$$\frac{\text{BCVA}(t,T)}{\text{CDS}_{\text{premium}}(t,T)} = -X_C^{CDS} \times \text{average EPE} - X_I^{CDS} \times \text{average ENE}$$

where:
X_I^{CDS} = institution's own CDS spread
ENE = expected negative exposure (the opposite of EPE)

Here the BCVA can be represented as a running spread. The formula implies that the institution may account for its own default through reduction of the unilateral CVA charge by its own credit spread multiplied by the ENE.

The calculation of this formula is identical to that for unilateral CVA. It differs only in that there is an additional identical subtractive calculation to reflect the BCVA of the financial institution.

EXAMPLE: Computing BCVA

A risk manager needs a quick calculation of the BCVA on a swap. Assume inputs are as follows: EPE = 5%, ENE = 3%, counterparty credit spread = 300 bps, and financial institution credit spread = 200 bps. **Compute** BCVA from the perspective of the financial institution.

Answer:

From the perspective of the financial institution:

–EPE × counterparty credit spread – ENE × institution credit spread

(–5% × 300) × – (–3% × 200) = –9 bps

This is what the financial institution may charge the counterparty for overall counterparty risk.

MODULE QUIZ 37.2

1. With respect to the CVA calculation, which of the following statements is correct when a risk manager wishes to understand which trades have the greatest impact on a counterparty's CVA? The manager would use:
   A. incremental CVA because it accounts for the change in CVA once the new trade is priced, accounting for netting.
   B. marginal CVA because he could break down netted trades into trade level contributions.
   C. incremental CVA because he could break down netted trades into trade level contributions.
   D. marginal CVA because it accounts for the change in CVA once the new trade is priced, accounting for netting.

MODULE 37.3: WRONG-WAY RISK

LO 37.k: Describe wrong-way risk and contrast it with right-way risk.

Wrong-way risk (WWR) is an outcome of any association, dependence, linkage, or interrelationship between exposure and counterparty creditworthiness that generates an overall increase in counterparty risk and, therefore, an increase in the amount of the credit value adjustment (CVA). WWR also results in a reduction of the debt value adjustment (DVA). WWR can be hard to determine due to difficulties assessing the relationship among variables and the lack of relevant historical data.

Right-way risk (RWR) is just the opposite of WWR. That is, any dependence, linkage, or interrelationship between the exposure and default probability of a counterparty producing an overall decrease in counterparty risk is described as RWR. RWR decreases the CVA and increases the DVA.

It is also worth mentioning that WWR has been the center of attention in historical context, while RWR has been paid relatively little attention. However, both risks are important, and financial institutions should strive to increase RWR and decrease WWR.

Another way to contrast WWR and RWR is to think that "normality" in derivatives markets is an example of RWR. That is, derivatives transactions produce intended results if the market is functioning in an expected manner. For instance, a coffee producer would sell (i.e., short) forward or futures contracts in order to protect against the downside risk of falling prices in the future, and a textile owner (that manufactures cotton cloth) would go long in cotton derivatives contracts if she anticipates a rise in cotton prices. Thus, RWR produces a favorable relation between default probability and exposure, reducing overall counterparty risk. Hedges, in normal functioning markets, should automatically generate RWR because the fundamental purpose of hedges is to curtail counterparty risk.

PROFESSOR'S NOTE
We are using derivatives markets just for illustration of wrong-way and right-way risks. By no means are these risks confined only to derivatives.

Markets and numerous interactions (e.g., market credit interaction) do not always produce normal behavior, as evidenced by the global financial crisis. Those who were seeking protection against the default of debt issuers (e.g., on collateralized debt obligations) became victims of WWR when unfavorable interaction between exposures and insurers' default probabilities (which were supposed to provide protection) intensified the amount of counterparty credit risk.

The amount of counterparty risk is roughly equal to the product of exposure and the counterparty's default probability at a specified loss rate given default. Counterparty risk is a kind of credit risk that is estimated as loss reserve for loans, and in over-the-counter (OTC) derivatives markets, it is similar to estimating loan reserves.

Loan exposure, however, is normally assumed to be a fixed amount for a specified time period, whereas in OTC derivatives, the exposure fluctuates depending on market conditions. An example of WWR (RWR) would be a change in exposure and counterparty credit quality, producing an unfavorable (favorable) dependence in exposure and counterparty credit quality and resulting in an increase (decrease) in the amount of overall counterparty risk. The change in exposure and credit quality could be due to numerous external factors such as interest rates, inflation, exchange rate movements, and global events. Note that credit quality increases actually increase

WWR. This is because counterparties with high credit quality are less likely to default. As a result, the occurrence of a default by a counterparty with high credit quality is less expected than a default by a counterparty with low credit quality.

Examples of Wrong-Way Risk and Right-Way Risk

LO 37.l: Identify examples of wrong-way risk and examples of right-way risk.

In this section, we'll create a few hypothetical examples of WWR and RWR. For example, what if Company XYZ (the borrower) and the guarantor on XYZ's loan, Company ABC, share ownership in a business (or are in the same industry)? Due to some market or economic factors, both may default together (WWR), whereas if the guarantor and the borrower are not in the same industry (nor have shared ownership), XYZ's loan guarantee may still be valid, even if XYZ defaults (RWR).

What if ABC has sold protection much higher than its capital in a concentrated area (business or industry) and XYZ has bought protection (insurance) from ABC? Macro factors may increase the "exposure" for the guarantor (ABC), and due to positive interaction between exposure and credit quality, the overall counterparty (guarantor) risk increases to the extent that XYZ's protection becomes meaningless (WWR). In contrast, the reverse of the situation may generate a favorable state—an increase in exposure may be sufficiently offset by an increase in creditworthiness.

The CVA, which is based on the amount of counterparty risk, is generally approximated by the product of exposure and the default probability of the counterparty (for a given recovery rate). This estimation is based on an underlying assumption that these events are independent. However, they may not be independent (as evidenced in the global financial crisis). Unfavorable (favorable) association between default probability (credit risk) and exposure (market risk) may produce WWR (RWR), increasing (decreasing) the overall CVA.

Quantifying WWR and RWR involves estimation of the CVA based on expected exposure, conditional on counterparty default (under the more realistic scenario of the presence of interconnected markets with systemic risk), whereas under the independence assumption, we use unconditional default probability.

It is estimated that conditional expected exposure will increase if the exposure (e.g., value of a forward contract) and the default probability of the counterparty are positively correlated, exhibiting WWR. On the other hand, negative correlation in this instance will lower the conditional expected exposure, showing RWR.

As discussed earlier, the overall counterparty risk stems from a situation in which the counterparty credit quality is linked with macro (and global) factors that also impact the exposure of transactions. The transaction can be any of the following: put options, call options, foreign currency transactions, forward contracts, credit default swaps, or interest rate and currency swaps. Let us examine WWR and RWR as they relate to some of these transactions.

Over-the-Counter Put Option

A put option gives the right to the long (buyer) to sell an underlying instrument at a predetermined price whereas the short (counterparty) is obligated to buy if the option is exercised. Out-of-the money put options have more WWR than in-the-money put options.

Macroeconomic events (such as interest rates, inflation, industry- and sector-specific factors, or global factors) may deteriorate the creditworthiness of the counterparty, increasing the default probability. The same factors may trigger a fall in the underlying (e.g., stock) asset's price, generating positive payoffs for the long but increasing the counterparty risk exposure. Before the long gets too excited to see an increase in payoffs, he is hit by the realization of increasingly becoming a victim of WWR, due to positive correlation between the risk exposure of the counterparty and probability of default of the counterparty producing an overall increase in counterparty risk. The payoffs may not materialize, although they are increasing. On the other hand, normalcy of the transaction would be termed as RWR if the counterparty is able to fulfill its obligation despite an increase in its position obligation.

PROFESSOR'S NOTE
We are assuming in the previous put option example that the counterparty and the underlying issuer are the same in order to clearly illustrate WWR. The positive association between default probability and exposure will still give rise to WWR if the counterparty and underlying issuer are not the same.

Over-the-Counter Call Option

A call option gives the right to the long (buyer) to buy an underlying instrument at a predetermined price whereas the short (counterparty) is obligated to sell at the agreed-upon price if the option is exercised. Like the put option, we are assuming the counterparty and the underlying issuer are the same.

Assume that due to changes in some macroeconomic and global factors, the default probability of the counterparty declines, and the price of the underlying asset (e.g., stock) increases, producing higher payoffs for the call buyer. In this instance, his excitement of making money will be appropriate because the counterparty will be in a strong position to pay off its obligation (due to the overall increase in creditworthiness). Such an outcome will be considered the "normalcy" of the transaction, and it is termed RWR. The short is able to fulfill its obligation despite the increase in its position obligation. On the other hand, if the counterparty is unable to fulfill its obligation due to the increase in its position obligation (higher value of underlying for the long, but higher obligation for the short—an increase in counterparty risk exposure), it would be an example of WWR (from the standpoint of the long position).

Credit Default Swaps (CDSs)

The 2007–2009 credit crisis offers a classic example of WWR from the perspective of the longs (i.e., the buyers) who had bought protection on issuers default on collateralized debt obligations (CDOs) or bonds backed by mortgage-backed securities (MBSs) via credit default swaps (CDSs).

As the real estate bubble burst and the market started taking a downward freefall, the value of MBSs started exhibiting a freefall as well. The monoline insurers, such as AMBAC and MBIA, had taken highly concentrated positions in offering protection against MBSs and CDOs. As the issuers of MBSs and CDOs started defaulting, the insurers were flooded by claims from the ones who had bought the protection (i.e., holders of CDSs).

The value of CDSs was rising, but this gain was generating an increase in risk exposure to the counterparty. Both the probability of default and the risk exposure of the insurers were rising. The unfortunate buyers of protection soon found out that the macrocredit and exposure linkage had produced unfavorable results for them. Despite huge gains on their positions, nothing materialized due to the deteriorating creditworthiness of the issuers, an example of WWR.

The normalcy of the transaction would be if the counterparty could fulfill its obligation despite an increase in position exposure (perhaps due to a negative association between risk exposure and probability of default). This would be an example of RWR. If insurance company ABC, for example, had taken a nonconcentrated exposure, it might not have experienced a decline in its creditworthiness (due to fewer claims) and would have been able to satisfy its obligations despite increasing risk exposure in the CDSs.

Foreign Currency Transactions

Consider a commercial bank in a developed economy (e.g., the United States) that enters into a cross currency agreement with a commercial bank (counterparty) in an emerging market (e.g., Uzbekistan), under which the counterparty will deliver developed market currency in return for local currency.

Macro conditions in the emerging country, such as a sovereign debt crisis, generate credit stress for the local bank, as well as a decline (depreciation) of local currency. The value of the transaction increases substantially for the financial institution in the developed economy due to the declining currency of the emerging economy. At the same time, the counterparty risk exposure increases as the gain for the financial institution in the developed economy increases.

Increases in default probability (due to credit stress) and risk exposure (due to declining currency) increase counterparty risk, resulting in WWR for the financial institution in the developed economy.

If the counterparty risk exposure and the credit quality are not unfavorably associated, then the risk exposure may increase, but the probability of default may decline (due to improvement in creditworthiness), producing a reduction in overall counterparty risk. This would be an example of RWR.

Foreign Currency Swaps

A real-world example will further clarify WWR in the foreign currency swaps market. Prior to the 2007–2009 credit crisis in the United States, numerous financial institutions in Japan had entered into swap agreements with U.S. financial institutions to obtain dollar funding by using yen. They pledged yen to get U.S. dollars. After the default of Lehman Brothers, the financial crisis reached its peak, raising grave concerns about the economic slowdown of the U.S. and European economies. The yen significantly appreciated against the U.S. dollar, resulting in a substantial gain to Japanese bank positions (the pledged yen will buy more dollars, and U.S. banks will have to surrender more dollars for the pledged yen), increasing the counterparty risk exposure for Japanese banks. At the same time, deteriorating macro conditions had a negative impact on U.S. banks and the economy. In addition, the default probabilities of the U.S. financial institutions increased. Positive (unfavorable) association between counterparty risk exposure and default probability generated an overall increase in counterparty risk for Japanese banks, and they experienced WWR.

If the risk exposure and default probabilities are not positively associated, the normalcy of the transaction would balance out the increase in risk exposure by improving the creditworthiness of the financial institutions (macro factors may be related to both events in a different manner), lowering overall counterparty risk. The counterparty is able to meet its obligation despite an increase in risk exposure (due to an appreciating yen). This would be an example of RWR.

Interest Rate Transactions

Interest rate swaps provide another good illustration of WWR. In an interest rate swap, one party (i.e., the long or fixed-rate receiver) enters into an agreement with a counterparty (i.e., the fixed-rate payer) to receive a fixed rate and pay a floating rate. The fixed-rate receiver gains if the market interest rate (the swap rate) falls.

Assume due to macroeconomic conditions (e.g., an economic downturn), policy interest rates are lowered. The fixed-rate receiver experiences a value gain to the extent that the swap rate declines against the counterparty with the fixed-rate payer and floating-rate receiver. However, this gain for the fixed-rate receiver also produces an increase in its counterparty risk exposure. Furthermore, if the economic downturn would also increase the default probability, then overall counterparty risk will increase, generating WWR for the fixed-rate receiver.

This is exactly what happened during the recent European sovereign debt crisis. Due to lower inflation and an economic recession, the policy interest rates were lowered. The euro (interest rate) swap rate declined, producing a gain for those who were holding fixed interest rate receiver positions against Italian financial institutions (fixed-rate payer). However, the decline in the euro swap rate also increased the counterparty risk exposure. Deteriorating economic conditions also increased the default probability of Italian financial institutions. An increase in both the risk exposure and default probability resulted in an overall increase in counterparty risk, generating WWR for the holder of fixed-rate receiver swaps.

In the absence of a positive association between risk exposure and default probability, the Italian financial institutions might have been able to fulfill their obligations comfortably, despite the increase in exposure, generating RWR.

Commodities

Airlines hedge against the risk of rising oil prices. For example, assume an airline is long an oil forward contract at a fixed price. The counterparty is a dealer who has taken heavy concentrated positions. If oil prices rise, the gains for the airline will rise. The airline will buy cheap oil because the spot price will be higher than the locked-in forward price, but at the same time, the risk exposure for the dealer will increase. Because the dealer had concentrated positions, there may be a flood of claims (several forward contract claims brought by various airlines), putting intense pressure on the credit quality of the counterparty. Thus, an increase in both the risk exposure and the default probability will increase overall counterparty risk, producing WWR.

On the other hand, a dealer with a nonconcentrated position may continue to have sound creditworthiness despite rising exposure. Thus, the dealer will be able to fulfill her obligation, lowering the overall expected amount of risk exposure from the standpoint of the airline. This would be an example of RWR.

WWR Modeling

LO 37.p: Describe the various wrong-way modeling methods including hazard rate approaches, structural approaches, parametric approaches, and jump approaches.

Wrong-way risk modeling techniques include:

- **Hazard rate approach:** This method (also known as the intensity approach) is the easiest approach to implement. It involves generating a stochastic process (i.e., simulation) for credit spreads. Conditional expected positive exposure is then computed for any path where default has occurred (i.e., typically with wider credit spreads). Despite its relative ease of use, this approach does not generally indicate a strong dependence between default and exposure, which may result in an underestimation of WWR.
- **Structural approach:** This is simpler method than the hazard rate approach and involves mapping the default distribution and exposure distribution onto a bivariate distribution. WWR would be identified with a combination of an early default time and a higher exposure. In this case, a positive correlation between default time and exposure is a sign of a higher CVA. An advantage to this approach is that it uses a pre-existing exposure distribution; however, this is also a disadvantage given that the information may not be relevant.
- **Parametric approach:** This method is a direct approach that examines the historical link between portfolio exposure and credit spreads. If high portfolio values are linked to above average credit spreads it would suggest the presence of WWR. A higher dependency parameter will indicate a higher CVA. For this method to be reliable, historical data must reflect current scenarios.
- **Jump approach:** This method may be the most applicable to WWR since it implies a jump at default. For example, at counterparty default, foreign exchange (FX) rates may jump (i.e., currency devalues). The jump factor is sometimes referred to as a

residual value (RV) factor. Therefore, the FX rate will depreciate by (1 – RV). Historical evidence suggests that the FX rate will jump more for large firms and highly rate sovereigns given that the shock will be more significant.

Impact of Collateral and CCPs on WWR

LO 37.m: Discuss the impact of collateral on wrong-way risk.

LO 37.n: Identify examples of wrong-way collateral.

Collateral can be viewed as a way to reduce exposure. Therefore, when exposure is increasing significantly, it's important to evaluate the overall impact of collateral on WWR. In cases where exposure is gradually increasing (before default), collateral is typically taken to minimize the impact of WWR. In this scenario, the benefit from collateral will increase as WWR increases, because additional collateral is relatively easy to request and receive. However, in cases where exposure jumps at a certain point in time, the benefits of collateral will be very limited. For example, with a jump in exposure, such as a currency devaluation associated with a sovereign default, it is much more difficult to receive collateral in a timely fashion.

An example of WWR collateral would be a payer interest rate swap collateralized with a high-quality government bond. In this case, the swap's value would increase when interest rates increase, leading to a decrease in margin. Another example would be a cross-currency swap that is collateralized by either currency in the transaction. Assuming margin is held in the currency being paid, any moves in the FX rate will increase exposure and reduce the collateral value. A more direct, but weaker, example of WWR collateral is a firm posting its own bonds as collateral.

LO 37.o: Discuss the impact of wrong-way risk on central counterparties (CCPs).

LO 37.q: Explain the implications of central clearing on wrong-way risk.

In the wake of the 2007–2009 financial crisis, the **central counterparty (CCP)** has risen as a solution for systemic risk mitigation. CCPs provide central clearing services for many different types of financial transactions between member firms, which means they essentially stand in the middle of previously bilateral OTC transactions and operate as the buyer for every seller and vice versa. Through this process, the original counterparty is no longer a direct risk as the CCP conceptually becomes the new counterparty.

CCPs remain market-neutral by netting all buy-side transactions with offsetting sell-side transactions. This multilateral netting process requires counterparties to post collateral through a margin account. The end result is less theoretical risk in the system due to the daily, or sometimes intra-daily, mark-to-market collateral system.

In the event of a default, a CCP must replace non-performing contracts. This process is formally called **novation**, and it involves closing out the non-performing side of a bilateral contract with a new counterparty who is capable of meeting the contractual obligations. If the CCP does need to step in to resolve a defaulted contract, they may need to access default funds that are held on reserve by the CCP and fed by contributions by all member firms. In this way, the losses are mutualized among all member firms by the nature of the CCP's structure. This loss mutualization process spreads any realized losses over a wide number of market participants rather than concentrating them with a singular party.

CCPs are particularly susceptible to WWR given their dependence on collateral and default fund contributions. The CCP relies on a defaulting member's posted margin (i.e., collateral) and default fund contribution to absorb potential losses. If these amounts do not cover losses, the CCP will need to use their own equity capital and/or default funds from non-defaulting members to help remain solvent.

However, the CCP's loss waterfall structure may be insufficient if member initial margins and default fund contributions fail to incorporate WWR. Since WWR tends to increase with increasing levels of credit quality, it could be argued that CCPs should demand higher levels of margin and default fund contributions from those members with higher credit quality. In addition, the collateral accepted by the CCP may also carry WWR. Some members may choose to post risky and illiquid assets as collateral, which may create higher levels of WWR for the CCP. One way to mitigate this practice is for the CCP to impose higher haircuts on specific assets that are accepted as collateral.

MODULE QUIZ 37.3

1. How many of the following statements regarding wrong-way risk (WWR) and right-way risk (RWR) are correct?
   - Co-movement in risk exposure and default probability producing a decline in overall risk is an example of wrong-way risk.
   - Co-movement in risk exposure and default probability producing an increase in overall counterparty risk is an example of right-way risk.
   - Co-movement in risk exposure and default probability producing neither a decline nor an increase in the overall counterparty risk is an example of wrong-way risk.
   - Co-movement in risk exposure and default probability producing a decline in risk exposure but an increase in counterparty default probability is an example of right-way risk.

   A. None.
   B. All.
   C. Two.
   D. Three.

2. Which of the following events would likely lead to an increase in WWR?
   I. The borrower and the guarantor are business partners.
   II. A monoline insurer sold protection concentrated in a business or industry.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following statements regarding WWR and RWR is correct?

A. A long put option is subject to WWR if both risk exposure and counterparty default probability decrease.
B. A long call option experiences RWR if the interaction between risk exposure and counterparty default probability produces an overall decline in counterparty risk.
C. Declining local currency can decrease the position gain in a foreign currency transaction, while increasing risk exposure of the counterparty.
D. The 2007–2009 credit crisis provides an example of WWR from the perspective of a long who had sold credit default swaps (CDSs) as protection against bond issuers' default.

4. How many of the following statements regarding counterparty risk are correct?
  - Speculation in normal-functioning derivatives markets automatically produces RWR.
  - RWR has been the center of attention in historical context, whereas WWR has not been paid much relative attention.
  - The counterparty default probability does not enter into the equation for estimating the overall counterparty risk.
  - Unlike exposure to OTC derivatives, which is normally assumed to be a fixed amount for a specified time period, exposure to bank loans fluctuates depending on market conditions.

A. None.
B. All.
C. Two.
D. Three.

5. Which of the following statements is correct?
I. Depreciation of the yen after the default of Lehman Brothers gave a substantial gain to Japanese bank foreign currency swaps positions to obtain dollar funding in interest rate swaps.
II. Fixed-rate receivers experience a value gain to the extent that the swap rate increases.

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

KEY CONCEPTS

LO 37.a

Motivations for pricing counterparty risk include (1) organization of responsibilities within the institution with respect to the pricing calculation and (2) determining whether a trade is sufficiently possible when factoring in counterparty risk charge.

LO 37.b

A credit value adjustment (CVA) is the price of counterparty risk. A positive value is a cost to the counterparty bearing the risk. The basic CVA formula assumes no wrong-way risk.

LO 37.c

CVA is calculated as follows:

$$\text{CVA} = -\text{LGD} \times \sum_{i=1}^{m} \text{EPE}(t_i) \times \text{PD}(t_{i-1}, t_i)$$

CVA as a spread is CVA divided by the risky annuity for the maturity of the contract in question, producing an annual spread or charge expressed in basis points:

$$\frac{\text{CVA}(t, T)}{\text{CDS}_{\text{premium}}(t, T)} = -X^{\text{CDS}} \times \text{average EPE}$$

LO 37.d

Credit spread levels, the shape of the credit spread curve, the impact of the recovery rate, and the basis risk that arises from different recovery rate assumptions must all be considered when evaluating the impact of the default probability and recovery on CVA.

LO 37.h

Netting reduces the CVA price because it nets exposure when trades are settled.

LO 37.i

Incremental CVA is used to calculate the cost of a new trade versus an existing one to determine the effect that the new trade has on CVA. Standalone CVA cannot do this. The formula for the incremental CVA calculation is identical to that for standalone CVA, except for the incremental expected exposure.

Marginal CVA is used for trade level attribution (i.e., to discover the determinants of the CVA). The formula for the calculation of marginal CVA is identical to that for standalone CVA, except for the substitution of marginal expected exposure for expected exposure.

LO 37.j

Collateralization reduces the CVA, changing only the counterparty's expected exposure.

LO 37.e and LO 37.f

Bilateral CVA is a collateral value adjustment that takes into account the possibility that both counterparties could default, though not simultaneously. The CVA of the financial institution is also known as the debt value adjustment (DVA). The BCVA is the sum of CVA and DVA components.

$$\text{BCVA} = \text{CVA} + \text{DVA}$$

$$\text{CVA} = -\text{LGD}_C \times \sum_{i=1}^{m} \text{EPE}(t_i) \times \text{PD}_C(t_{i-1}, t_i)$$

$$\text{DVA} = -\text{LGD}_I \times \sum_{i=1}^{m} \text{ENE}(t_i) \times \text{PD}_I(t_{i-1}, t_i)$$

Implications of the BCVA model include:

- BCVA can be positive if DVA is higher than CVA. Standalone CVA may only be negative (representing a cost).

- Parties in agreement on the BCVA settle in accordance with the BCVA equation's symmetry.
- Netting may be disadvantageous where the financial institution's counterparty risk exceeds that of the counterparty. Without it, the institution can pick which contracts to settle.
- Parties in agreement will have in theory all BCVAs net out to zero due to the symmetrical nature of the BCVA formula.

LO 37.g

The BCVA formula as a credit spread is:

$$\frac{\text{BCVA}(t, T)}{\text{CDS}_{\text{premium}}(t, T)} = -X_C^{\text{CDS}} \times \text{average EPE} - X_I^{\text{CDS}} \times \text{average ENE}$$

To price BCVA: −(credit spread of Counterparty A × EPE) − (credit spread of Counterparty B × ENE) = either negative number that stronger counterparty charges the weaker one or positive number that the stronger counterparty may owe the weaker one if its ENE is greater than the counterparty's EPE.

Negative BCVA: The counterparty has a higher chance of defaulting and will owe money (CVA cost).

Positive BCVA: the counterparty has a lower chance of defaulting and will be owed money (recipient of CVA fee).

LO 37.k

Financial institutions should pay more attention to wrong-way risk and right-way risk for planning purposes. The global financial crisis and European sovereign debt crisis have illustrated the significance of these risks.

Numerous macroeconomic events can impact exposure risk and default probability, producing an overall increase in counterparty credit risk. Position gains may not materialize due to an increase in the counterparty's overall risk. This is an example of wrong-way risk.

On the other hand, favorable associations between exposure risk and default probability resulting from changes in macro factors may produce a decline in overall counterparty credit risk. This is an example of right-way risk.

LO 37.l

Wrong-way risk and right-way risk can be identified in numerous investment products and transactions, such as call options, put options, credit default swaps, foreign currency transactions, interest rate products, currency swaps, and forward contracts.

The key to identify wrong-way and right-way risk is to assess the impact on overall counterparty risk. If the co-movement between risk exposure and default probability generates an overall increase (decrease) in counterparty risk, it would be an example of wrong-way risk (right-way risk).

During the global financial crisis, credit default swaps offered a classic example of wrong-way risk. The buyers of credit default swaps (protection against the default of bond issuers) experienced a substantial gain as the values of the bonds backed by mortgage-backed securities started tumbling. However, the collapse of the mortgage market not only increased the risk exposure but also the default probability, leading to an overall increase in counterparty risk. There were many buyers of credit default swaps whose gains remained paper gains due to the deteriorating creditworthiness of the counterparty.

LO 37.m

When exposure is increasing gradually, the impact of collateral on WWR will be beneficial. As WWR increases, more collateral can be taken. However, when there is a jump in exposure, the impact of collateral on WWR will be limited due to the inability to receive collateral in a timely fashion.

LO 37.n

An example of WWR collateral would be a payer interest rate swap collateralized with a high-quality government bond. Another example would be a cross-currency swap that is collateralized by either currency.

LO 37.o and LO 37.q

Central counterparties (CCPs) may be impacted by WWR if they fail to incorporate this risk into their member's initial margins and default fund contributions. To mitigate the impact of WWR, CCPs should require higher margin (i.e., collateral) and default funds from members with better credit quality. CCPs should also impose higher haircuts on any posted collateral that may increase WWR.

LO 37.p

Wrong-way risk modeling methods include the intensity approach (i.e., hazard rate approach), the structural approach, the parametric approach, and the jump approach. The intensity approach uses a stochastic process for credit spreads, the structural approach considers the combination of default and exposure distributions, the parametric approach looks at the link between portfolio values and credit spreads, and the jump approach considers that FX rates may jump upon default.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 37.1

1. **A** Accurate pricing should account for not only the cost of the trade, but also the cost of counterparty risk. (LO 37.a)

2. **A** Calculation of the CVA as a running spread entails multiplying the counterparty's EPE by its credit spread:

   $-7\% \times 4.75\% = -33$ bps

   (LO 37.c)

3. **C** Increasing the recovery rate will increase the implied probability of default but reduce the resulting CVA. The CVA will most often increase given an increase in the credit spread. When considering the shape of the credit spread curve, the CVA will be lower for an upward-sloping curve compared to a downward-sloping curve. Finally, a higher actual recovery rate will most likely lead to a lower CVA compared to a situation where the recovery assumptions are the same for both actual and settled rates. (LO 37.d)

4. **C** Both statements are incorrect. Netting reduces the CVA price as it reduces exposure when trades are settled. Collateralization also reduces the CVA, changing only the counterparty's expected exposure (EE), but not its default probability. (LO 37.j)

Module Quiz 37.2

1. **B** Understanding which trades have the greatest impact on a counterparty's credit value adjustment requires use of the marginal CVA. Incremental CVA, by contrast, is useful for pricing a new trade with respect to an existing one. (LO 37.i)

Module Quiz 37.3

1. **A** A decline in overall counterparty risk is an example of right-way risk. An increase in overall counterparty risk is an example of wrong-way risk. An increase in overall counterparty risk is a condition for the emergence of wrong-way risk. A decline in risk exposure but increase in counterparty default probability may or may not lower overall counterparty risk. (LO 37.l)

2. **C** WWR will increase if the borrower and guarantor are business partners. The guarantees offered by a monoline insurer may turn out to be worthless if the risk exposure increases and the guarantor is hit by a flood of claims due to a concentrated position in an industry or business. (LO 37.l)

3. **B** A long call option experiences RWR if risk exposure and counterparty default probability results in decreased counterparty risk. A long put option is subject to WWR if both risk exposure and counterparty default probability *increase*. Declining local currency can *increase* the position gain in a foreign currency transaction, while increasing counterparty risk exposure. The 2007–2009 credit crisis provides an example of WWR from the perspective of a long who had *bought* CDSs as protection against bond issuers' default. (LO 37.l)

4. **A** Hedging, and not speculation, in normal functioning markets automatically produces RWR. Historically, RWR was relatively neglected by institutions for planning purposes. The counterparty default probability is one of the key elements in estimating overall counterparty risk. OTC exposures fluctuate based on market conditions. (LO 37.l)

5. **D** Appreciation, and not depreciation, of the yen generated a substantial gain for Japanese banks with foreign currency swaps positions. A fixed-rate receiver experiences a value gain to the extent that the swap rate declines. (LO 37.l)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Siddique and Hasan, Chapter 4.

READING 38

THE EVOLUTION OF STRESS TESTING COUNTERPARTY EXPOSURES

Study Session 6

EXAM FOCUS

In this reading, we take a detailed look at counterparty credit risk measurement and management. We begin by differentiating between the various measures of exposure. Next, we look at the treatment of counterparty credit risk, both as a credit risk and as a market risk. We then review the credit valuation adjustment (CVA) and stresses to the CVA. For the exam, be able to describe a stress test that can be performed on both a loan portfolio and a derivatives portfolio. In addition, ensure that you are able to calculate the stressed expected loss. Finally, be able to calculate stressed CVA and understand how the debt value adjustment (DVA) differs from the CVA.

MODULE 38.1: COUNTERPARTY CREDIT RISK

Counterparty Credit Risk Exposure Measures

LO 38.a: Differentiate among current exposure, peak exposure, expected exposure, and expected positive exposure.

The concept of **counterparty credit risk (CCR)** and its measurement and management gained prominence in the 1990s, and it now forms a critical part of most organizations' risk governance. Financial institutions incorporated CCR through analyzing their derivatives exposures and by tracking the current exposure to their counterparties. Institutions measured regulatory capital for CCR as add-ons to current exposures, calculated as a percentage of gross notional derivatives values.

With the rise in importance of measuring CCR, modeling CCR also evolved. Initially, potential exposure models were used to measure and limit CCR. This approach evolved into expected positive exposure models, which allowed derivatives to be incorporated into portfolio risk models along with loans. The measurement of CCR also formed the basis for regulatory capital under Basel II and allowed for the incorporation of credit mitigants into risk modeling, including netting agreements.

There are four important definitions of exposure measures:

- **Current exposure.** Also called replacement cost, current exposure is the greater of (1) zero or (2) the market value of a transaction (or a portfolio of transactions) that would be lost if the counterparty defaulted and no value was recovered during bankruptcy.
- **Peak exposure.** Peak exposure measures the distribution of exposures at a high percentile (95% or 99%) at a given future date prior to the maturity of the longest maturity exposure in the netting group. Peak exposure is usually generated for many future dates.
- **Expected exposure.** Expected exposure measures the mean (average) distribution of exposures at a given future date prior to the maturity of the longest maturity exposure in the netting group. Expected exposure is also typically generated for many future dates.
- **Expected positive exposure (EPE).** EPE is the weighted average of expected exposures over time. The weights represent the proportion of individual expected exposures of the entire time interval. For the purposes of calculating the minimum capital requirement, the average is measured over the first year or over the length of the longest maturing contract.

One of the issues with CCR is **wrong-way risk**. Wrong-way risk is the risk that the exposure from a counterparty grows at the same time that the risk of default by the counterparty increases. Note that wrong-way risk does not arise with fixed-rate loans.

CCR Treatment

LO 38.b: Explain the treatment of counterparty credit risk (CCR) both as a credit risk and as a market risk and describe its implications for trading activities and risk management for a financial institution.

The treatment of CCR as a market risk was historically done through pricing in a **credit valuation adjustment (CVA)**. CVA represents the market value of the CCR. Before the 2007–2009 financial crisis, institutions saw stable credit spreads and CVAs that made up only a small component of a derivatives portfolio. When the financial crisis resulted in unusual losses and gains, institutions began to pay closer attention to risk managing the CVA.

Financial institutions may view CCR as either a credit risk or a market risk and may manage the credit portfolio accordingly, but looking at it as only one type of risk (in a silo) exposes the institution to the risk from the other side.

Treating CCR as a credit risk exposes the institution to changes in CVA; therefore, CVA must be included when valuing a derivatives portfolio. Not including the CVA could lead to large swings in market value. Credit risk is managed at inception or typically through collateral arrangements, but it is not actively managed once the trades are set up. Since at default all trades need to be replaced in the market, emphasis is on risk mitigation and credit evaluation.

Treating CCR as a market risk allows an institution to hedge market risk losses but leaves it exposed to declines in counterparty creditworthiness and default. However, CCR can be hedged through replacing contracts with a counterparty instead of waiting for default to occur. This can be achieved by buying positions in proportion to the counterparty's probability of default (PD). A counterparty with a low PD will only have a small component of its trades replaced this way, while counterparties with deteriorating credit quality will see their trades replaced faster and moved to other counterparties.

The treatment of CCR as both a credit risk and a market risk creates a large variety of measurements that can be complex to interpret. For example, credit risk uses current exposure, peak exposure, and expected exposure, while market risk uses CVA and variability in CVA (measured by VaR of CVA). When stress testing the portfolio, the number of stress results can be very large. By classifying CCR as both a credit risk and a market risk, the number of stress results would equal at least twice the number of counterparties plus one (stresses are run for each counterparty as well as the aggregate portfolio), and would be at least double that amount again if instantaneous shocks were considered in addition to stressed risk measures.

MODULE QUIZ 38.1

1. Which of the following exposure measures reflects the average distribution of exposures at a specific future date prior to the maturity of the longest maturity transaction within a netting set?
   A. Peak exposure.
   B. Current exposure.
   C. Expected exposure.
   D. Expected positive exposure.

2. Is the following statement on the treatment of counterparty credit risk (CCR) correct?
   "Treating CCR as a market risk does not allow an institution to hedge market risk losses, and it exposes the institution to declines in counterparty creditworthiness and default."
   A. The statement is correct with regard to both hedging market risk losses and counterparty creditworthiness and default.
   B. The statement is incorrect with regard to both hedging market risk losses and counterparty creditworthiness and default.
   C. The statement is correct with regard to hedging market risk losses only.

D. The statement is correct with regard to counterparty creditworthiness and default only.

MODULE 38.2: STRESS TESTING

Stress Testing Current Exposure

LO 38.c: Describe a stress test that can be performed on a loan portfolio and on a derivative portfolio.

Stress testing current exposure is the most common stress test. Financial institutions apply current exposure stresses to each counterparty by repricing portfolios under a scenario of risk-factor changes. Counterparties with the largest current exposures and largest stressed current exposures are typically reported to senior management.

For example, an institution that is stress testing current exposure using an equity crash involving a 25% decline in equity markets may create a table of the top counterparties with the largest stressed current exposure and include their credit ratings, mark-to-market values, collateral values, and current exposures. In effect, the table would indicate to management which counterparties are most vulnerable to a large scale equity market decline and how much the counterparties would owe the financial institution. Of course, financial institutions could construct tables for other stresses as well, including credit events and interest-rate shocks. The different stress scenarios would likely include different counterparties.

However, stress tests of current exposure suffer from two main shortcomings: (1) aggregating results is challenging and (2) it does not provide information on wrong-way risk.

Aggregating stress results needs to incorporate additional information for it to be meaningful. Simply taking the sum of all exposures only looks at a loss that would occur if all counterparties were to simultaneously default, which is an unlikely scenario. In addition, the stressed current exposures do not factor in the credit quality of the counterparty. The stress results, therefore, only look at the trade values and not the counterparty's capacity or willingness to repay its obligations. This difference becomes especially relevant when comparing the exposures between high-risk early stage companies and highly rated mature companies. Nevertheless, the task of incorporating counterparty credit quality into each stress scenario is onerous.

The stress results of current exposure also do not provide information on wrong-way risk. Since the stress measures already omit the credit quality of the counterparty, they cannot provide meaningful information on the correlation of exposure with credit quality.

Stress Testing Expected Loss

LO 38.d: Differentiate between stressed expected loss and stress loss of a credit portfolio, and calculate the stress loss on a loan portfolio and the stress loss on a

derivative portfolio.

Loan Portfolios

The **expected loss (EL)** for any counterparty in a loan portfolio is a function of the probability of default (PD_i), exposure at default (EAD_i), and loss given default (LGD_i). The EL for a portfolio is the sum of the individual exposures:

$$EL = \sum_{i=1}^{N} PD_i \times EAD_i \times LGD_i$$

Stress testing the EL could involve stressing the PD, which is a function of several other variables, including the unemployment rate or a relevant exchange rate. The **stressed expected loss (ELS)** is, therefore, conditional on the impact of these variables on the PD. The EL_S can be expressed as:

$$EL_S = \sum_{i=1}^{N} PD_i^S \times EAD_i \times LGD_i$$

The *stress loss* for the loan portfolio is the difference between EL_S and EL. The financial institution could create different stress scenarios by increasing the PDs or by stressing the various variables. Note that the variables tend to be macroeconomic or balance sheet values.

Derivatives Portfolios

The EL and EL_S for a derivatives portfolio are derived similarly to the loan portfolio in that they both use the PD and LGD. However, exposure at default, which is stochastic and depends on market factors, is replaced with the expected positive exposure (EPE_i) multiplied by an alpha factor (α). This allows CCR exposures to be used in a portfolio credit model. We can then measure EL and EL_S for derivatives portfolios as:

$$EL = \sum_{i=1}^{N} PD_i \times (EPE_i \times \alpha) \times LGD_i$$

$$EL_S = \sum_{i=1}^{N} PD_i^S \times (EPE_i^S \times \alpha) \times LGD_i$$

Stress losses are done on a portfolio of derivatives counterparties. Similar to the loan portfolio, the financial institution could create different stress scenarios by increasing the PDs, or by stressing macroeconomic variables, balance sheet values, or values of financial instruments.

In the context of EPE, institutions could also stress market variables including swap rates and equity prices. The stresses to these variables may either increase or decrease EL. Their overall impact will depend in part on the directional bias of the financial institution's portfolio, which counterparties are margined, and which have excess margin. This differs from stresses on the loan portfolio, which tend to be directionally the same and, therefore, have similar effects across counterparties. It is important to note that an institution that conducts EPE stresses does not need to separately consider aggregating them with its loan portfolio, since loans are not sensitive to market variables and will not change exposures due to changes in these variables.

Financial institutions typically shock a series of market variables instantaneously. During these instantaneous shocks, the institution shocks the initial value of a derivative prior to running the EPE simulation. How much this affects EPE will depend in part on the degree of collateralization and the portfolio's moneyness. A series of shocks could also be performed over time; however, the common approach is to perform shocks to current exposure only.

Financial institutions could also consider joint stresses between credit quality and market variables. Although this is conceptually easy, it is challenging in practice since the variables are not tied by any meaningful connection. Equity-based approaches may be the closest to modeling joint stresses; however, the link between a shock to exposure and the equity-based default probability is unclear. It is also difficult to model the connection between exposure and PD in calculating wrong-way risk. Currently, the best way to identify wrong-way risk is to stress current exposure and identify the counterparties most exposed to wrong-way risk.

Treating CCR as a credit risk allows an institution to improve the management of its loan portfolio. Performing stress tests to CCR allows aggregating losses with loan portfolios and allows considering counterparty credit quality. On the other hand, treating CCR as a market risk allows for easier joint stresses of credit quality and exposure, and allows an institution to derive the PD from market variables.

Stress Testing Credit Valuation Adjustment

LO 38.e: Describe a stress test that can be performed on CVA.

LO 38.f: Calculate the stressed CVA and the stress loss on CVA.

Stress testing CCR for market risk events looks at the losses in market value of a counterparty exposure due to market risk events or credit spread changes. Financial institutions typically only consider the **unilateral CVA** for stress testing, which looks at a counterparty's default to the institution under various market events. However, financial institutions should also consider the possibility that they could default to their counterparties, and, as a result, should consider their **bilateral CVA (BCVA)**, which is discussed in LO 38.g.

To calculate the stressed CVA and the stress loss, let's first look at the formula for calculating CVA. The following is a simplified formula for CVA that does not factor in wrong-way risk:

$$\text{CVA}_n = -\text{LGD}_n^* \times \sum_{j=1}^{1} \text{EPE}_n^*(t_j) \times \text{PD}_n^*(t_{j-1}, t_j)$$

where:

LGD_n^* = risk-neutral loss given default

$\text{EPE}_n^*(t_j)$ = risk-neutral discounted expected positive exposure

$\text{PD}_n^*(t_{j-1}, t_j)$ = risk-neutral marginal default probability

When aggregating across *N* counterparties in a portfolio, the formula for CVA becomes:

$$\text{CVA} = -\sum_{n=1}^{N} \text{LGD}_n^* \times \sum_{j=1}^{T} \text{EPE}_n^*(t_j) \times \text{PD}_n^*(t_{j-1}, t_j)$$

The components of this formula all depend on market variables, including credit spreads, market spreads, and derivatives values. Calculating a stressed CVA involves applying an instantaneous shock to these market variables, which could affect the discounted expected exposure or the risk-neutral marginal default probability. The **stressed CVA** can then be calculated as:

$$\text{CVA}_S = -\sum_{n=1}^{N} \text{LGD}_n^* \times \sum_{j=1}^{T} \text{EPE}_n^S(t_j) \times \text{PD}_n^S(t_{j-1}, t_j)$$

The *stress loss* is simply the difference between CVA_S and CVA.

Stress testing CCR in a credit-risk framework has similarities with stress testing in a market-risk framework. Both rely on EL as a function of LGD, exposure, and PD. Nevertheless, their values will differ depending on whether the view is from a market-risk or credit-risk perspective. The two primary differences include the use of risk-neutral values for CVA (versus physical values for ELs), and the use of ELs over the transaction's life for CVA (versus a specific time horizon for ELs).

In addition, CVA uses a market-based model for calculating the PD. The market-based approach has the advantage of being able to incorporate a correlation between the exposure and the PD. This correlation can significantly influence the CVA. Because there is uncertainty regarding the correlation, financial institutions should run stress tests to determine the effects on profit and loss from incorrect correlation assumptions.

Stress Testing Debt Value Adjustment

LO 38.g: Calculate the DVA and explain how stressing DVA enters into aggregating stress tests of CCR.

Financial institutions should include the liability effects in their stress calculations to properly calculate the CVA profit and loss. As a result, institutions could adequately incorporate the value of their option to default to a counterparty through the bilateral CVA. This component is often called the **debt value adjustment (DVA)**.

The BCVA formula is similar to the CVA formula with two differences. First, BCVA incorporates **expected negative exposure (ENE)**, which is calculated from the counterparty's perspective. Second, the option that the financial institution can default on its counterparty is dependent on the counterparty surviving first; therefore, the probability of the counterparty's survival must be included in the BCVA formula (we denote this as S_I, with *I* representing the financial institution). This change must also be reflected in the CVA formula. The BCVA formula can therefore be set up as:

$$\begin{aligned}\text{BCVA} = &-\sum_{n=1}^{N} \text{LGD}_n^* \times \sum_{j=1}^{T} \text{EPE}_n^*(t_j) \times \text{PD}_n^*(t_{j-1}, t_j) \times S_I^*(t_{j-1}) \\ &-\sum_{n=1}^{N} \text{LGD}_I^* \times \sum_{j=1}^{T} \text{ENE}_n^*(t_j) \times \text{PD}_I^*(t_{j-1}, t_j) \times S_n^*(t_{j-1})\end{aligned}$$

The probability of survival depends on credit default swap (CDS) spreads, and the losses depend on the financial institution's own credit spread. Institutions should be aware that this may result in counterintuitive results, for example, implying that losses occur because the institution's credit quality has improved. In any case, the financial institution should consider stress results for the BCVA and calculate stress losses by subtracting the current BCVA from the stressed BCVA.

The benefit of incorporating BCVA is that it allows CCR to be treated as market risk, which enables CCR to be included in market risk stress testing consistently. Any gains or losses from the BCVA stress could then be added to the institution's stress tests from market risk.

Shortcomings of Stress Testing CCR

LO 38.h: Describe the common pitfalls in stress testing CCR.

Stress testing CCR includes the following pitfalls:

- Stress testing CCR is a relatively new method, and institutions typically do not aggregate CCR with loan portfolio or trading position stress tests.
- Institutions typically stress test *current exposure* when incorporating the losses with loan or trading position. This is a mistake because institutions should instead use *expected exposure* or *expected positive exposure*.
- Using current exposure can lead to significant errors, which is particularly evident in at-the-money exposures when measuring derivatives market values.
- When calculating changes in exposures, using delta sensitivities is also challenging for CCR since delta is nonlinear. The linearization of delta sensitivities in models can lead to significant errors.

MODULE QUIZ 38.2

1. An analyst notes that stress testing current exposure is problematic because aggregating results is typically not meaningful, although it is easy to account for the credit quality of the counterparty. Are the analyst's statements correct?
   A. The analyst is correct with regard to both aggregating results and credit quality.
   B. The analyst is correct with regard to aggregating results only.
   C. The analyst is correct with regard to credit quality only.
   D. The analyst is incorrect with regard to both aggregating results and credit quality.

2. Which of the following statements best reflects the reason why a financial institution does not need to consider aggregating stresses to the expected positive exposure (EPE) with its loan portfolio?
   A. Loans are not sensitive to market variables.
   B. Stresses to EPE are not sensitive to market variables.
   C. The EPE and the loan portfolio are negatively correlated.
   D. The EPE and the loan portfolio are positively correlated.

3. Is the following statement on bilateral credit valuation adjustment (BCVA) correct?
   "The formula for BCVA is similar to the formula for CVA, except that the BCVA formula uses expected positive exposure (EPE) and it incorporates the probability of the

counterparty's survival."

A. The statement is correct with regard to both EPE and probability of survival.
B. The statement is correct with regard to EPE only.
C. The statement is correct with regard to probability of survival only.
D. The statement is incorrect with regard to both EPE and probability of survival.

KEY CONCEPTS

LO 38.a

The four definitions of counterparty credit risk (CCR) exposure measures are:

- Current exposure, or replacement cost, is the greater of zero or the market value of a transaction (or transactions) upon counterparty default, assuming no recovery in value.
- Peak exposure measures the distribution of exposures at a high percentile (95% or 99%) at a given future date before the maturity of the longest maturity exposure in the netting group.
- Expected exposure measures the mean distribution of exposures at a given future date prior to the maturity of the longest maturity exposure in the netting group.
- Expected positive exposure (EPE) is the weighted average of expected exposures over time, where the weights represent the proportion of individual expected exposures of the entire time interval.

LO 38.b

Credit valuation adjustment (CVA) represents the market value of the CCR. Financial institutions could view CCR as either credit risk or market risk, although it should consider both risks.

Treating CCR as credit risk exposes an institution to changes in CVA. CVA should, therefore, be included in valuing a derivatives portfolio, otherwise the portfolio could experience large changes in market value.

Treating CCR as market risk allows an institution to hedge market risk losses; however, it leaves the institution exposed to declines in counterparty creditworthiness and default.

Treating CCR as both credit risk and market risk is prudent, but this approach is complex and difficult to interpret.

LO 38.c

The most common stress test is stress testing current exposure. Stresses may include equity crash simulations, other credit events, or interest-rate shocks. Counterparties with the largest current exposures are generally reported to senior management.

Stress tests of current exposure have two primary shortcomings. First, aggregating results is challenging and stresses do not factor in the credit quality of the counterparty. Second, they do not provide information on wrong-way risk.

LO 38.d

In a loan portfolio, the expected loss (EL) for any one counterparty is a function of the probability of default (PD_i), exposure at default (EAD_i), and loss given default (LGD_i). The EL for a portfolio is the sum of the individual exposures.

The stressed expected loss (EL_S) is determined by stressing the PD. The stress loss for the loan portfolio is, therefore, the difference between the stressed EL and EL.

In a derivatives portfolio, the EL for any counterparty is a function of PD_i, LGD_i, and expected positive exposure (EPE_i) multiplied by an alpha factor (α).

LO 38.e

Currently, institutions typically only consider a counterparty's probability of default (PD) to the institution (i.e., unilateral CVA). A financial institution should instead consider its bilateral CVA, or the possibility that counterparties could default to the institution and the possibility that the institution could default to its counterparties.

LO 38.f

The formula for calculating CVA across all counterparties is a function of the discounted expected exposure, the risk-neutral marginal probability for a counterparty, and the risk-neutral LGD. The formula depends on market variables, including credit spreads, market spreads, and derivatives values. To calculate a stressed CVA (CVA_S), an instantaneous shock is applied to these market variables. The stress loss is the difference between CVA_S and CVA.

LO 38.g

Financial institutions should incorporate the value of their option to default to a counterparty through the bilateral CVA, also known as the debt value adjustment (DVA).

The BCVA formula differs from the CVA formula in that BCVA incorporates expected negative exposure (ENE) and the probability of the counterparty's survival.

The probability of survival depends on credit default swap spreads, and the losses depend on the institution's own credit spread. The financial institution should consider stress results for the BCVA. Stress losses are calculated by subtracting the value of the current BCVA from the stressed BCVA.

LO 38.h

Shortcomings of stress testing CCR include:

- CCR is not aggregated with loan portfolio or trading position stress tests.
- Stress testing current exposure is not optimal. Instead, institutions should use expected exposure or positive expected exposure.
- Using current exposure can lead to significant errors, especially for at-the-money exposures, when measuring derivatives market values.

- The linearization of delta sensitivities in models can lead to significant errors.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 38.1

1. **C** Expected exposure measures the mean distribution of exposures at a given future date prior to the maturity of the longest maturity exposure in the netting group. (LO 38.a)

2. **D** Treating CCR as a market risk allows an institution to hedge market risk losses; however, it leaves the institution exposed to declines in counterparty creditworthiness and default. CCR can be hedged by the ongoing replacement of contracts with a counterparty instead of waiting for default to occur. (LO 38.b)

Module Quiz 38.2

1. **B** The analyst is correct to state that aggregating stress results is not meaningful. Simply taking the sum of all exposures only considers the loss that would occur if all counterparties were to simultaneously default. This is an unlikely scenario. The analyst's statement on credit quality of the counterparty is incorrect since stresses do not factor in the credit quality of the counterparty. (LO 38.c)

2. **A** A financial institution does not need to consider aggregating stresses to the EPE with its loan portfolio, because loans are not sensitive to market variables and, therefore, will not have any exposure changes from changes in market variables. (LO 38.d)

3. **C** The BCVA formula differs from the CVA formula in that BCVA incorporates expected negative exposure (ENE), and the probability of the counterparty's survival must be included in the BCVA formula. (LO 38.g)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Choudhry, Chapter 12.

READING 39

AN INTRODUCTION TO SECURITIZATION

Study Session 6

EXAM FOCUS

Securitization is the process of selling cash-flow producing assets to a third party special purpose entity (SPE), which in turn issues securities backed by the pooled assets. Mortgage-backed securities (MBSs) securitize residential mortgages where the property serves as the collateral. For the exam, be prepared to discuss the securitization process of selling cash flow-producing assets to a special purpose vehicle (SPV) and contrast the differences between amortizing, revolving, and master trust structures. Also, be familiar with the different types of credit enhancements, and be prepared to define and calculate the various performance tools for securitized structures discussed.

MODULE 39.1: SECURITIZATION PROCESS

LO 39.a: Define securitization, describe the securitization process, and explain the roles of participants in the process.

Securitization is the process of transforming the illiquid assets of a financial institution or corporation into a package of asset-backed securities (ABSs) or mortgage-backed securities (MBSs). A third party uses careful packaging, credit enhancements, liquidity enhancements, and structuring to issue securities backed by the pooled cash flows (i.e., principal and interest) of the same underlying assets. Cash is transferred to the selling party, and the obligation is effectively removed from the seller's balance sheet if the sale is made without recourse. Hence, securitization represents an off-balance-sheet transaction.

A wide range of assets can be securitized (e.g., mortgages, credit card receivables, auto loans, etc.). The common feature of all ABS and MBS is that the underlying assets generate cash flows. It is important to note that the third party in the securitization process is not involved in the origination of the assets underlying the securitized product.

The two key participants in the securitization process are the originator and the issuer. The **originator** is the entity that seeks to convert its credit-sensitive assets into cash. The credit risk is then transferred away from the originator. The **issuer** is a third party who stands between the originator and the eventual investor that purchases the securities. The issuer buys the assets from the originator. The issuer must be a distinct legal entity from the originator in order for the sale of the assets to be considered a *true sale*. In a true sale, the assets are transferred off the originator's balance sheet and there is no recourse. A **special purpose vehicle (SPV)** (also sometimes referred to as a special purpose entity [SPE]) is a separate legal trust or company that is set up specifically for the purpose of securitization.

The SPV separates the underlying asset pool supporting the securitized issues from the other assets of the originator. This is an important step in the process because it ensures that the securitized assets are not affected if the originator becomes insolvent. This process of securitization provides *credit enhancement* to the newly issued securities as the third party SPV guarantees the credit quality of the issues. Thus, the investors purchasing the securitized issues are not concerned about the financial strength or creditworthiness of the originator. Investors are only concerned about the credit quality of the securitized issues and the SPV guaranteeing them. Thus, in the event that the originating financial institution becomes financially insolvent, it would not impact the SPV (except for any consideration on the first-loss piece which will be discussed later in this reading).

As stated previously, the SPV may be designated as a corporation or a trust. For tax purposes, SPVs are often incorporated in offshore locations such as the Cayman Islands, Dublin, or the Netherlands, which are regions that have SPV-friendly legislation. If the SPV is set up as a *corporation*, the originator sells the assets to the SPV in exchange for cash. The SPV, in turn, issues claims directly against the assets of the SPV. In European countries, accounting regulations allow SPVs to be structured as corporations. However, this method may not distance the originator from the assets enough for accounting purposes in the United States. Therefore, in the United States, the SPV *trust* is the most common structure.

The most common application for an SPV is to set up cash flow securitization where the originator sells the assets to the SPV who funds the purchase of the assets by issuing notes to investors. However, SPVs are also used to convert the currency of underlying assets through currency swaps, issue credit-linked notes (CLNs), and transfer illiquid assets into liquid assets (e.g., accounts receivables from equipment leases).

The **structuring agent** is the *de facto* advisor for the securitization issue. This agent is largely responsible for the security design (e.g., maturity, desired credit rating, credit enhancement, etc.) and forecasting the interest and principal cash flows. The structuring agent may also be the **sponsor** as the two roles have natural overlap.

In the event of a default, a **trustee** is charged with the fiduciary responsibility to safeguard the interests of the investors who purchase the securitized products. The trustee will monitor the assets based on pre-specified conditions of the asset pool such as minimum credit quality and delinquency ratios.

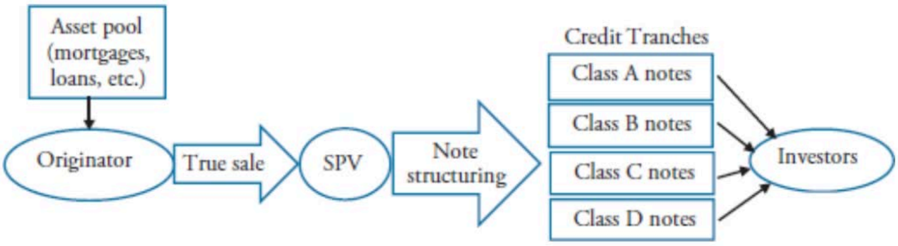
An insurance company referred to as the **financial guarantor** is sometimes used to wrap the deal by providing a guarantee of financial support in the event the SPV defaults. Financial guarantors are more common in a master trust arrangement, which we will cover later in this reading.

The **custodian** was initially responsible for safeguarding the physical securities. This role has evolved to also include the collection and distribution of the cash flows of assets like equities and bonds.

Credit rating agencies such as Moody's, Standard & Poor's, or Fitch also play an important role in the securitization process. The rating agencies provide formal credit ratings for each securitization. The rating agencies quantify the corporate credit quality of the originator. In addition, they provide analysis on competitors, the industry, regulatory issues, the legal structure of the SPV, and cash flows. If the credit rating is too low, the securitization deal may be restructured by the structuring agent to offer additional credit enhancements.

Figure 39.1 illustrates how the SPV purchases assets from an originator. The purchase of these assets is funded by issuing notes and selling them to investors. The structure of the issues is often customized to meet the credit quality needs of the investors via tranches. As mentioned, the process of securitization allows the originator to remove credit risk and assets from their balance sheet.

Figure 39.1: Securitization Process



MODULE QUIZ 39.1

1. Which of the following statements most accurately describes the effect of selling a loan without recourse?
   A. The bank that sells the loan retains a contingent liability.
   B. The bank that sells the loan bears a specified percentage of the credit risk.
   C. The loan is removed from the balance sheet of the bank that sells the loan.
   D. The purchaser has the right to sell the loan back to the bank that originated the loan.

MODULE 39.2: CASH WATERFALL, SPV STRUCTURES, SECURITIZATION BENEFITS, CREDIT ENHANCEMENTS, AND PERFORMANCE

ANALYSIS TOOLS

Cash Waterfall Process

LO 39.b: Explain the terms over-collateralization, first-loss piece, equity piece, and cash waterfall within the securitization process.

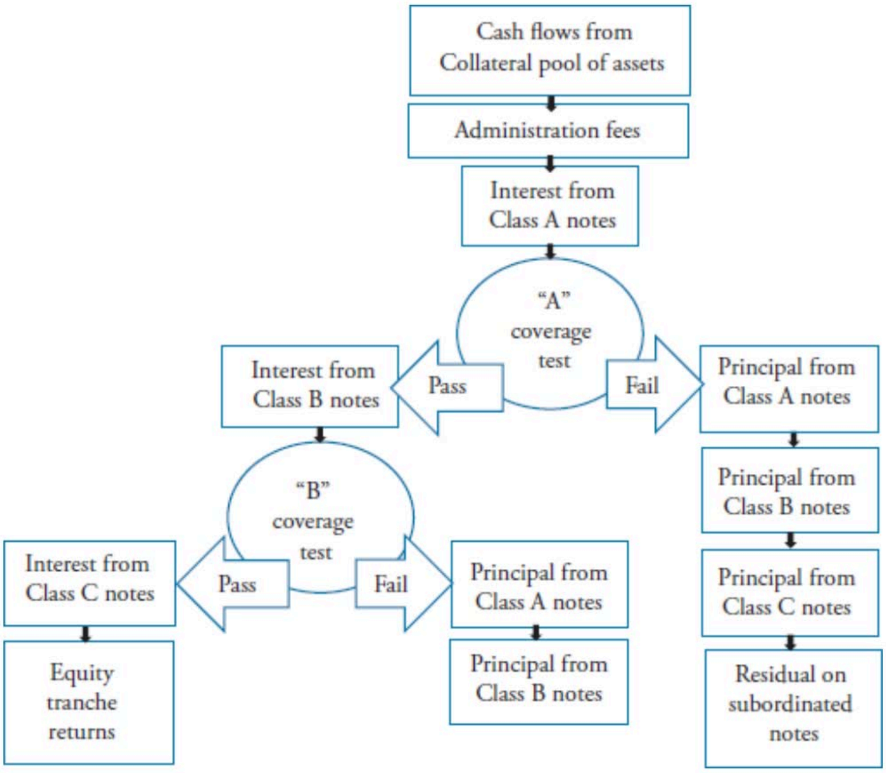
The securitization process issues notes that are structured to meet specific needs of investors by pooling the assets into different classes referred to as **tranches**. The quality of credit on the lowest rated assets can be enhanced by a method known as **overcollateralization**. The lowest class of notes is often overcollateralized by issuing notes with a principal value that is less than the principal value of the original underlying assets purchased from the originator. For example, assume a mortgage pool was securitized based on 100 mortgages, but the originator included 101 mortgages in the pool. This issue is overcollateralized by one mortgage. Thus, investors in the mortgage pool can absorb one default before suffering any economic losses.

The **first-loss piece** is the class of assets with the lowest credit quality. This is the most junior level where losses are first absorbed in the event of a default. The originator often maintains ownership of the first-loss piece. Because the originator still has ownership of this first-loss piece, it is also sometimes referred to as the **equity piece** (or the **equity tranche**). The first-loss piece or equity piece is often non-rated and absorbs initial losses.

The **cash waterfall** process of securitization refers to the order in which payments from the asset pool are paid to investors. Senior tranches are paid prior to making payments to junior tranches. A third party is hired to run “tests” in order to ensure cash flows are sufficient to pay all outstanding liabilities.

Figure 39.2 illustrates how cash flows are allocated to the different tranches in the cash waterfall process. If the first coverage test passes, then interest payments are made to subordinate tranche levels. However, if a coverage test fails, then the principal of the notes will begin to be paid off starting with the most senior tranche.

Figure 39.2: Cash Flow Waterfall



SPV Structures

LO 39.c: Analyze the differences in the mechanics of issuing securitized products using a trust versus a special purpose vehicle (SPV) and distinguish between the three main SPV structures: amortizing, revolving, and master trust.

The three main special purpose vehicle (SPV) structures used in the securitization process are amortizing, revolving, and master trust. The master trust is a special type of structure that is used for frequent issuers. The difference in how payments are received over the asset-backed security's life determines whether the ABS is better suited to the amortizing or revolving structure.

In an **amortizing structure**, principal and interest payments are made on an amortizing schedule to investors over the life of the product. Because payments are made as coupons are received, this type of structure is referred to as a *pass-through structure*. Amortizing structures are very common with the securitization of products that have amortization schedules such as residential mortgages, commercial mortgages, and consumer loans. Amortizing structures are valued based on the expected maturity and the *weighted-average life (WAL)* of the asset. The WAL is the time-weighted period that the underlying assets are outstanding. Because borrowers of mortgages and consumer loans often have the option to pay off the loans early, the WAL must include pre-payment assumptions to estimate the rate at which principal is repaid over the life of the loans.

Revolving structures are used with products that are paid back on a revolving basis. Thus, under the revolving structure, principal payments of the assets are paid in large lump sums rather than a pre-specified amortization schedule. Credit card debt and auto loans are examples of products that are securitized using a revolving structure due to their short time horizon and high rate of pre-payments. Under a revolving structure, payments are not simply passed through. Rather, principal payments are often used to purchase new receivables with criteria similar to assets already in the pool. Investors are repaid by principal payments through controlled amortization or in single lump sum payments referred to as soft bullet payments.

PROFESSOR'S NOTE

The term revolving structure is similar in nature to a revolving loan issued by a commercial bank to a corporation. Under the terms of a revolving loan, the corporation has a line of credit and is required to pay down that line of credit to zero every year. Thus, the loan does not amortize to reduce the balance, rather the balance is reduced in large lump sum payments.

A **master trust structure** allows an SPV to make frequent issues or multiple securitizations. The originator transfers assets to the master trust SPV who in turn issues new notes from this asset pool. Master trusts are often used in the securitization of mortgages and credit card debt.

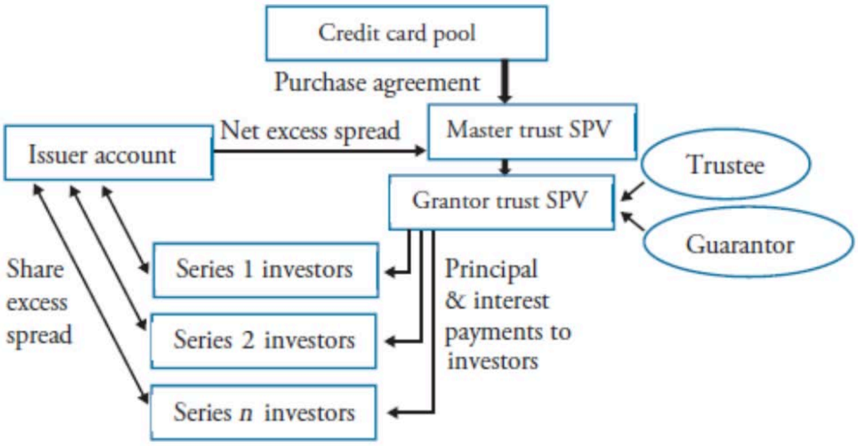
Figure 39.3 illustrates the securitization process for credit card asset-backed securities (ABSs) using the SPV master trust structure. The pool of credit card receivables is changing over time. However, the master trust structure enables the SPV to issue multiple ABS through the single trust. Investors from different series receive payments from the entire pool of credit card ABS.

Excess spread is created from the high yield credit card debt less the cost of issuing the ABS. The excess spread is the difference between the cash inflows from the underlying assets and the cash outflows in the form of interest payments on the ABS issues. After administration expenses are covered, any remaining excess spread is held in a reserve account to protect against future losses. If there are no future losses, the remaining excess spread is returned to the originator.

As illustrated by Figure 39.3, under a trust arrangement two distinct SPVs are created. The additional entity is created to further distance the originator from the issuer and the underlying assets. A common arrangement will involve a master trust, or special purpose vehicle (SPV), and a grantor trust. In contrast to the previous approach (i.e., corporation), the assets do not serve directly as collateral. Under this arrangement, the originator sells the assets to the master trust (SPV 1) for cash, but the master trust in turn deposits the assets in the grantor trust (SPV 2). The master trust receives a beneficial interest in the grantor trust, which represents the same economic position as if only one SPV was employed. Now the claims of the securitized products are backed by the beneficial claim on the master trust rather than on the assets themselves.

Credit card debt is not collateralized and typically suffers from a low rate of recovery in the event of default. Therefore, a **financial guarantor** is used as a credit enhancement. If there are payment defaults for a series, the excess spread is shared to cover the losses. The ability of SPV master trust structures to sell multiple issues to investors that share excess spreads over these multiple series makes this structure very different from the amortizing and revolving structures.

Figure 39.3: Master Trust Structure



Securitization Benefits

LO 39.d: Explain the reasons for and the benefits of undertaking securitization.

Benefits to Financial Institutions

The three main reasons for a financial institution to use securitization are for funding assets, balance sheet management, and risk management. Typically a financial institution specializes in financing specific assets such as residential mortgages, automobile loans, commercial loans, or credit card debt. Securitization of these assets provides funding for the financial institution that helps support growth, diversifies the funding mix, and reduces maturity mismatches. The diversification of the funding mix reduces risk and the cost of funding. The originator separates the assets from its balance sheet by going through a third party (i.e., the SPV).

Asset-backed securities (ABSs) issued by a SPV often have higher credit ratings than the bonds issued by the originator. If the SPV has a higher credit rating, then the originating institution benefits by lowering the cost of issuing debt when going through the SPV. ABS markets are not as liquid as bond markets, but the higher credit ratings of SPVs typically make them a more cost effective funding option. Thus, the cost savings from securitization creates a cash surplus for the originator. In addition, financial institutions often use short-term liabilities (such as savings and checking account balances) to fund long-term assets (such as residential mortgages). Securitization allows notes to be issued by the SPV that match the time horizon of the underlying asset. At the same time, the originator is able to remove the risk of mismatched durations on the balance sheet.

Another reason financial institutions securitize assets is to manage the capital on their balance sheets. The Basel I Accord set capital requirements for banks based on the riskiness of the assets. Basel I capital requirements provided a big incentive for banks to securitize assets in order to gain regulatory capital relief. SPVs are not categorized as banks, so they are not subject to the same capital requirements as banks. For example, regulators may require banks to hold capital of 8% of the bank's total asset value. An originating bank is able to reduce capital requirements by selling assets to an SPV. As mentioned earlier, originators often keep a portion of the capital exposure by retaining the first-loss piece. Therefore, the capital requirements from the securitization are significantly reduced, but not completely eliminated. By reducing capital requirements, securitization is a form of raising capital. Banks need to issue less preferred stock and other forms of equity when they securitize assets. The reduction of capital also increases the return on equity (ROE) which is a key ratio for investors.

In addition to providing regulatory relief, securitization provides additional risk management benefits in the form of removing non-performing assets from the balance sheet. Securitization removes the credit risk as well as the negative sentiment associated with non-performing assets. Furthermore, the originator may receive surplus profit from the SPV in the event these non-performing assets start returning cash flows in the future.

Benefits to Investors

Securitization also provides benefits for investors. As a result of securitizations, investors have access to new liquid assets that were previously not available to them. This allows investors to create different risk-reward profiles and diversify into new sectors. Securitized notes often provide higher risk-reward incentives than corporate bonds with the same credit rating. The improved performance results from the originator maintaining the equity tranche. In addition, holding a securitized asset diversifies the risk exposure because the securitized asset is purchased from an SPV with a pool of assets as opposed to a corporate bond from one entity. Securitization broadens the market for buyers and sellers through diversification and customization of new liquid products. The increased liquidity reduces transaction costs, which benefits both borrowers and investors.

Credit Enhancements

LO 39.e: Describe and assess the various types of credit enhancements.

Credit enhancements play an important role in the securitization process by improving the credit rating for the asset-backed security (ABS) or mortgage-backed security (MBS) tranches. The benefits of improved credit quality are even greater for the lowest-rated assets. The different types of credit enhancements used in securitization include: overcollateralization, pool insurance, subordinating note classes, margin step-up, and excess spread.

The first two types of credit enhancements are designed to increase the ability of collateral to absorb losses associated with defaults in the underlying asset pool. The lowest class of notes often exhibit **overcollateralization** where the principal value of the notes issued are valued less than the principal value of the original underlying assets. The additional collateral of the ABS issues absorbs initial losses with no impact to investors. The credit rating can also be enhanced by offering **pool insurance**. A composite insurance company provides pool insurance on the ABS issues that covers the loss of principal in the collateral pool in the event an SPV defaults.

Other types of credit enhancements are designed to control the cash flows from the collateral pool. **Subordinating note classes** of a collateral pool into different tranches is another type of credit enhancement. Junior or class B notes are subordinate to more senior class A notes. Therefore, investors in class B do not receive payments of principal until the class A notes are fully redeemed or until rating agency requirements are met. The collateral pool is required to pass certain performance tests over a period of time before making principal payments on subordinate notes.

Two other cash flow related credit enhancements are margin step-up and excess spread. ABS issues sometimes use a **margin step-up** that increases the coupon structure after a call date. The issuer has the option to redeem the notes after this call date. The margin step-up provides investors with an extra incentive to invest in the issues. However, the issuer may refinance if the increased coupons are greater than market rates.

The **excess spread** is the difference between the cash inflows from the underlying assets and the cash outflows in the form of interest payments on the ABS issues. The securitization is structured such that the liability side of the SPV (issued notes) has a lower cost than the asset side of the SPV (receivables from mortgages, loans, or credit card debt). After administration expenses are covered, any remaining excess spread is held in a reserve account to protect against future losses. If there are no future losses, the remaining excess spread is returned to the originator.

Performance Measures for Securitized Structures

LO 39.f: Explain the various performance analysis tools for securitized structures and identify the asset classes they are most applicable to.

There are a number of performance tools designed to analyze the collateral pool of asset-backed security (ABS) and mortgage-backed security (MBS) products. MBS products were first created to provide cheaper financing for residential homes by issuing pass-through securities. Investors benefited from a new liquid asset class and lenders benefited by removing interest rate risk off the balance sheet. In addition, MBS were backed by a government-sponsored entity with "Ginnie Mae" issues. Auto loans and credit card ABS products also became more popular with investors during the low interest rate environment of 2002–2007. Investor demand grew for ABSs because they provided diversification benefits and offered higher returns than the corporate bond market.

The portfolio performance of ABS and MBS products is largely dependent on the ability of individuals to pay off their obligations in the form of consumer debt and mortgages. Performance measures serve as trigger methods to accelerate amortization. ABS structures also have reserve accounts to protect against losses resulting from interest shortfalls. A key difference between the collateralized debt obligations (CDOs) and ABS structures is the number of underlying loans. A CDO portfolio typically consists of less than 200 loans, while ABS or MBS structures often have much greater diversity with thousands of obligors.

Auto Loan Performance Tools

There are specific performance measures that are used for different asset class types. *Auto loans* have features that are very favorable for investors in this ABS product. Auto loans are collateralized with assets that are highly liquid in the event of default. In addition, most loans have a short three to five year horizon. Thus, prepayment risk is very stable and losses are relatively low compared to other ABS.

A good measure of performance for auto loan ABS is the **loss curve**. The loss curve shows the expected cumulative loss for the life of the collateral pool. The expected losses based on the loss curve are compared to actual losses. Originators of prime loans typically have evenly distributed losses. Subprime or non-prime loan originators have higher initial losses resulting in a steeper loss curve. Losses for all types of loans typically decline in later years of the curve.

Another important performance tool for the auto loan ABS is the **absolute prepayment speed (APS)**, which indicates the expected maturity of the issued ABS. The APS measures prepayment by comparing the actual period payments as a percentage of the total collateral pool balance. The APS is an important measure that is used to determine the value of the implicit call option of the ABS issue at any time.

Credit Card Performance Tools

Another type of ABS product is collateralized by pools of *credit card debt*. The fact that credit cards have no predetermined term for outstanding balances differentiates this class from other ABS products. Despite having no predetermined term, most credit card debt is repaid within six months. The repayment speed of a credit card ABS is controlled by scheduled amortization or a revolving period under a master trust framework. Recall that the master trust allows multiple issues and principal collections to be used to purchase new receivables.

Three important performance tools for credit card receivables of ABS are the delinquency ratio, default ratio, and monthly payment rate (MPR). These three ratios serve as triggers to signal early amortization of the receivable pool. The delinquency ratio and default ratio measure the credit loss on credit card receivables pools. An early indication of the overall quality of the credit card ABS collateral pool is the delinquency ratio. The **delinquency ratio** is computed by dividing the value of credit card receivables that are 90 days past due by the total value of the credit card receivables pool. The **default ratio** is calculated by dividing the amount of written off credit card receivables by the total credit card receivables pool. The **monthly payment**

rate (MPR) is calculated as the percentage of monthly principal and interest payments divided by the total credit card receivables pool. Rating agencies require every non-amortizing ABS (such as credit cards) to set a minimum MPR as a trigger for early amortization.

MODULE QUIZ 39.2

1. A major benefit of securitization for a financial institution is the ability to remove assets from the balance sheet, which lowers risk and the required regulatory capital. While a large portion of the risk is removed from the balance sheet the originating financial institution often maintains a portion of the risk. Which of the following terms best identifies the risk that is maintained by the originator?
   A. Correlation.
   B. Excess spread.
   C. First-loss piece.
   D. Guarantor of collateral value.

2. Securitized products are often customized to meet the needs of the investor as well as the originator. What type of asset-backed securities (ABSs) typically uses a revolving structure?
   A. Residential mortgage.
   B. Credit card debt.
   C. Commercial mortgage.
   D. Commercial paper.

3. Which of the following statements regarding credit enhancements in the process of structuring a securitization through a special purpose vehicle (SPV) is correct?
   A. The securitization process is structured such that the asset side of the SPV has a lower cost than the liability side of the SPV.
   B. Credit enhancements are typically only associated with mortgage-backed securities (MBSs) and are not used in other types of asset-backed securities (ABSs).
   C. The most senior class of notes is often overcollateralized in order to reduce the risk of the asset-backed security (ABS).
   D. A margin step-up is sometimes used by an asset-backed securities (ABSs) where the coupon structure increases after a call date.

MODULE 39.3: SECURITIZED STRUCTURE RATIOS

LO 39.g: Define and calculate the delinquency ratio, default ratio, monthly payment rate (MPR), debt service coverage ratio (DSCR), the weighted average coupon (WAC), the weighted average maturity (WAM), and the weighted average life (WAL) for relevant securitized structures.

As described previously, the delinquency ratio, default ratio, and monthly payment rate (MPR) serve as triggers to signal early amortization of the receivables pool for an ABS.

EXAMPLE: Delinquency ratio, default ratio, monthly payment rate

Suppose an ABS has a total outstanding balance of credit card receivables of $57,800,000. $49,900,000 of the total receivables are current, $5,750,000 of the receivables are over 30 days past due, $1,270,000 of the receivables are over 60 days past due, and $880,000 are over 90 days past due. In addition, $1,100,000 of receivables were written off. Total monthly principal and interest payments per month are $1,560,000. **Calculate** the delinquency ratio, default ratio, and monthly payment rate for this ABS.

Answer:

The delinquency ratio 1.522%, computed by dividing the value of credit card receivables over 90 days past due by the total credit card receivables pool ($880,000 / $57,800,000).

The default ratio is 1.903%, calculated by dividing the amount of written off credit card receivables by the total credit card receivables pool ($1,100,000 / $57,800,000).

The monthly payment rate (MPR) is 2.699%, calculated as the percentage of monthly principal and interest payments divided by the total credit card receivables pool ($1,560,000 / $57,800,000).

MBS Performance Tools

The debt service coverage ratio (DSCR), weighted average coupon (WAC), weighted average maturity (WAM), and weighted average life (WAL) are performance tools used to analyze MBS. The **debt service coverage ratio (DSCR)** is calculated by dividing net operating income (NOI) by the total amount of debt payments. Net operating income is the income or cash flows that are left over after all of the operating expenses have been paid. The DSCR is a performance tool that measures the ability of a borrower to repay the outstanding debt associated with commercial mortgages. A DSCR less than one indicates that the underlying asset pool of commercial mortgages do not generate sufficient cash flows to cover the total debt payment. Total debt service refers to all costs related to servicing a company's debt. This often includes interest payments, principal payments, and other obligations. As investors' confidence levels in the securitization increase, the required DSCR decreases, and vice versa. For residential mortgages, this ratio is typically between 2.5 and 3.0. However, higher DSCRs are needed with more risky receivables where the value of the receivables is highly discounted in the event of a default.

EXAMPLE: **Debt service coverage ratio**

Suppose an MBS has net operating income from commercial mortgaged properties equal to $89,572,500. The total debt payments for notes issued against these mortgages is equal to $87,958,000. **Calculate** the debt service coverage ratio (DSCR).

Answer:

The DSCR is equal to 1.02, calculated as $89,572,500 / $87,958,000. A DSCR greater than one implies that there is sufficient cash flows generated from the underlying mortgage pool to meet debt payments. However, this is a very low DSCR for mortgages.

The **weighted average coupon (WAC)** is calculated by multiplying the mortgage rate for each pool of loans by its loan balance and then dividing by the total outstanding loan balance for all pools. Thus, it measures the weighted coupon of the entire mortgage pool. The WAC is compared to the net coupon payable to investors as an indication of the mortgage pool's ability to pay over the outstanding life of the MBS.

EXAMPLE: Weighted average coupon

Suppose an MBS is composed of three different pools of mortgages: $6 million of mortgages that yield 7.8%, $10 million of mortgages that yield 6.0%, and $4 million of mortgages that yield 5%. **Calculate** the weighted average coupon (WAC).

Answer:

The WAC is calculated as follows:

$$\begin{aligned} \text{WAC} &= [0.078(6 \text{ million}) + 0.06(10 \text{ million}) \\ &\quad + 0.05(4 \text{ million})] \,/\, (6 \text{ million} + 10 \text{ million} + 4 \text{ million}) \\ &= (0.468 \text{ million} + 0.6 \text{ million} + 0.2 \text{ million}) \,/\, 20 \text{ million} \\ &= 1.268 \text{ million} \,/\, 20 \text{ million} \\ &= 0.0634 \text{ or } 6.34\% \end{aligned}$$

If notes issued by the SPV are for 5.5%, for example, then an excess spread will be generated if there are no defaults on the original mortgages.

The **weighted average maturity (WAM)** is the weighted average months remaining to maturity for the pool of mortgages in the MBS. To calculate the WAM, the weight of each MBS pool is multiplied by the time until maturity of each MBS pool, and then all the values are added together. (Note that the weight is determined by taking the total value of the pool for one maturity and dividing that by the total value of all loans.)

The volatility of an MBS is directly related to the length of maturity of the underlying securities. The WAM is calculated based on stated maturity dates or reset dates. A WAM calculated based on stated maturity dates includes the liquidity risk of all mortgage securities in the portfolio by using the actual maturity date. A WAM calculated based on reset dates captures the effect of prepayments on the maturity of the loans.

EXAMPLE: Weighted average maturity

Suppose an MBS is composed of three different pools of mortgages: $6 million of mortgages that have a maturity of 180 days, $10 million of mortgages that have a maturity of 360 days, and $4 million of mortgages that have a maturity of 90 days. **Calculate** the weighted average maturity (WAM).

Answer:

The WAM is calculated as follows:

$$\begin{aligned} \text{WAC} &= [180(6\text{ million}) + 360(10\text{ million}) \\ &\quad + 90(4\text{ million})] \,/\, (6\text{ million} + 10\text{ million} + 4\text{ million}) \\ &= (1{,}080\text{ million} + 3{,}600\text{ million} + 360\text{ million}) \,/\, 20\text{ million} \\ &= 5{,}040\text{ million} \,/\, 20\text{ million} \\ &= 252\text{ days} \end{aligned}$$

The **weighted average life (WAL)** of the mortgage notes issued is calculated by summing the time to maturity multiplied by a pool factor using the following formula:

$$\text{WAL} = \sum(a/365) \times \text{PF}(t)$$

Figure 39.4 illustrates how WAL is calculated for an MBS with an initial outstanding balance for the entire pool of $89,530,000. The pool factor, *PF(t)*, is the outstanding notional value adjusted by the repayment weighting. The actual days, *a*, until the next payment are stated in Column B. This amount in Column B is then divided by 365 in Column F to calculate the time to maturity. The amount in Column F is multiplied by Column C to compute each individual note's weighted life and this is recorded in Column G. WAL is then determined as the summation of Column G.

Figure 39.4: Calculation of WAL

A	B	C	D	E	F	G
Payment Date	Actual Days (a)	PF(t)	Paid Principal	Outstanding Balance (000s)	a / 365	(a / 365) × PF(t)
11/21/2008	66	1.00		89,530	0.1808	0.1808
1/26/2009	90	0.94	5,059	84,471	0.2466	0.2318
4/26/2009	91	0.89	4,941	79,530	0.2493	0.2219
7/26/2009	91	0.83	4,824	74,706	0.2493	0.2069
10/25/2009	91	0.75	4,706	70,000	0.2493	0.1870
1/24/2010	91	0.73	4,588	65,412	0.2493	0.1820
4/25/2010	91	0.68	4,471	60,941	0.2493	0.1695
7/25/2010	91	0.63	4,353	56,588	0.2493	0.1571
10/24/2010	92	0.58	4,235	52,353	0.2521	0.1462
1/24/2011	90	0.54	4,118	48,235	0.2466	0.1332
4/24/2011	91	0.49	4,000	44,235	0.2493	0.1222
7/24/2011	92	0.45	3,882	40,353	0.2521	0.1134
10/24/2011	92	0.41	3,765	36,588	0.2521	0.1033
1/24/2012	91	0.37	3,647	32,941	0.2493	0.0922
4/24/2012	91	0.33	3,529	29,412	0.2493	0.0823
7/24/2012	92	0.29	3,412	26,000	0.2521	0.0731
10/24/2012	92	0.25	3,294	22,706	0.2521	0.0630
1/24/2013	90	0.22	3,176	19,530	0.2466	0.0542
4/24/2013	91	0.18	3,058	16,472	0.2493	0.0449
7/24/2013		0	16,472	0	0	0
					WAL =	2.565

MODULE QUIZ 39.3

1. Assume an MBS is composed of the following four different pools of mortgages:
   - $2 million of mortgages that have a maturity of 90 days.
   - $3 million of mortgages that have a maturity of 180 days.
   - $5 million of mortgages that have a maturity of 270 days.
   - $10 million of mortgages that have a maturity of 360 days.

   What is the weighted average maturity (WAM) of these mortgage pools?

   A. 167 days.
   B. 225 days.
   C. 252 days.
   D. 284 days.

MODULE 39.4: PREPAYMENT FORECASTING

LO 39.h: Explain the prepayment forecasting methodologies and calculate the constant prepayment rate (CPR) and the Public Securities Association (PSA) rate.

Common methodologies used to estimate prepayments for an MBS or ABS collateralized by mortgages or student loans are the **constant prepayment rate (CPR)** and the **Public Securities Association (PSA)** method. Assumptions regarding the rate of prepayment are required to estimate the cash flows for an MBS. Prepayments will reduce the yield of an MBS, assuming principal payments remain unchanged.

The CPR is calculated as: $CPR = 1 - (1 - SMM)^{12}$. The **single monthly mortality (SMM)** is the single-month proportional prepayment. Factors that influence the CPR are market environment, characteristics of the underlying mortgage pool, and the outstanding balance of the pool.

EXAMPLE: Constant prepayment rate

Suppose an ABS has an SMM of 1.5%. This implies that the approximate prepayment for the month is equal to 1.5% of the remaining mortgage balance for the month less the scheduled principal repayment. **Calculate** the CPR for this MBS.

Answer:

The CPR for this MBS equals 16.59%, calculated as:

$$CPR = 1 - (1 - 0.015)^{12} = 0.1659$$

The PSA typically assumes that prepayments will increase as a pool approaches maturity. The MBS pool of mortgages has a 100% PSA if its CPR begins at 0 and increases 0.2% each month for the first 30 months. A graph of the CPR for an ABS as it approaches maturity is illustrated in Figure 39.5.

Note that the middle line in Figure 39.5 represents 100% PSA where the prepayments are assumed to start at 0 and increase 0.2% each month up until month 30. After 30 months, the 100% PSA is assumed to be at a constant 6% (calculated as 0.2 times 30 months) until maturity. Other prepayment scenarios are then calculated as a percentage of this 100% base case. Thus, a 50% PSA assumes 50% of the initial increase for the first 30 months. In Figure 39.5, the bottom line represents the 50% PSA scenario where prepayments are assumed to increase 0.1% each month for the first 30 months, before reaching a constant prepayment rate of 3%. Similarly, the top line represents the 150% PSA scenario where prepayments are assumed to increase 0.3% each month for the first 30 months, before reaching a constant prepayment rate of 9%.

Figure 39.5: Different Prepayment Scenarios

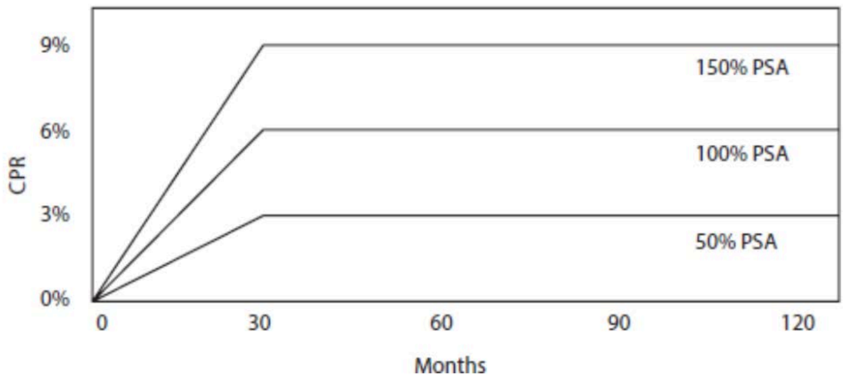


Figure 39.6 summarizes performance tools discussed in this topic based on the type of ABS or MBS.

Figure 39.6: ABS and MBS Performance Tools

Performance Analysis Tools	Asset Type	Calculation
loss curves	auto loans	expected cumulative losses
absolute prepayment speed (APS)	auto loans	prepayments / pool balance
delinquency ratio	credit cards	past due receivables / pool balance
default ratio	credit cards	defaults / pool balance
monthly payment rate (MPR)	credit cards	receivables collected / pool balance
debt service coverage ratio (DSCR)	commercial mortgages	NOI / debt payments
weighted average coupon (WAC)	mortgages	weighted pool coupon payments
weighted average maturity (WAM)	mortgages	weighted pool maturity
weighted average life (WAL)	mortgages	$\sum(a/365) \times PF(t)$
single monthly mortality (SMM)	mortgages, home-equity, student loans	prepayment / pool balance
constant prepayment rate (CPR)	mortgages, home-equity, student loans	$1 - (1 - SMM)^{12}$
Public Securities Association (PSA)	mortgages, home-equity, student loans	[CPR / (0.2)(months)] × 100

MODULE QUIZ 39.4

1. Which of the following measures are most likely to be used by a securitized product backed by student loans?
   A. Single monthly mortality (SMM), constant prepayment rate (CPR), and Public Securities Association (PSA).
   B. Loss curves and absolute prepayment speed (APS).
   C. Weighted average life (WAL), weighted average maturity (WAM), and weighted average coupon (WAC).
   D. Debt service coverage ratio (DSCR) and monthly payment rate (MPR).

KEY CONCEPTS

LO 39.a

Securitization is the process of issuing securities against an asset pool. The proceeds of the security sale collateralize the purchase of the assets from the originator, thereby removing the liability and involvement of the originator. A special purpose vehicle (SPV) is used to separate the assets from the originator and customize the products for investors.

LO 39.b

A common credit enhancement of securitized assets is overcollateralization where the principal value of the notes issued by the SPV are valued less than the principal value of the original underlying assets. The first-loss piece or equity piece absorbs initial losses. This non-rated junior tranche is often held by the originator.

LO 39.c

The master trust is a special type of structure that is used for frequent issuers. The difference in how payments are received from the underlying collateral over the asset-backed security's life determines whether the ABS is better suited to the amortizing or revolving structure.

LO 39.d

Financial institutions benefit from securitization by funding assets, balance sheet management, and risk management. Securitization benefits investors by providing access to liquid assets that were previously not available to them.

LO 39.e

Credit enhancements such as overcollateralization, pool insurance, subordinating note classes, margin step-up, and excess spread enable originators to lower costs while providing investors customized products that meet their needs.

LO 39.f

The portfolio performance of ABS and MBS products is largely dependent on the ability of individuals to pay off their obligations in the form of consumer debt and mortgages. Performance measures serve as trigger methods to accelerate amortization.

The loss curve shows the expected cumulative loss for the life of the collateral pool. The absolute prepayment speed (APS) measures prepayment by comparing the actual period payments as a percentage of the total collateral pool balance.

LO 39.g

The delinquency ratio is computed by dividing the value of credit card receivables more than 90 days past due by the total credit card receivables pool. The default ratio is calculated by dividing the amount of written off credit card receivables by the total credit card receivables pool. The monthly payment rate (MPR) is calculated as the percentage of monthly principal and interest payments divided by the total credit card receivables pool.

The debt service coverage ratio (DSCR) is calculated by dividing net operating income (NOI) by the total amount of debt payments. The weighted average coupon (WAC) is calculated by multiplying the mortgage rate for each pool of loans by its loan balance and then dividing by the total outstanding loan balance for all pools. The weighted average maturity (WAM) is the weighted average months remaining to maturity for the pool of mortgages in the MBS. The weighted average life (WAL) of the mortgage notes issued is calculated by summing the time to maturity multiplied by a pool factor, which is the outstanding notional value adjusted by the repayment weighting.

LO 39.h

Common methodologies used to estimate prepayments for securitized products collateralized by mortgages or student loans are the constant prepayment rate (CPR) and the Public Securities Association (PSA) method.

The CPR is calculated as:

$$CPR = 1 - (1 - SMM)^{12}$$

The PSA typically assumes that prepayments will increase as a pool approaches maturity. The MBS pool of mortgages has a 100% PSA if its CPR begins at 0 and increases 0.2% each month for the first 30 months.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 39.1

1. **C** When a bank originates a loan and then sells it without recourse, the loan is removed from the bank's balance sheet, and the purchaser bears all of the credit risk. (LO 39.a)

Module Quiz 39.2

1. **C** The originator often maintains ownership of the first-loss piece, which is the class of assets with the lowest credit quality and is the most junior level where losses are first absorbed in the event of a default. (LO 39.b)

2. **B** Revolving structures are used with products that are paid back on a revolving basis, such as credit card debt or auto loans. Credit card debt does not have a pre-specified amortization schedule; therefore the principal paid back to investors is in large lump sums rather than amortizing schedules. (LO 39.c)

3. **D** ABS issues may use a margin step-up that increases the coupon structure after a call date. Credit enhancements play an important role in the securitization process for both the asset-backed security (ABS) and mortgage-backed security (MBS) issues. The liability side of the SPV has a lower cost than the asset side of the SPV to create an excess spread prior to administration costs. The lowest class of notes are often overcollateralized where the principal value of the notes issued are valued less than the principal value of the original underlying assets. (LO 39.e)

Module Quiz 39.3

1. **D** The WAM is calculated as follows:

$$\begin{aligned} WAC &= [90(2\text{ million}) + 180(3\text{ million}) + 270(5\text{ million}) + 360(10\text{ million})] / (2\text{ million} + 3\text{ million} + 5\text{ million} + 10\text{ million}) \\ &= (180\text{ million} + 540\text{ million} + 1{,}350\text{ million} + 3{,}600\text{ million}) / 20\text{ million} \\ &= 5{,}670\text{ million} / 20\text{ million} \\ &= 284\text{ days} \end{aligned}$$

(LO 39.g)

Module Quiz 39.4

1. **A** The constant prepayment rate (CPR) and the Public Securities Association (PSA) method are common methodologies used to estimate prepayments for student loans and mortgages. (LO 39.h)

FORMULAS

Reading 19

expected loss: $EL(\$) = PD(\%) \times LGD(\%) \times EAD(\$)$

Reading 20

unexpected loss: $UL = EA \times \sqrt{PD \times \sigma^2{}_{LR} + LR^2 \times \sigma^2{}_{PD}}$

risk contribution: $RC_i = \dfrac{UL_i \sum_j UL_j \rho_{ij}}{UL_P}$

economic capital: economic $capital_P = UL_P \times CM$

Reading 21

capital adequacy ratio (CAR): $CAR = \dfrac{\text{capital}}{\text{risk-weighted assets}} \geq \alpha$

risk-weighted assets (RWA): $RWA = K \times 12.5 \times EAD$

risk-adjusted return on capital (RAROC): $RAROC = \dfrac{\text{loan revenues}}{\text{capital at risk}}$

loan revenues $=$ loan value $\times (s + f - 1 - c)(1 - x)$

Reading 25

Altman's Z-score:

X_1: working capital / total assets

X_2: retained earnings / total assets

X_3: earnings before interest and taxes (EBIT) / total assets

X_4: market value of equity / book value of total liabilities

X_5: sales / total assets

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

probability of default:

$Q(t) = 1 - e^{-\bar{\lambda}(t) \times t}$

where:

$\bar{\lambda}(t)$ = average hazard rate between time 0 and time t

CDS-bond basis = CDS spread − bond yield spread hazard rate:

$$\bar{\lambda} = \frac{s(T)}{1 - RR}$$

where:

$\bar{\lambda}$ = average hazard rate
s(T) = credit spread for maturity T
RR = recovery rate

Merton model: $E_T = \max(V_T - D, 0)$

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}}$$

$d_2 = d_1 - \sigma_v \sqrt{T}$
N = cumulative normal distribution function

distance to default: $d_2 = \dfrac{\ln(V_0) - \ln(D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$

Reading 27

correlation with default probabilities: $\rho_{12} = \dfrac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1 - \pi_1)}\sqrt{\pi_2(1 - \pi_2)}}$

Reading 30

Probability of survival at time t = $PS_t = e^{-\lambda \times t}$

Probability of default in year t = $PD_t = (PS_{t-1} - PS_t)$

up-front premium % = $D \times (s - c)$

where:
D = CDS payment duration
s = CDS spread
c = coupon rate

CDS price: price = $100 - [100 \times D \times (s - c)]$

Gaussian copula model: $Q(t \mid F) = N\left(\dfrac{N^{-1}[Q(t)] - \sqrt{\rho}F}{\sqrt{1 - \rho}}\right)$

Reading 36

netting factor $= \frac{\sqrt{n + n(n-1)\overline{\rho}}}{n}$

where:
n = number of exposures
$\overline{\rho}$ = average correlation

Reading 37

credit value adjustment: $CVA = -LGD \times \sum_{i=1}^{m} EPE(t_i) \times PD(t_{i-1}, t_i)$

where:
LGD = loss given default or how much of the exposure one expects to lose in the event of a counterparty default; equal to 1 minus the recovery rate (1 − RR)
EPE = discount expected positive exposure for future dates
PD = marginal default probability

CVA as a spread: $\frac{CVA(t,T)}{CDS_{premium}(t,T)} = -X^{CDS} \times \text{average EPE}$

where:
$CDS_{premium}(t, T)$ = unit premium value of a credit default swap
X^{CDS} = CDS premium at maturity date *T*; this amount can be thought of as a credit spread
EPE = expected positive exposure that is the average of the expected exposure over a preset time period, typically from the present to the maturity date of the transaction

bilateral credit value adjustment: $BCVA = CVA + DVA$

$$CVA = -LGD_C \times \sum_{i=1}^{m} EPE(t_i) \times PD_C(t_{i-1}, t_i)$$

$$DVA = -LGD_I \times \sum_{i=1}^{m} ENE(t_i) \times PD_I(t_{i-1}, t_i)$$

where:
ENE = expected negative exposure (EPE from the counterparty's perspective)

BCVA as spread: $\frac{BCVA(t,T)}{CDS_{premium}(t,T)} = -X_C^{CDS} \times \text{average EPE} - X_I^{CDS} \times \text{average ENE}$

where:
X_I^{CDS} = institution's own CDS spread
ENE = expected negative exposure (the opposite of EPE)

Reading 38

loan portfolio expected loss: $EL = \sum_{i=1}^{N} PD_i \times EAD_i \times LGD_i$

derivatives portfolio expected loss: $EL = \sum_{i=1}^{N} PD_i \times (EPE_i \times \alpha) \times LGD_i$

Reading 39

weighted average life (WAL): $WAL = \sum(a/365) \times PF(t)$

constant prepayment rate: $CPR = 1 - (1 - SMM)^{12}$

APPENDIX

USING THE CUMULATIVE Z-TABLE

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of \$5.00 and a standard deviation of \$1.50. What is the approximate probability of an observed EPS value falling between \$3.00 and \$7.25?

If EPS = x = \$7.25, then $z = (x - \mu) / \sigma$ = (\$7.25 − \$5.00) / \$1.50 = +1.50.

If EPS = x = \$3.00, then $z = (x - \mu) / \sigma$ = (\$3.00 − \$5.00) / \$1.50 = −1.33.

For z*-value of 1.50:* Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z*-value of −1.33:* Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of −1.33 is 1 − 0.9082 = 0.0918.

The area between these critical values is 0.9332 − 0.0918 = 0.8414, or 84.14%.

Hypothesis Testing—One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0%. Assume the population standard deviation is 20.0%. Can we say with 95% confidence that the mean return is greater than 0%?

H_0: $\mu \leq 0.0\%$, H_A: $\mu > 0.0\%$. The test statistic = z-statistic = $\dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
= (2.0 − 0.0) / (20.0 / 6) = 0.60.

The significance level = 1.0 − 0.95 = 0.05, or 5%.

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative z-table. The closest value is 0.9505, with a corresponding critical z-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0.

Hypothesis Testing—Two-Tailed Test Example

Using the previous assumptions, suppose that the analyst now wants to determine with 99% confidence that the stock's return is not equal to 0.0%.

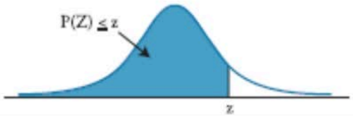
H_0: $\mu = 0.0\%$, H_A: $\mu \neq 0.0\%$. The test statistic (z-value) = (2.0 − 0.0) / (20.0 / 6)
= 0.60. The significance level = 1.0 − 0.99 = 0.01, or 1%.

Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 (1.0 − 0.005) in the table. The closest value is 0.9951, which corresponds to a critical z-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0%.

Cumulative Z-Table

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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