considered "real-world" (physical) default probabilities. Risk-neutral probabilities tend to be much higher than real-world probabilities, although in theory they would be the same if there were no expected excess returns.

These differences in risk-neutral and real-world probabilities may stem from the relative illiquidity of corporate bonds. Another reason may be higher subjective default probabilities from bond traders. The biggest reason for the difference is that bond defaults are not independent of one another. Some years have low default rates across the board, while others have substantially higher default rates.

Systematic risk, which cannot be mitigated through diversification, stems from the year-to-year variation in default risk. While conditions in the economy can drive systematic risk, credit contagion may also create risk as the default of one company can drive the defaults of others. Nonsystematic risks unique to each bond are also an element to consider, as bond returns have a high skew with limited upside. This makes diversification difficult, thereby driving bond trader returns for bearing both systematic and nonsystematic risks.

In terms of which default probability estimate to use, the guidance is as follows:

- **Risk-neutral default probabilities.** Use these when estimating the impact of default risk on instrument pricing or when valuing credit derivatives.
- **Real-world default probabilities.** Use these when carrying out scenario analyses to calculate potential future losses from defaults.

Equity Prices for Default Probability Estimates

Because credit rating revisions tend to be infrequent, equity prices may be a better source for more up-to-date information on default probabilities. The **Merton model** shows that equity, E_T, is essentially a call option on the value of the company's assets, V_T, with a strike price equivalent to the required debt repayment, D:

$$E_T = \max(V_T - D, 0)$$

Based on the Black-Scholes-Merton formula, the value of equity today is equal to:

$$E_0 = V_0 N(d_1) - D e^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}}$$

$$d_2 = d_1 - \sigma_v \sqrt{T}$$

The value of the debt today is then computed as:

$$D_0 = V_0 - E_0$$

Suppose that the market value of the debt (using the equation $V_0 - E_0$) is 8.25, and that the present value of the promised payment on the debt is 8.41. In this case, the expected loss on the debt is approximating (8.41 – 8.25)/8.41 = 1.90% of its no-default value.

Historical data suggests that the Merton model produces solid rankings of default probabilities under both risk-neutral and real-world scenarios.

MODULE QUIZ 29.1

1. If the unconditional default probability of a Ba-rated bond during Year 3 is 1.914% and the probability of survival through Year 2 is 97.413%, the probability of a default during Year 3, conditional on no earlier default, is closest to:
   A. 0.673%.
   B. 1.914%.
   C. 1.965%.
   D. 2.587%.
2. A 2-year corporate bond yields 190 basis points above the risk-free rate. With a recovery rate of 35%, the average hazard rate for Years 1 and 2 is closest to:
   A. 0.67%.
   B. 1.55%.
   C. 2.92%.
   D. 5.43%.

MODULE 29.2: CREDIT RISK IN DERIVATIVES

LO 29.a: Assess the credit risks of derivatives.

An **International Swaps and Derivatives Association (ISDA) Master Agreement** is typically used to govern bilaterally cleared derivatives between two companies. Initial and variation margin requirements must be stated in the agreement for transactions between financial institutions. For variation margin calculations only, transactions are netted.

The agreement also provides the situations where an "event of default" occurs. Two situations exist where the nondefaulting party will likely incur a loss:

1. Total value for the nondefault party is positive and exceeds the default party posted collateral. The nondefault party then assumes the role of an unsecured creditor for an amount equivalent to the value of the transactions less the value of collateral.
2. Total value for the default party is positive and is less than the nondefault party posted collateral. The nondefault party then assumes the role of an unsecured creditor for the return of the excess collateral posted.

CVA and DVA

LO 29.b: Define credit valuation adjustment (CVA) and debt valuation adjustment (DVA).

From the perspective of a bank, the **credit valuation adjustment (CVA)** and **debt (debit) valuation adjustment (DVA)** are defined as follows for a given counterparty:

- CVA is the present value of the expected cost to the bank if the counterparty defaults.
- DVA is the present value of the expected cost to the counterparty if the bank defaults.

For each of its counterparties, the bank will have one CVA and one DVA. These can be considered derivatives, which change in value due to market variables, counterparty credit spreads, and bank credit spreads. Because a bank default implies the bank will not have to make payments on its derivatives position, the DVA (which is of course a cost to the counterparty) is a benefit and an increase to the value of the derivatives portfolio for the bank.

The no-default value to a bank for its outstanding derivatives transactions (f_{nd}) assumes neither side will default. As such, the value of derivatives transactions, accounting for defaults, is reflected as:

$$f_{nd} - \text{CVA} + \text{DVA}$$

The CVA and DVA formulas are expressed as:

$$\text{CVA} = \sum_{i=1}^{N} q_i \times v_i$$

$$\text{DVA} = \sum_{i=1}^{N} q_i^* \times v_i^*$$

where:

q_i = risk-neutral probability of the counterparty defaulting during the ith interval

v_i = present value of the expected loss to the bank if the counterparty defaults at the midpoint of the ith interval

q_i^* = risk-neutral probability (calculated from the bank's credit spreads) of the bank defaulting during the ith interval

v_i^* = present value of the expected loss to the counterparty (gain to the bank) if the bank defaults at the midpoint of the ith interval

Note that the calculation for the q_i term, which represents the probability of the counterparty defaulting during the ith interval, is:

$$q_i = \exp\left(-\frac{s(t_{i-1})t_{i-1}}{1 - RR}\right) - \exp\left(-\frac{s(t_i)t_i}{1 - RR}\right)$$

where:

$s(t_i)$ = counterparty's credit spread

Extensive Monte Carlo simulations are often needed to perform these calculations. Peak exposure at the midpoint of each interval in the event of default is an additional calculation banks perform. The impacts of new transactions on CVA and DVA can be calculated quickly because banks will often store all sampled paths from their simulations for all market variables. The incremental effects on CVA and DVA from the new transaction are relatively straightforward to discern. A new transaction, which is positively (negatively) correlated to existing transactions, will likely increase (decrease) CVA and DVA.

Two terms related to CVA and DVA calculations are *wrong-way risk* and *right-way risk*. Wrong-way risk occurs when the probability of default is positively correlated with exposure, and right-way risk occurs when the probability of default is negatively correlated with exposure.

Credit Risk Mitigation

LO 29.d: Describe, compare, and contrast various credit risk mitigants and their role in credit analysis.

Banks have several ways in which they can attempt to reduce credit risk in bilaterally cleared transactions. These methods include netting, collateral agreements, and downgrade triggers:

- **Netting.** As an example, four uncollateralized transactions with a counterparty are valued at: +$5 million, –$7 million, +$10 million, and –$2 million. Independently, the bank's exposure is $5 million and $10 million together (or, $15 million). Note that the negative amounts do not represent exposures for the bank. By netting, the total exposure is reduced to $6 million (i.e., 5 – 7 + 10 – 2).
- **Collateral agreements.** Cash or marketable securities are often used as collateral. In the event of a default, the party that did not default can keep the collateral posted by the defaulting party.
- **Downgrade triggers.** These are clauses in Master Agreements between banks and nonfinancial counterparties. The clause states that if the counterparty credit rating falls below a certain level, the bank can choose to close out outstanding transactions at market value or require collateral to be posted. The value of these triggers is reduced if there are big downgrades in counterparty credit ratings or if the counterparty has triggers in place with multiple dealers.

As noted earlier, extensive Monte Carlo simulations are often needed to calculate CVA and DVA. This is actually not necessary in a situation where the transaction portfolio between the bank and counterparty is just a single (uncollateralized) derivative with a payoff to the bank at time *T*. The bank exposure in the future is the no-default derivative value at that time, where the present value is the no-default value today.

If the DVA is zero, the value *f* of the derivative today (accounting for credit risk) is equal to:

$$f = f_{nd} - (1 - RR) f_{nd} \sum_{i=1}^{N} q_i$$

For the value of a *T*-year zero-coupon bond issued by the counterparty with *nd* again representing a no-default situation, the value of the bond is equal to:

$$B = B_{nd} - (1 - RR)B_{nd}\sum_{i=1}^{N} q_i$$

These two equations can be rearranged such that:

$$\frac{f}{f_{nd}} = \frac{B}{B_{nd}}$$

Note that the value of the derivative can be determined by increasing the applied discount rate on the expected payoff (in a risk-neutral environment) by the counterparty's *T*-year credit spread.

MODULE QUIZ 29.2

1. In a bilaterally cleared derivatives transaction between two companies (Company A and Company B), Company B defaults. The value for Company A is a positive $50,000 and the collateral posted by Company B is $30,000. In this situation, Company A is a(n):
   A. secured creditor in the amount of $20,000.
   B. secured creditor in the amount of $80,000.
   C. unsecured creditor in the amount of $20,000.
   D. unsecured creditor in the amount of $80,000.

2. A bank is assessing the impact of a new transaction on CVA and DVA. If the new transaction is negatively correlated to existing transactions, the impact will likely be a(n):
   A. increase to both CVA and DVA.
   B. decrease to both CVA and DVA.
   C. increase to CVA and decrease to DVA.
   D. decrease to CVA and increase to DVA.

3. Bank TGF has three uncollateralized transactions with JL Co. The transactions have values to the bank of: +$12 million, −$4 million, and −$3 million. If the bank mitigates its credit risk through netting, the impact of this technique will result in an exposure reduction of:
   A. $5 million.
   B. $7 million.
   C. $8 million.
   D. $12 million.

MODULE 29.3: DEFAULT CORRELATION AND CREDIT VaR

LO 29.e: Describe the significance of estimating default correlation for credit portfolios and distinguish between reduced form and structural default correlation models.

Default correlation represents the likelihood that two companies will default around the same time. This can happen for a myriad of reasons:

1. External events tend to impact companies in the same geographic region or industry.
2. Overall economic conditions can lead to higher average default rates in some years relative to other years.

3. Contagion where one party defaulting may cause another to default. Because of default correlation, credit risk cannot be fully diversified away.

Two models used to evaluate default correlation include reduced-form models and structural models:

- **Reduced-form models.** These models assume hazard rates are correlated with macroeconomic variables and follow random processes. They are relatively straightforward from a mathematical sense and reflect trends in default correlations due to economic cycles. However, the range of achievable default correlations is low as it is unlikely any two companies will default over the same short period of time.
- **Structural models.** These models are based on models similar to the Merton model (described earlier), and default correlations between companies are introduced by assuming correlated stochastic processes for the assets of both companies. Unlike reduced-form models, the correlation can be set as high as needed. However, the computation time is significant.

Gaussian Copula Model

LO 29.f: Describe the Gaussian copula model for time to default and calculate the probability of default using the one-factor Gaussian copula model.

To determine the time to default, a **Gaussian copula model** may be used. This model is used to quantify the correlation between the probability distributions of times to default for multiple companies, under the assumption that all companies will eventually default. Both real-world and risk-neutral default probabilities may be used:

- *Real-world probabilities.* The left tail is estimated from rating agency data.
- *Risk-neutral probabilities.* The left tail is estimated from bond prices.

Although the probability distributions of company times to default are not "normal," the Gaussian copula model transforms the times to default into normal variables with means of zero and unit standard deviations. The assumption that the joint distribution of the variables is bivariate normal is considered the *Gaussian copula*, which allows the correlation structure of the variables to be estimated independent of their unconditional (marginal) distributions. The default correlation is called the **copula correlation**.

A one-factor model may be used to avoid the need for different correlations between each company pair in the Gaussian copula model. This assumption is given in the following formula:

$$x_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

where:
a_i = correlation of company i equity returns with a market index
F = common factor, which impacts defaults for all companies
Z = factor impacting only one company i

Conditioned on the common factor (F), the default probability, Q_i, is equal to:

$$Q_i(T \mid F) = N\left(\frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1 - a_i^2}}\right)$$

When the probability distributions of default are the same for all companies, and the correlations are the same for all companies (i.e., common correlation ρ), the default probability equation can be restated as:

$$Q(T \mid F) = N\left(\frac{N^{-1}[Q(T)] - \sqrt{\rho}F}{\sqrt{1 - \rho}}\right)$$

Credit VaR

LO 29.g: Describe how to estimate credit VaR using the Gaussian copula and the CreditMetrics approach.

Credit value at risk (VaR) is defined within a specific confidence level as the credit risk loss that will not be exceeded over a given period of time. So, a credit VaR with a one-year time horizon and a 98.5% confidence level is the credit loss we are 98.5% confident will not be exceeded over that one-year period of time. Given *X*% certainty over *T* years, the percentage of losses on a large portfolio will be less than *V(T, X)*, where:

$$V(T, X) = N\left(\frac{N^{-1}[Q(T)] + \sqrt{\rho}N^{-1}(X)}{\sqrt{1 - \rho}}\right)$$

This formula uses *Q(T)* as the probability of default by time *T* and the copula correlation as ρ. Credit VaR can then be estimated as: L × (1 − RR) × V(T, X), with the loan portfolio as *L* and the recovery rate as *RR*.

Another approach to estimating credit VaR is with the **CreditMetrics model** (from JPMorgan), where a probability distribution of credit losses is determined by applying a Monte Carlo simulation to the credit rating changes of all counterparties. Although this can be time consuming from a computation standpoint, it does account for both defaults and downgrades. Also, credit mitigation strategies such as the ones described earlier can be incorporated into the model.

For CreditMetrics, a rating transition matrix can be used as a basis for performing Monte Carlo simulation. However, credit rating changes for counterparties are not assumed to be independent when sampling for credit loss estimates. A joint probability distribution for rating changes can be constructed using a Gaussian copula model, with the copula correlation between rating transitions for two companies set equal to their equity return correlations.

MODULE QUIZ 29.3

1. Which of the following statements is most accurate regarding reduced-form versus structural models used to estimate default correlation?
   A. Structural models tend to take a long time to process.
   B. Reduced-form models are more computationally intensive.
   C. Reduced-form models allow for very high default correlations.

D. Economic cycle impacts on default correlation trends are best reflected in structural models.

2. The Gaussian copula model transforms which of the following factors into normal variables?
   A. Recovery rates.
   B. Times to default.
   C. Credit spread risk.
   D. Default probabilities.

3. Assuming a loan portfolio of L, a recovery rate of RR, and the percentage of losses on a portfolio less than $V(T, X)$, which of the following formulas is used to estimate credit VaR?
   A. $L \times (RR) \times V(T, X)$.
   B. $L \times (1 - RR) / V(T, X)$.
   C. $V(T, X) / [L \times (1 - RR)]$.
   D. $L \times (1 - RR) \times V(T, X)$.

KEY CONCEPTS

LO 29.a

An International Swaps and Derivatives Association (ISDA) Master Agreement is typically used to govern bilaterally cleared derivatives between two companies. Initial and variation margin requirements must be stated in the agreement for transactions between financial institutions.

Two situations exist where the nondefault party will likely incur a loss and will become an unsecured creditor: (1) total value for the nondefault party is positive and exceeds default party posted collateral and (2) total value for the default party is positive and less than the nondefault party posted collateral.

LO 29.b

From the perspective of a bank, the credit valuation adjustment (CVA) and debt (debit) valuation adjustment (DVA) are defined as follows for a given counterparty:

- CVA is the present value of the expected cost to the bank if the counterparty defaults.
- DVA is the present value of the expected cost to the counterparty if the bank defaults.

For each of its counterparties, the bank will have one CVA and one DVA.

The no-default value for outstanding derivatives transactions assumes neither side will default. As such, the value of derivatives transactions for the bank, accounting for defaults, is reflected as:

$$f_{nd} - \text{CVA} + \text{DVA}$$

A new transaction, which is positively (negatively) correlated to existing transactions, will likely increase (decrease) CVA and DVA.

Wrong-way risk occurs when the probability of default is positively correlated with exposure, and right-way risk occurs when the probability of default is negatively correlated with exposure.

LO 29.c

The yield spread of a bond is the difference between the promised yield on the bond and the risk-free rate. Default probabilities can be estimated using bond yield spreads.

The risk-free rate, which is a critical factor in default probability calculations, is typically a Treasury rate. Although a better measure to use may be credit default swap (CDS) spreads, which do not depend on the risk-free rate. An asset swap spread may also be used, as the present value of the spread is the difference between the price of the risk-free bond and the price of the corporate bond.

Bond yield spreads tend to produce higher estimated default probabilities than those estimated from historical data. For investment-grade companies, the ratio of the hazard rate derived from bond prices to the hazard rate derived from historical data is very high. However, this ratio subsequently declines as ratings decline such that the spread between the hazard rates increases.

The hazard rates (default probabilities), which are implied from credit spreads, are considered "risk neutral," which means they can be used to calculate forecasted cash flows when there is credit risk in a risk-neutral world. When historical data is used, hazard rates are considered "real-world" (physical) default probabilities. Risk-neutral probabilities tend to be much higher than real-world probabilities. The biggest reason for the difference is that bond defaults are not independent of one another. Some years have low default rates across the board, while others have substantially higher rates.

Systematic risk, which cannot be mitigated through diversification, stems from the year-to-year variation in default risk. Nonsystematic risks unique to each bond are also an element to consider, as bond returns have a high skew with limited upside. This makes diversification difficult, thereby driving bond trader returns for bearing both systematic and nonsystematic risks.

Because credit rating revisions tend to be infrequent, equity prices may be a better source for more up-to-date information on default probabilities. The Merton model shows that equity is essentially a call option on the value of the company's assets with a strike price equivalent to the required debt repayment. Historical data suggests that the Merton model produces solid rankings of default probabilities under both risk-neutral and real-world scenarios.

LO 29.d

Banks have several ways in which they can try to reduce credit risk in bilaterally cleared transactions. These methods include netting, collateral agreements, and downgrade triggers.

Although Monte Carlo simulations are often needed to calculate CVA and DVA, this is not needed when the transaction portfolio between the bank and counterparty is just a single (uncollateralized) derivative with a payoff to the bank at time T. The bank exposure in the future is the no-default derivative value at that time, where the present value is the no-default value today.

LO 29.e

Default correlation represents the likelihood that two companies will default around the same time, which happens due to: (1) external events impacting companies in the same geographic region or industry, (2) overall economic conditions leading to higher average default rates, or (3) contagion where one party defaulting may cause another to default.

Two models used to evaluate default correlation are reduced-form models (easy to calculate, lower default correlations) and structural models (higher correlations, but slower to process).

LO 29.f

To determine the time to default, a Gaussian copula model can be used to quantify the correlation between the probability distributions of times to default for multiple companies. Both real-world and risk-neutral default probabilities may be used.

The Gaussian copula model transforms the times to default into normal variables with means of zero and unit standard deviations. The assumption that the joint distribution of the variables is bivariate normal is considered the Gaussian copula. The default correlation is called the copula correlation. A one-factor model may be used, which assumes that a common factor impacts defaults for all companies.

LO 29.g

Credit risk VaR is defined within a specific confidence level as the credit risk loss that will not be exceeded over a given period of time. So, a credit VaR with a one-year time horizon and a 98.5% confidence level is the credit loss we are 98.5% confident will not be exceeded over that one-year period of time.

One approach to estimating credit VaR is the CreditMetrics model, where a probability distribution of credit losses is determined by applying a Monte Carlo simulation to the credit rating changes of all counterparties. A rating transition matrix can be used as a basis for performing Monte Carlo simulation. A joint probability distribution for rating changes can be constructed using a Gaussian copula model, with the copula correlation between rating transitions for two companies set equal to their equity return correlations.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 29.1

1. **C** The probability of a default during Year 3, conditional on no earlier default, is equal to: 0.01914 / 0.97413 = 1.965%. (LO 29.c)

2. **C** A 2-year bond yielding 190 basis points above the risk-free rate with a recovery rate of 35% would yield an average hazard rate of: 0.0190 / (1 − 0.35) = 0.0292, or 2.92% per year. (LO 29.c)

Module Quiz 29.2

1. **C** The total value for Company A (as the nondefault party) is positive, and at \$50,000, it exceeds the \$30,000 collateral posted by Company B (as the default party). Company A will be an unsecured creditor for an amount equal to \$50,000 – \$30,000 = \$20,000. (LO 29.a)

2. **B** If a new transaction is negatively correlated to existing transactions for the bank and the counterparty, the new transaction will likely decrease both the CVA and the DVA. (LO 29.b)

3. **B** If the bank treats all three transactions as individual transactions, the exposure is \$12 million. If the transactions are netted, the exposure is \$5 million. This equates to an exposure reduction of \$7 million. (LO 29.d)

Module Quiz 29.3

1. **A** Structural models are computationally intensive (relative to reduced-form models), and, therefore, take a long time to process. However, an advantage to structural models is that they allow for higher default correlations. Economic cycle impacts on default correlation trends are best reflected in reduced-form models. (LO 29.e)

2. **B** The Gaussian copula model transforms the times to default into normal variables with means of zero and unit standard deviations. The other choices are all relevant in the credit risk world, but they are not transformed into normal variables via this model. (LO 29.f)

3. **D** The appropriate formula for estimating credit VaR is: $L \times (1 - RR) \times V(T, X)$. (LO 29.g)

- The linearization of delta sensitivities in models can lead to significant errors.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 38.1

1. **C** Expected exposure measures the mean distribution of exposures at a given future date prior to the maturity of the longest maturity exposure in the netting group. (LO 38.a)

2. **D** Treating CCR as a market risk allows an institution to hedge market risk losses; however, it leaves the institution exposed to declines in counterparty creditworthiness and default. CCR can be hedged by the ongoing replacement of contracts with a counterparty instead of waiting for default to occur. (LO 38.b)

Module Quiz 38.2

1. **B** The analyst is correct to state that aggregating stress results is not meaningful. Simply taking the sum of all exposures only considers the loss that would occur if all counterparties were to simultaneously default. This is an unlikely scenario. The analyst's statement on credit quality of the counterparty is incorrect since stresses do not factor in the credit quality of the counterparty. (LO 38.c)

2. **A** A financial institution does not need to consider aggregating stresses to the EPE with its loan portfolio, because loans are not sensitive to market variables and, therefore, will not have any exposure changes from changes in market variables. (LO 38.d)

3. **C** The BCVA formula differs from the CVA formula in that BCVA incorporates expected negative exposure (ENE), and the probability of the counterparty's survival must be included in the BCVA formula. (LO 38.g)