$$E = AN(d_1) - Le^{-rT}N(d_2)$$

Where A is the current market value of the firm's assets, r is the risk-free rate, and N(.) is the cumulative distribution function of the standard normal distribution. The formulas that follow show the two instances of N(.) in which σ_A denotes the volatility of asset returns:

$$d_1 = \frac{\ln\left(\frac{A}{L}\right) + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Additionally, the volatility of equity σ_E can be computed assuming that equity is a function of both assets and time:

$$\sigma_E = \frac{\sigma_A AN(d_1)}{E}$$

In these provided formulas, the unknowns are the current value of assets (A) and volatility of asset returns σ_A. All other variables are known, which includes the following:

- Market value of equity (E)—the market cap of the borrower
- Volatility of equity (σ_E)—estimated using historical market data
- Time (T) until full repayment of the loan
- Face value of the debt (L)—in practice, this is often the short-term debt on the borrower's balance sheet
- Risk-free rate (r)—the current yield on Treasury bills

The **risk-neutral probability of default**, $N(-d_2)$, is effectively the probability that shareholders will not exercise their option to repay the loan. This analysis assumes that the firm's assets grow at the risk-free rate. For the **real-world probability of default**, the expected return on the firm's assets (μ) is substituted for the risk-free rate (r):

$$PD = N\left[-\frac{\ln\left(\frac{A}{L}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}\right]$$

For this formula to derive a PD estimate, the asset return is required. A common approach is to solve for the market value of equity (E) and the standard deviation of equity σ_E using historical market data. Then, define μ as the historical rate of change in the value of assets (A). Additionally, the real-world PD estimate can be used to measure the **distance to default (DD)**, which is the number of standard deviations the company's assets are away from the face value of the debt (L):

$$DD = \frac{\ln\left(\frac{A}{L}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$

Additional Credit Risk Modeling Approaches

LO 21.f: Compare and contrast different approaches to credit risk modeling, such as those related to the Merton model, Credit Risk Plus (CreditRisk+), CreditMetrics, and the Moody's-KMV model.

As shown in the previous section, the Merton model uses an option pricing model to estimate the PD for a borrower's loans. This financial model presents a very useful calculation, but it only focuses on the short-term debt of the firm. There is a related but alternative model known as **Moody's-KMV Expected Default Frequency (EDF) model**. This model has two key differences from the Merton model. First, rather than using the standardized normal distribution, the Moody's-KMV model uses historical data to develop the distribution of default frequencies. Second, the default point (L) is defined as all short-term debt plus half of the long-term debt. The inclusion of some long-term debt more closely approximates the actual loan obligations of the borrower.

The **CreditMetrics model** computes the PD through peer set analysis (i.e., comparing a borrower to the default records of others). This method uses a rating transition matrix to account for changes in default rates over time. The CreditMetrics method is effectively a mark-to-market model. The primary model inputs include the borrower's credit rating, a transition matrix of credit ratings and default rates, historical recovery rates, and the yield margins in the bond market. Controversially, the credit grades have been assumed to be homogenous. The time horizon for a CreditMetrics analysis is typically one year, but some analysts have used up to 10 years. Another point of controversy is that recovery rates are calculated by considering fixed interest rates, which does not reflect the historical volatility of this key variable. The CreditMetrics formula is shown as follows:

$$PV_{it} = \frac{CF_{it}}{(1 + r_t + s_{it})^t}$$

This equation calculates the value of a bond in credit grade i (PV_{it}). Credit grade i is the assumed credit rating for the next period. This is essentially an adjusted present value formula, at time t, given the cash flows (CF_{it}) (i.e., coupon payments), the risk-free rate (r_t), and the annual risk premium for a loan in credit grade i at time t (s_{it}).

The CreditMetrics model can be used to derive market value estimates for individual nonmarketable loans or for an entire loan portfolio. The overall output generates a distribution of loan value changes over time. The method promotes transparency in credit risk management and can be used across a wide range of financial products. It is important to note that the CreditMetrics method is not a rating tool. It is intended to make credit risk management more systematic by providing risk measures that capture the relationship between individual assets. It evaluates risk, at both the individual loan and portfolio levels, in the following three stages:

1. *Reporting profiles*. CreditMetrics can integrate reports from a wide range of loans including fixed-rate bonds, floating-rate bonds, nonperforming loans, non-interest-bearing loans, letters of credit, and market-based tools like swaps.

2. *Volatility from rating changes and defaults*. The transition matrix explicitly considers the impact of upgrades, downgrades, and defaults. Each rating transition adjusts the assumed risk premium, which influences the estimated value of each loan.
3. *Correlations*. Individual loan valuations are aggregated at the portfolio level. Simple correlation estimates are then made between the volatility of the portfolio's value and the asset price volatility.

Developed by Credit Suisse in 1997, the **CreditRisk+ model** does not account for the capital structure of the borrower because it is not directly related to its default probability (in contrast to Moody's-KMV model). While the CreditMetrics method estimates the influence of all individual loans on a firm, the CreditRisk+ method only considers loans in either bankruptcy or a bad credit scenario. The end result is focusing on negative shocks only with an estimated default frequency that is a continuous random variable. This model estimates the PD to be very small and independent of other credit events. A Poisson distribution is the most appropriate option because it sets the average default rate equal to its variance. The CreditRisk+ method only calculates default events, not credit ratings. This points to one of the key benefits of this approach, which is the small amount of data needed for application.

Risk-Adjusted Return on Capital

LO 21.g: Apply risk-adjusted return on capital (RAROC) to measure the performance of a loan.

Profitability is a significant concern for all businesses. Banks need to consider the risk-reward tradeoff related to the loans accepted. This process is a complement to traditional credit risk analysis. The most used metric in this case is the **risk-adjusted return on capital (RAROC)**, which is a ratio of the income on a loan relative to the capital required to obtain the loan:

$$\text{RAROC} = \frac{\text{loan revenues}}{\text{capital at risk}}$$

A loan is considered to be profitable if the calculated RAROC is higher than the bank's cost of capital. The RAROC numerator is the revenue (i.e., interest and principal repayments) coming from the loan. It explicitly considers the following factors:

- The spread (s) between the loan rate and the bank's cost of capital
- The fees (f) attributable to the loan
- The estimated loan losses (l)
- The associated operating costs (c)
- Relevant taxation (x)

In this context, **loan revenues** can be formalized using the following formula:

$$\text{loan revenues} = \text{loan value} \times (s + f - l - c)(1 - x)$$

The RAROC denominator is the **capital at risk** (i.e., the capital a bank would need to cover the risk of default on the loan in question). One way to quantify capital at risk is

to use the change in value of the loan over an observation period, which is often one year. This is intended to capture the impact of changing interest rates, which is why duration comes into play. It is possible to estimate the change in a loan's value (ΔL) given the loan's value (L), its duration (D), the current interest rate (i), and the estimated change in interest rates (Δi):

$$\Delta L \approx -L \times D \times \frac{\Delta i}{1+i}$$

For example, assume that XYZ Bank has a loan with a value of $1,000,000. There is an associated commission of 0.1%, the spread between the loan's interest rate and the bank's cost of capital is 0.65%, the estimate of operating costs is 0.2%, and their estimated tax rate is 11.1%. This loan is expected to be a performing loan with no risk of default. Given this information, the estimated loan revenue is:

$$\$1,000,000(0.0065 + 0.001 - 0 - 0.002)\,(1 - 0.111) = \$4,889.50$$

Additionally, this loan has a duration of 3.75 and an interest rate of 10.5%. Interest rates are expected to increase by 1.5%. The capital at risk can be estimated using the change in loan value as an approximation:

$$\Delta L = -\$1,000,000(3.75)\left(\frac{0.015}{1.105}\right) = -\$50,904.98$$

In this example, RAROC is equal to 9.61%. As long as the bank's own interest rate is less than 9.61%, then this loan is profitable for the bank:

$$\text{RAROC} = \frac{\$4,889.50}{\$50,904.98} = 9.61\%$$

There is an alternative method for estimating the capital at risk that uses historical values and not market-based metrics like expected changes in interest rates. This method uses the **unexpected loan loss** in the RAROC denominator using the following formula:

$$\text{unexpected loan loss} = \alpha \times \text{LGD} \times \text{EAD}$$

This method considers the loss given default (LGD), the exposure at default (EAD), and an adjustment (α) for unexpected default rates. This adjustment factor is derived from the distribution of historical default rates. If the distribution of default rates is assumed to be normal, then at the 99.5% confidence level, an analyst should set $\alpha = 2.6\sigma$. However, most loan distributions have some degree of skewness. For this reason, it is more common to use a higher value (e.g., 5 or 6) based on the characteristics of the loan.

MODULE QUIZ 21.2

1. A financial analyst and a risk manager are discussing ways to derive the probability of default for a bank's loan portfolio. The analyst says that he prefers models that rely on the perspective of internal experts. The manager adds that while internal experts have good perspectives in the risk management process, she prefers a model that is data-driven and one that can assess loans for both consumer and corporate customers. The analyst and manager are referring to which type of credit risk assessment?

	Analyst	Manager
A.	Financial	Empirical
B.	Judgmental	Financial
C.	Judgmental	Empirical
D.	Endogenous	Financial

2. The Merton model is different from Moody's-KMV Expected Default Frequency approach in two key areas. Which of the following statements refers to one of those differences?
   A. The Merton model factors the market value of the borrower's equity, while Moody's-KMV model uses book value.
   B. The Merton model uses the risk-free rate to calculate the distance to default, while Moody's-KMV model uses a customized value.
   C. Moody's-KMV model uses a default point with short-term debt and half of long-term debt, while the Merton model only uses short-term debt.
   D. The Merton model uses historical data to derive a distribution of default frequencies, while Moody's-KMV model uses standardized expectations.

3. A credit risk manager is looking for a method to estimate default risk that considers volatility, credit scores, and comparisons to similar borrowers to enhance predictions. Which of the following methods should this manager select?
   A. Merton model.
   B. CreditRisk+ model.
   C. CreditMetrics model.
   D. Moody's-KMV model.

4. ABC Bank has a loan with a value of $750,000. There is an associated commission of 0.15%, the spread between the loan's interest rate and the bank's cost of capital is 0.45%, the estimate of operating costs is 0.3%, and their estimated tax rate is 12%. This loan is expected to be a performing loan with no risk of default. Additionally, this loan has a duration of 2.85 and an interest rate of 9.5%. Interest rates are expected to increase by 1.25%. What is this loan's risk-adjusted return on capital (RAROC)?
   A. 4.92%.
   B. 6.01%.
   C. 7.64%.
   D. 8.11%.

KEY CONCEPTS

LO 21.a

The CAMEL system is used as a credit risk management tool. More specifically, this method analyzes capital adequacy, asset quality, management, earnings, and liquidity. The goal is to score each area to provide a view into the bank's credit risk exposure.

LO 21.b

An important part of credit risk modeling is estimating expected losses, which is typically calculated over a one-year time horizon. The subcomponents are the probability of default (PD), the exposure at default, and the loss given default (LGD). The PD is the likelihood that a borrower will end up delinquent for more than 90 days. The exposure at default is the dollar risk exposure at the time of default for a given

borrower. The LGD is the percentage of the loan that is estimated to be lost in the event of a default scenario.

LO 21.c

The capital adequacy ratio (CAR) is an integral part of credit risk management. There are two ways to calculate the CAR. The first way is using standardized weights, which can cause a bank to hold more capital because external (i.e., noncustomized) data is used. The second way uses internal historical data to empirically derive risk-weighted asset (RWA) values. This second method involves estimating PD, LGD, and EAD. Basel III enhances the risk management calculations by increasing the CAR threshold to 10.5% (from 8%), adding liquidity and leverage requirements, emphasizing counterparty risks, and introducing stress testing for extreme market conditions.

LO 21.d

There are three primary approaches to finding the PD. They are the judgmental (i.e., expert), empirical, and financial (i.e., market) models. The judgmental approach relies on the experience of experts to assess the risk of default. The empirical model uses historical data to look for important risk relationships. Machine learning is now involved in this method. Financial models for credit risk modeling include structural models and reduced-form models.

LO 21.e

The Merton model uses option pricing theory to define the PD. and the distance to default for a borrower. A strength of this approach is that it uses observed historical data in the analysis. One noteworthy limitation is that the default point (L) is only defined as the short-term debt of the firm.

LO 21.f

While the Merton model uses option pricing theory with only short-term debt as the default point, Moody's-KMV model uses both short-term debt and half of long-term debt. Moody's-KMV model also uses historical data to personalize a default frequency distribution as opposed to the standardized normal distribution used by the Merton model.

The CreditMetrics model evaluates similar companies that have default in its analysis. This is effectively a mark-to-market model that considers the borrower's credit rating and a transition matrix of default rates for different credit ratings. While the CreditMetrics model includes all loans with a borrower, the CreditRisk+ model discards the use of credit ratings and focuses only on nonperforming loans and firms in bankruptcy. Thus, this model requires less data to operate.

LO 21.g

The risk-adjusted return on capital (RAROC) can be used to estimate if a loan will be profitable for a bank. The RAROC numerator is the estimated revenue from the loan. The RAROC denominator is the capital at risk, which can either be estimated as the

change in loan value given a change in interest rates or as the unexpected loan loss. The goal is for the RAROC to be higher than the bank's own interest rate.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 21.1

1. **C** Asset quality focuses on bank assets that are performing or showing signs of delinquency. Capital adequacy relates to minimum capital reserves as set by regulation. Earnings reviews are focused on core earnings with an emphasis on stability, net interest margin, return on assets, and future earnings potential. Liquidity focuses more on short-term liquidity risk than on longer-term focused interest rate risk although both are considered. (LO 21.a)

2. **A** Expected loss has three subcomponents (probability of default, exposure at default, and loss given default). The probability of default is the likelihood that a borrower is delinquent for more than 90 days. The exposure at default is the expected dollar loss if the loan defaults. The loss given default is the percentage of the loan that is estimated to be lost if a default occurs. (LO 21.b)

3. **D** The IRB approach considers several elements based on the bank's customized historical record. The parameters do not reflect regulatory norms because they are customized to each bank. This approach assumes that the credit portfolio is well diversified and that no idiosyncratic risk remains. The maturity adjustment has two components. First, capital requirements should increase with maturity because the risk of default increases as maturity increases. Second, borrowers with low probabilities of default (PD) should have higher adjustments than high PD borrowers. A borrower with a high PD already has a higher instance of default factored into their estimates. (LO 21.c)

Module Quiz 21.2

1. **C** A judgmental model uses the perspectives of experts to assess the probability of default. It can be applied to both consumer and corporate loans. An empirical model uses historical data to estimate relationships between variables and risk outcomes. This can also be applied to both consumer and corporate customers. The financial model uses market data to detect trends and risk estimates. Because financial market data is the primary input, this method can only be used for corporate customers. (LO 21.d)

2. **C** One key difference is that Merton sets the default point as only short-term debt and Moody's-KMV model adds in half of long-term debt. Another key difference is that Moody's-KMV model uses historical data to derive a personalized distribution of default probabilities, while the Merton model uses standardized normal distribution values. In the distance to default formula, the rate of change in historical assets replaces the risk-free rate, and both models use the market value of equity. (LO 21.f)

3. **C** The CreditMetrics model is the option for peer comparisons. It considers bond issuer credit ratings (unlike the CreditRisk+ model). The Merton model uses option pricing theory to calculate the default rate, but it only considers short-term debt. Moody's-KMV model adds some long-term debt to the theory behind the Merton model, but the CreditMetrics model is the best option due to peer comparison demands. (LO 21.f)

4. **D** The estimated loan revenue is $1,980 and the estimated capital at risk is $24,400.68. This results in a RAROC of 8.11%. As long as the bank's own interest rate is less than this value, then the loan will be profitable.

$$\begin{aligned}\text{loan revenue} &= \$750,000(0.0045 + 0.0015 - 0 - 0.003)(1 - 0.12)\\ &= \$1,980\end{aligned}$$

$$\Delta L = -\$750,000(2.85)\left(\frac{0.0125}{1.095}\right) = -\$24,400.68$$

$$\text{RAROC} = \frac{\$1,980}{\$24,400.68} = 8.11\%$$

(LO 21.g)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Doumpos et al., Chapter 2.

READING 22

CREDIT SCORING AND RATING

Study Session 4

EXAM FOCUS

This topic focuses on credit scoring and rating systems. For the exam, understand the differences between these two systems as well as the four benefits offered by the implementation of these systems. Also, understand the differences between and applications of through-the-cycle and point-in-time assessments. Be aware of how behavioral and profit scoring are used in the context of consumer lending and the innovative introduction of social lending. In addition, there are many details you need to know regarding the credit rating agency process. Pay attention to the details considered for corporate borrowers as well as the overall score/rating development process (i.e., data collection and preprocessing, model fitting, model validation, definition and validation of the risk rating, and implementation). Finally, understand the common criticisms of the credit rating agency process.

MODULE 22.1: CREDIT SCORING AND RATING SYSTEMS

LO 22.a: Compare the credit scoring system to the credit rating system in assessing credit quality and describe the different types of each system.

Lenders, investors, and regulators all need a way to quantify the creditworthiness of a borrower. Smaller businesses and individuals are often graded using a **credit score**, which is a number ranging from 300 to 850. Higher numbers are associated with a higher estimated ability to repay. Larger businesses and governments are typically graded with a **credit rating**, which is a letter-grade tabulation (e.g., AAA, AA, A, BBB). While a rating of AAA is generally the rating showing the strongest expectation of repayment ability, each rating agency can deploy their own lettering system. Some ratings use both uppercase and lowercase letters (e.g., Aaa), and some use pluses and minuses (e.g., BBB+).

There are four primary benefits to using a credit scoring/rating system:

1. Reduces subjectivity in loan evaluation and acceptance

2. Enables analysis in credit risk management (e.g., scenario analysis and stress testing)
3. Promotes transparency and consistency through a common framework
4. Reduces time for and cost of appraisal

Credit scoring/rating systems can be developed either internally or externally. Banks often develop their own proprietary system that they use to accept or reject loans. This system allows banks to use internal historical data and apply an understanding of what has worked well in the past. Credit rating agencies, credit bureaus, and consulting firms may create external scores/ratings. The three external vendors (Standard & Poor's, Moody's, and Fitch) sell credit scores/ratings to credit institutions, corporations, and investors.

The next two sections will underscore the important aspects to consider with respect to credit scoring/rating systems.

Types of Credit Rating Systems

LO 22.b: Distinguish between through-the-cycle and point-in-time credit rating systems.

The Basel Committee on Banking Supervision (BCBS) has distinguished two high-level types of credit rating philosophies as follows:

- **Through-the-cycle approach.** This category is used by major credit rating agencies because it has a long-term orientation. It covers at least one full business cycle. Therefore, the ratings are not updated frequently, which makes them less sensitive to short-term events. Thus, this approach may be best suited for longer-term loans.
- **Point-in-time approach.** This category is primarily used by internal processes at a credit institution or a bank. This approach typically focuses on the current scenario and may widen the time period up to one year. These ratings are more volatile than through-the-cycle assessments because they are heavily influenced by short-term events. The point-in-time method may capture default risk close to real-time. Thus, this approach may be best suited for short-term loans.

Developing Credit Scoring and Rating Models

LO 22.c: Describe the process for developing credit risk scoring and rating models.

Credit scores and ratings can be determined for either a specific issuer (i.e., borrower) or for a specific issuance (i.e., loan/bond). An issuer is most likely to seek a credit score/rating when they need to access capital markets to borrow money.

There are two different scoring methods commonly used:

1. **Behavioral scoring.** This approach applies to consumer loans. It refers to the process of observing the historical financial behavior of existing customers. Behavioral scoring is a short-term measure. Unlike the static nature of standard

credit ratings and scores, behavioral scoring updates in real-time to new behavioral information observed (e.g., payment and purchasing behavior, updated default estimates). This approach may be used with credit limits, managing debt collection, and marketing new products.

2. **Profit scoring.** This approach focuses only on the estimated profitability of a loan. It considers loan pricing, operational decisions, credit risk assessments, and default predictions. There are two types of profit scoring models:
   - *Account-level*. This method calculates profit for each individual account and ignores any connections between multiple accounts.
   - *Customer-level*. This method explicitly considers all accounts owned by a customer. It enables the profit perspective to be aggregated across any accounts associated with a specific customer.

Advances in artificial intelligence (AI) and the fintech industry have enabled **social lending** (a.k.a. peer-to-peer lending), which is a transaction directly between a lender and a borrower without the need for a financial intermediary. Credit decisions can be made very quickly, but credit risk modeling in social lending is still an emerging topic. More needs to be done to understand the risks in this lending channel, which often has higher default rates than intermediated loans. The responsible use of big data and transparency are material concerns with this innovation.

MODULE QUIZ 22.1

1. ABC Bank needs to conduct a risk assessment for a large manufacturing firm that is publicly traded. Which of the following statements regarding this assessment is correct? The bank should use a:
   A. credit score to reduce subjectivity.
   B. credit score to enhance transparency.
   C. credit rating to reduce the cost of appraisal.
   D. credit rating despite the limitations for stress testing.

2. A lender is considering using a through-the-cycle approach when assessing the creditworthiness of a potential borrower. Which of the following statements regarding this approach is correct? This method:
   A. will capture default risk in the best way possible.
   B. should be used if the loan is short-term in nature.
   C. should be used if the loan is long-term in nature.
   D. is a robust option because it relies on internal data only.

3. A risk analyst is considering the best method to generate a credit score that updates in near real-time while considering the payment and purchase history of a customer. Which of the following methods for developing credit risk scoring and rating models should he use?
   A. Behavioral scoring.
   B. Social lending scoring.
   C. Account-level profit scoring.

D. Customer-level profit scoring.

MODULE 22.2: CREDIT RATING AGENCY METHODOLOGIES

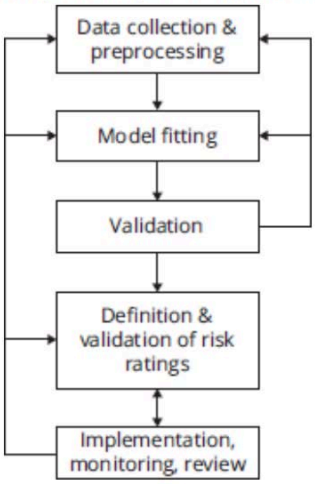
LO 22.d: Describe rating agencies' assignment methodologies for issue and issuer ratings, and identify the main criticisms of the credit rating agencies' ratings.

Credit scoring and rating systems are governed and imposed by regulators and central banks. The Basel Committee has outlined the following five basic specification requirements:

1. *Meaningful differentiation of risk*. There are three subcomponents to this specification. First, credit grades must be established based on risk differences and not as an attempt to manage regulatory capital requirements. Second, borrowers in the same credit grade may be treated differently based on transaction-level differences. Third, there must be a reasonable dispersion across credit grades to avoid concentration in any specific credit grade.
2. *Continuous evaluation of borrowers*. Borrower ratings should be updated on a reasonable basis. In practice, this means no less frequent than an annual rerating review.
3. *Operational oversight*. The goal is to ensure adequate system integrity. The credit scoring/rating system should be continuously monitored for proper functioning, efficient controls (i.e., stress testing), and adequate feedback from associated parties.
4. *Correct selection of risk assessment metrics*. The risk factors analyzed should adequately predict the creditworthiness of the borrower. The goal is to make estimates that are as close as possible to the actual outcomes.
5. *Collecting substantial data*. The data used must represent reality. The credit grading must incorporate historical data, past credit scores/ratings, prior default estimates, and payment history.

Figure 22.1 provides an overview of the rating development process. Each category will be discussed in greater detail in the following sections.

Figure 22.1: Credit Risk Scoring/Rating Development Process



Source: Michalis Doumpos, Christos Lemonakis, Dimitrios Niklis, and Constantin Zopounidis, *Analytical Techniques in the Assessment of Credit Risk: An Overview of Methodologies and Applications* (Springer, 2019). Chapter 2.

Data Collection and Preprocessing

The first step in the credit scoring/rating development process is to collect the appropriate data and consider any data preprocessing needs. The dataset will include performing loans and loans that have been nonperforming for more than 90 days. Some of the data requires preprocessing to eliminate outliers, selecting the most appropriate risk attributes for analysis, and transforming data as needed. Algorithms, statistical tests, and expert judgment will likely be involved in this step.

The data involved in collection and preprocessing will be both quantitative and qualitative. It is important to remove subjectivity from this process and introduce as much consistency as possible. For consumer loans, relevant inputs could include the borrower's income, their assets, any existing loan obligations, their payment history, employment status, use of collateral/guarantees, and the type of loan (i.e., auto loan, credit card, mortgage).

For corporate customers, a different set of information will need to be considered. These factors are discussed as follows:

- *Financial data*. Corporate financial statements (i.e., balance sheet, income statement, and statement of cash flows) are reviewed. Financial ratios are tabulated to better visualize and compare the data. These ratios provide a static view of the firm at a point-in-time as well as their past trend:
  - *Profitability ratios*. The goal is to estimate the ability to repay. These ratios include return on assets (ROA), return on equity (ROE), and profit margins.
  - *Solvency ratios*. The goal is to consider the ability to manage existing debts. These ratios include the current debt load (e.g., debt-to-equity [D/E] ratio) and the ability to repay (e.g., interest coverage).
  - *Liquidity measures*. The goal is to assess the ability to cover short-term obligations. The metrics include working capital, the current ratio, and the quick

ratio.

  - *Management efficiency ratios*. The goal is to measure the operational performance of the firm. These ratios include turnovers for inventories, receivables, accounts payable, and total assets.
- *Transaction data*. While financial ratios are point-in-time estimates, transactions can be reviewed to provide a more updated view of a corporate borrower. This could reflect business transactions or payments made to suppliers or other lenders. This review of borrower behavior could also consider current debt balances, delinquencies, and credit limits.
- *Size and age*. Large firms that are well-established often have a lower default risk than smaller and younger firms. This makes intuitive sense considering access to capital and the ability to absorb adverse events. Market cap is used as the size barometer for publicly traded firms. Total assets or revenues can be used for privately held businesses.
- *Market conditions and competitive position*. An assessment needs to be made of the prevailing market conditions, which are constantly changing. From the perspective of competition, the following questions should be considered:
  - Are there many other competitors? What is the market share of rivals relative to the firm under analysis?
  - Is the market open to new entrants? Do regulations provide any barriers to entry?
  - Is the general outlook for the economy (or sector) favorable?
  - Does the firm have a competitive edge in technology, supply chain access, or some other key resource?
  - Do the firm's suppliers have any level of control over the firm in question?
- *Financial market data*. Publicly traded firms can offer real-time, market-based data like stock prices, volatility of return data, and valuations (e.g., price-to-earnings, price-to-book, earnings per share). Financial market data is best for shorter-term periods and accounting data is better for longer-term loans.
- *Corporate governance*. Several studies have shown that understanding managerial oversight and ethical standards is an important value-added review for corporate success. Items considered include Board of Directors composition and independence, qualifications of top executives, accountability, and protection of shareholder rights.
- *Corporate news and analytics*. An emerging area of consideration is the vast array of available news. This includes press releases, mainstream media, social media, and alternative media sources. There is a tremendous amount of information available that could provide an edge in evaluating a borrower's creditworthiness when done responsibly.

Model Fitting

The second step in the process is **model fitting**, which involves identifying model parameters with the best descriptive ability (i.e., fit) for the model training data. Assume the following linear model:

$$f(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n$$

In this formula, the vector $\beta = (\beta_0, \beta_1, \ldots, \beta_n)$ includes a constant term (β_0) and a series of coefficients $(\beta_1, \ldots, \beta_n)$ for various risk attributes. The goal is to provide the best fit (i.e., optimized estimates for β) such that:

$$\beta^* = \arg\min_{\beta \in \mathbb{R}} L(\beta, X)$$

In this general optimization problem, β^* is the optimal parameter vector and *L* is a loss function representing the difference between the model's output and classification in the training data. This is an empirical proxy for the true problem, which is to minimize default losses. This formula is a regression-like process with a binary (e.g., default/nondefault) dependent variable rather than a continuous variable normally used in standard regression models. Using algorithms, model fitting can involve statistical tools, data mining, and operations research techniques.

Model Validation

The next step in the process is **model validation**, which checks the fitted model against a new sample (i.e., the validation data) not included in the model training data. In other words, this is an out-of-sample test. The validation data should have the same risk attributes as the training data, but use different cases (i.e., borrowers/loans). This process involves statistical tests, like backtesting, using both out-of-sample and out-of-time tests. Both are critical, because if the validation step does not provide good results, then either model fitting or data collection and preprocessing need to be repeated with different variables. The first three steps in the credit scoring/rating development process (i.e., collection/preprocessing, fitting, and validation) often require multiple iterations to achieve success.

To further enhance the validation step, analysts often use **walk-forward testing**, which is a systematic process of rotating forward the time period under evaluation. Testing period *t* involves data beginning at *t–1* and *t–2*. The observation window moves forward for one time period. The new *t+1* testing period includes periods *t*, *t–1*, and *t–2*. Further enhancement can be made using resampling techniques (e.g., bootstrapping and cross-validation).

Benchmarking is an alternative to backtesting, and it involves comparing a credit model's output to an external source (e.g., the rating already published by a credit rating agency). In contrast to the quantitative nature of model fitting and validation, this approach is qualitative. The goal of benchmarking is not to replicate the existing credit rating. Instead, it is to identify any deviation and then seek to understand why it exists. The reason for the deviation may require recalibration and model improvement.

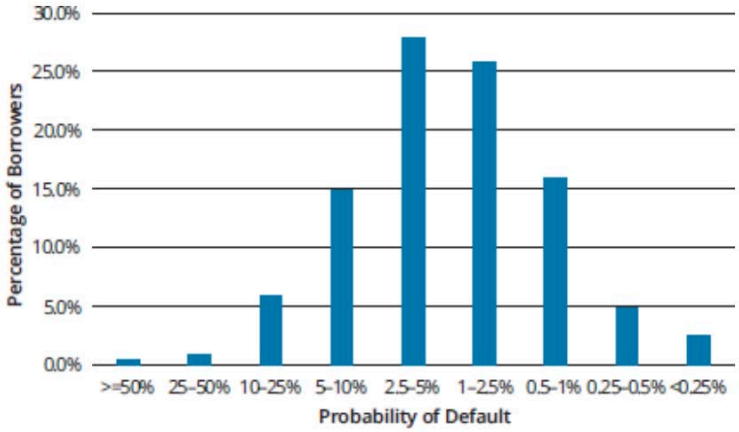
Definition and Validation of Ratings

After data collection/preprocessing, model fitting, and model validation, the derived credit scores should be mapped to a risk rating class. Each rating class should be associated with empirical probability of default (PD) estimates. In addition, each rating

class should be well-diversified relative to its neighboring class, and the distribution of borrowers in the class should not be highly concentrated in a single risk grade.

Consider Figure 22.2, which provides an example distribution of borrowers given their level of risk. In this example, borrowers are organized into nine risk grades. The lowest risk has less than a 0.25% PD and the highest risk has more than a 50% PD. The middle four risk grades have the highest concentration with two of them having over 25% concentration of borrowers. This is considered an acceptable risk concentration (i.e., no excess concentration).

Figure 22.2: Example Distribution of Borrower Risk Grades



This mapping process should also be checked for consistency over time given that rating migration is a concern. If this grouping does not produce adequate results, then the process needs to be re-evaluated and adjusted.

Implementation, Monitoring, and Review

The last stage in the process is implementation, monitoring, and review. At this point, the credit scoring/rating system that was developed will be used to derive real-time risk estimates for loan applications. The system should be continuously monitored, reviewed, and recalibrated as needed. The risk management focus should be on identifying any observed degradation in the quality of the generated risk estimates.

Criticisms of Credit Rating Agency Scores

Common criticisms of credit rating agency (CRA) scores/ratings include the following:

- *Lack of transparency*. The process underlying any rating agency's system is typically proprietary. Therefore, the fairness and validity of the model cannot be publicly verified.
- *Potential conflicts of interest*. It is understood that CRAs harvest fees from the entities for which they are providing a rating. This presents a potential conflict of interest.
- *Promoting a debt explosion*. The work of CRAs has enhanced risk management at lending institutions. This has led to reduced risk premiums. Lower borrowing costs

make debt more accessible, which may increase systemic risk due to credit bubbles and crunches.

- *Poor predictive ability*. The CRAs have collectively done a poor job of predicting recent credit failures. Examples include Enron, WorldCom, and Lehman Brothers. Several empirical studies have found CRA scores/ratings to be less predictive of default than accounting and market-based methods.
- *Procyclicality*. CRAs claim that their models are based on the through-the-cycle approach. The idea is that the ratings should be unaffected by movement through the business cycle. In practice, it has been observed that ratings are often too optimistic during periods of economic growth and too pessimistic during recessions. This suggests some level of procyclicality.

Despite these common criticisms, credit scores/ratings provided by CRAs add useful information for investors and lenders to consider. This should be a complementary part of overall credit risk analysis.

MODULE QUIZ 22.2

1. Moody's is considering a revision to their credit rating model. When they checked the model derived from model training data against out-of-sample data, they found a few issues to address. Which of the following steps in the credit rating development process were they likely on when they noticed this issue?
   A. Model fitting.
   B. Preprocessing.
   C. Model validation.
   D. Validation of the risk rating.

KEY CONCEPTS

LO 22.a

Credit scoring systems feature a three-digit numeric code ranging from 300 to 850. They are typically used to show the creditworthiness of small businesses and individual borrowers. Credit ratings are a lettered code (e.g., AAA, AA, A, BBB) used for larger firms and governments. These models may be developed internally or externally. There are four primary benefits to using these methods, including reduced subjectivity, reduced time/cost for appraisal, enabling analytical tests, and promoting transparency and consistency.

LO 22.b

There are two high-level types of credit rating philosophies. Through-the-cycle approaches are long-term in nature and capture an entire business cycle. They are primarily used by credit ratings agencies, and they are best suited for long-term loans. Point-in-time approaches are used internally by banks and credit institutions. They are short-term in nature, which means they will capture risk events and possibly default risk in almost real-time. This method is best suited for loans that are short-term in nature.

LO 22.c

Credit scores for consumers may be tabulated using either a behavioral or a profit scoring approach. The behavioral method uses historical data to derive a credit score using the customer's behavior (e.g., payments, purchases, personalized default estimates). Using internal data enables scores to adjust in almost real-time.

The profit scoring method ignores customer behavior and instead focuses only on the profitability of the lending relationship. One option is to consider profitability at the account-level and the second is consideration at the customer-level. The recent innovation of social lending also has scoring implications. This is an emerging area that will need further evolution in scoring methods.

LO 22.d

The credit rating agency score/rating development process is complex and data intensive. Different inputs are used for individual borrowers versus corporate clients. The process involves the following five steps:

1. Data collection and preprocessing
2. Model fitting
3. Model validation
4. Definition and validation of the risk rating
5. Implementation

The first three steps may be performed multiple times as the process seeks reliable output. Monitoring, which is inherently involved in the implementation stage, may also produce a need for re-evaluation.

There are five common criticisms for credit rating scores/ratings. They include a lack of transparency, the related potential for conflicts of interest, promotion of higher debt loads, poor predictive ability, and procyclicality.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 22.1

1. **C** Borrowers that are large firms (especially those that are publicly traded) should use credit ratings. Smaller firms and private individuals should be assessed using credit scores. The advantages to using credit scores/ratings include reduced subjectivity, reduced time/cost of appraisal, enabling analysis (e.g., scenario analysis or stress testing), and promoting transparency and consistency. (LO 22.a)

2. **C** Through-the-cycle approaches are used by credit ratings agencies, not by internal models. They consider data from an entire business cycle, which is long-term in nature. For this reason, this method is best suited for long-term loans. Point-in-time assessments use short-term data, and they capture real-time default risk better than the long-term focused through-the-cycle approach. (LO 22.b)

3. **A** Behavioral scoring updates a credit score in near real-time with factors such as a customer's payments, purchases, and probability of default. The two profit scoring approaches (account-level and customer-level) offer different vantage points on a method focused on profitability rather than on the behavior of a customer. Social lending is an emerging innovation with a credit risk scoring system still in its infancy. (LO 22.c)

Module Quiz 22.2

1. **C** The credit score/rating process has five steps: (1) data collection and preprocessing, (2) model fitting, (3) model validation, (4) definition and validation of the risk rating, and (5) implementation. Step 3, model validation, involves checking the model developed in Step 2 with out-of-sample data. (LO 22.d)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Crouhy, Galai, and Mark, Chapter 9.

READING 23

CREDIT SCORING AND RETAIL CREDIT RISK MANAGEMENT

Study Session 4

EXAM FOCUS

This reading examines credit risk management, primarily from the perspective of the retail credit lender. For the exam, focus on the risks incurred by a lender and how credit scoring models can be used to incorporate variables into an effective risk evaluation model. While estimating risk and evaluating model performance is critical, assessing credit applicants for potential profitability is also important. Be familiar with the role of a credit applicant as both a borrower and a potential client for other lender products. Also, understand the concept of risk-based pricing and how it has changed the way that lenders price their products to different customers.

MODULE 23.1: CREDIT RISK

Retail Banking Risks

LO 23.a: Analyze the credit risks and other risks generated by retail banking.

The retail banking industry revolves around receiving deposits from and lending money to consumers and small businesses. Loans can take the form of home mortgages, home equity lines of credit (HELOCs), installment loans (revolving loans covering automobiles, credit cards, etc.), and small business loans (SBLs). From the perspective of the lending institution, these individual loans constitute small pieces of large portfolios designed to reduce the incremental risk to any one exposure.

The biggest risk associated with retail banking is **credit risk**, which is the likelihood that a borrower will default on debt. Throughout the five years preceding the 2007 subprime mortgage crisis, banks offered customers products they could not afford with risks that were more than customers could bear. **Loan-to-value (LTV) ratios** on mortgaged properties were very high and borrowers with weaker credit were given

mortgages. These strategies backfired when housing prices collapsed, which resulted in mortgages often exceeding the value of the properties themselves.

Although credit risk is the primary risk in retail banking, several other risks also impact the industry. These risks include:

- **Operational risks:** day-to-day risks associated with running the business.
- **Business risks:** strategic risks associated with new products or trends and volume risks associated with measures like mortgage volume when rates change.
- **Reputation risks:** the bank's reputation with customers and regulators.
- **Interest rate risks:** the bank provides specific interest rates to its assets and liabilities and rates change in the marketplace.
- **Asset valuation risk:** a form of market risk associated with the valuation of assets, liabilities, and collateral classes. An example includes prepayment risk associated with mortgages in decreasing rate environments. Valuation risk also exists in situations when car dealers assume a residual value for a vehicle at the end of the life of a lease.

Retail Credit Risk vs. Corporate Credit Risk

LO 23.b: Explain the differences between retail credit risk and corporate credit risk.

There are several features that distinguish retail credit risk from **corporate credit risk**. As mentioned earlier, retail credit exposures are relatively small as components of larger portfolios such that a default by any one customer will not present a serious threat to a lending institution. A commercial credit portfolio often consists of large exposures to corporations that can have a significant impact on their industry and the economy overall.

Due to the inherent diversification of a retail credit portfolio and its behavior in normal markets, estimating the default percentage allows a bank to effectively treat this loss as a cost of "doing business" and to factor it into the prices it charges its customers. A commercial credit portfolio is subjected to the risk that its losses may exceed the expected threshold, which could have a crippling effect on the bank.

Banks will often have time to take preemptive actions to reduce retail credit risk as a result of changes in customer behavior signaling a potential rise in defaults. These preemptive actions may include marketing to lower risk customers and increasing interest rates for higher risk customers. Commercial credit portfolios typically don't offer these signals, as problems might not become known until it is too late to correct them.

The Dark Side of Retail Credit Risk

LO 23.c: Discuss the "dark side" of retail credit risk and the measures that attempt to address the problem.

An unexpected, systematic risk factor may cause losses to rise beyond an estimated threshold, damaging a bank's retail portfolio through declines in asset and collateral values and increases in the default rate. This represents the "dark side" of retail credit risk.

Primary causes include:

- The lack of historical loss data due to the relative newness of specific products.
- An across the board increase in risk factors impacting the economy overall that causes retail credit products to behave unexpectedly.
- An evolving social and legal system which may inadvertently "encourage" defaults.
- An operational flaw in the credit process due to its semi-automated structure that results in credit granted to higher risk individuals.

The **Consumer Financial Protection Act (CFPA)**, in an attempt to manage the dark side of retail credit risk, requires credit originators to evaluate **qualified mortgages** and **ability to repay**.

A borrower with a "qualified mortgage" is assumed to have the capacity to repay. A qualified mortgage will put a limit on the amount of income allocated to debt repayments (e.g., debt-to-income ratio less than 45%). A qualified mortgage cannot have excess upfront fees and points, may not be balloon payment loans or interest-only loans, may not be for longer than 30 years, and may not be negative amortization loans.

When a lender is evaluating a customer's "ability to repay," the following underwriting standards must be considered:

- Credit history.
- Current income and assets.
- Current employment status.
- Mortgage monthly payments.
- Monthly payments on mortgage-related items such as insurance and property taxes.
- Monthly payments on other associated property loans.
- Additional debt obligations of the borrower.
- The monthly debt-to-income ratio resulting from the mortgage.

Due to the predictable and relative safety of retail credit, banks must set aside a relatively small amount of risk capital compared to requirements associated with corporate loans. Banks must provide regulators with specific statistics associated with differentiated segments of their portfolios. These statistics include: **probability of default (PD)**, **exposure at default (EAD)**, and **loss given default (LGD)**.

Credit Risk Scoring Models

LO 23.d: Define and describe credit risk scoring model types, key variables, and applications.

A **credit risk scoring model** takes information about an applicant and converts it into a number for the purpose of assessing risk; the higher the number, the higher the probability of repayment by the borrower and the lower the overall risk. Credit scoring models facilitate the gathering of an enormous amount of information into a single automated process.

A credit risk scorecard will gather information from applications and credit bureau reports and weight it depending on the type of questions answered. The question/entry will ask for a specific characteristic like "number of years with current employer," and the attribute will be the response (e.g., 10 years). Credit scoring models will determine positive and negative values and weight each attribute according to past history and the associated probability of repayment.

Three model types exist in regard to scoring applications for consumer credit:

- **Credit bureau scores:** this refers to an applicant's FICO score, and is very fast, easy, and cost effective to implement and evaluate. Scores will typically range from a low of 300 to a high of 850, with higher scores associated with lower risk to the lender and lower interest rates for the borrower.
- **Pooled model:** this model, built by outside parties, is more costly than implementing a credit bureau score model; however, it offers the advantage of flexibility to tailor it to a specific industry.
- **Custom model:** created by the lender itself using data specifically pulled from the lender's own credit application pool. This model type allows a lender to evaluate applicants for their own specific products.

Every individual with a credit history will have credit files containing the following information:

- Personal (identifying) information which doesn't factor into scoring models.
- Records of credit inquiries when a file is accessed. Requests for new credit will be visible to credit grantors.
- Data on collections, reported by entities that provide credit or agencies that collect outstanding debts.
- Legal (public) records on bankruptcies, tax liens, and judgments.
- Account and trade line information gathered from receivables information sent to credit bureaus by grantors.

MODULE QUIZ 23.1

1. Which of the following statements is most accurate regarding risks incurred by retail lenders?
   A. Reputation risk is more of a concern for the borrower rather than the lender.
   B. Business risk relates to the day-to-day operational risks of the business.

C. Credit risk relates to the potential for a lender to default on their obligation.
D. Refinancing a mortgage when rates decrease is an example of asset valuation risk.

2. The dark side of retail credit risk is perpetuated by all of the following factors except:
A. capital set aside to protect a bank in the event of default.
B. process flaws resulting in high risk applicants receiving credit.
C. new products which do not have sufficient historical loss data.
D. a social acceptance of bankruptcy and borrowers "walking away" from their obligations.

3. Which of the following statements is correct regarding credit risk scoring models?
A. A pooled model will result in scores ranging from 300 to 850.
B. A custom model is cheaper to implement than credit bureau scores.
C. Multiple requests for new credit will reduce an applicant's credit score.
D. An example of a characteristic in a scoring model is the applicant's current gross salary of $50,000.

MODULE 23.2: CREDITWORTHINESS

Mortgage Credit Assessment

LO 23.e: Discuss the key variables in a mortgage credit assessment and describe the use of cutoff scores, default rates, and loss rates in a credit scoring model.

In assessing an application for mortgage credit, the key variables include:

- **FICO score:** a numerical score serving as a measure of default risk tied to the borrower's credit history.
- **Loan-to-value (LTV) ratio:** the amount of the mortgage divided by the associated property's total appraised value.
- **Debt-to-income (DTI) ratio:** the ratio of monthly debt payments (mortgage, auto, etc.) to the monthly gross income of the borrower.
- **Payment (pmt) type:** dictates the type of mortgage (adjustable rate, fixed, etc.).
- **Documentation (doc) types**, which include:
  - *Full doc*: a loan which requires evidence of assets and income.
  - *Stated income*: employment is verified but borrower income is not.
  - *No ratio*: similar to stated income, employment is documented but income is not. The debt-to-income ratio is not calculated.
  - *No income/no asset*: income and assets are provided on the loan application but are not lender verified (other than the source of income).
  - *No doc*: no documentation of income or assets is provided.

Cutoff Scores

Cutoff scores represent thresholds where lenders determine whether they will or will not lend money (and the terms of the loan) to a particular borrower. As noted earlier, the higher the score, the lower the risk to the lending institution. Setting the cutoff

score too low presents a higher risk of default to the lender. Setting the cutoff score too high may limit potential profitable opportunities by unintentionally eliminating low risk borrowers.

Once the cutoff score is established, historical experience can be used to establish the estimated profitability for a specific product line and the associated loss rate. As estimates are made from longer time horizons (which hopefully capture a full economic cycle), a bank may adjust its cutoff score to maximize the appropriate balance between risk and profitability.

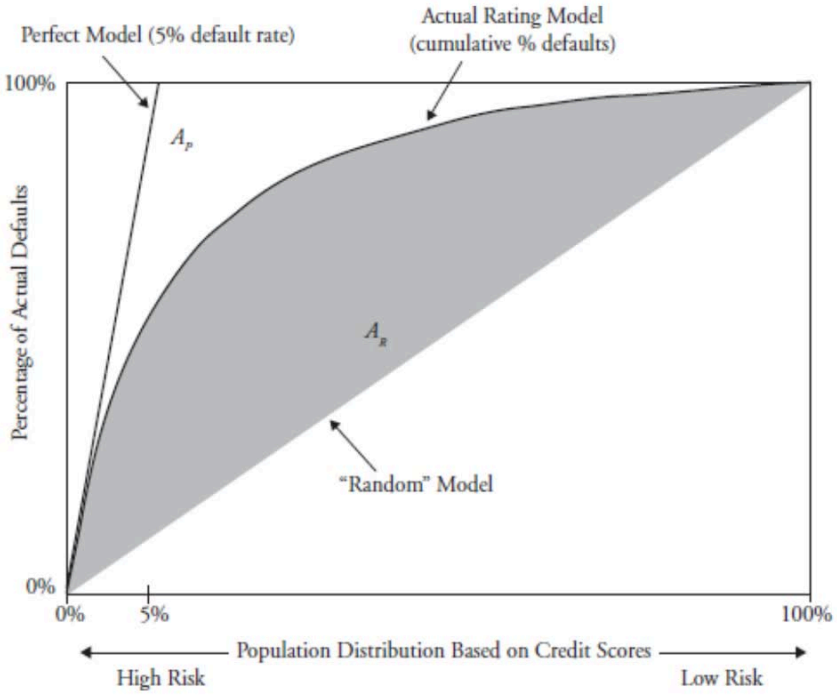
Banks are required by the Basel Accord to group their portfolios into subgroups that share similar loss attributes, with score bands used to differentiate the groups by risk levels. For each of these subgroups, banks are required to estimate the PD and the LGD. The implied PD is a by-product of the historical loss rate and the LGD such that if a portfolio has a loss rate of 3% with a 75% LGD, then the PD is 4% (i.e., 3% / 75% = 4%).

Scorecard Performance

LO 23.f: Discuss the measurement and monitoring of a scorecard performance including the use of cumulative accuracy profile (CAP) and the accuracy ratio (AR) techniques.

Credit scoring is used as a means of predicting default risk, such that high (low) scores on the scorecard are assigned to strong (weak) credits. In assessing the performance of the scorecard, a **cumulative accuracy profile (CAP)** and the **accuracy ratio (AR)** are often used. The CAP shows the population distribution based on credit scores (and therefore risk) versus the percentage of actual defaults.

Figure 23.1: Cumulative Accuracy Profile and Accuracy Ratio



Lines plotted on the graph include the perfect model line, random model line, and observed cumulative default percentage line defined as follows:

- In a perfect model, if the bank predicts, for example, 5% of its accounts will default over a specific period, 100% of those defaults will come from the riskiest 5% of the population.
- A random model will assume 5% of the defaults will come from the riskiest 5%, 20% will come from the riskiest 20%, etc.
- The observed cumulative default line represents the actual defaults observed by the bank.

The area between the perfect model and the random model is represented by A_P, while the area between the observed cumulative default percentage line and the random line is represented by A_R. The accuracy ratio (AR) is defined as A_R/A_P, with a ratio close to 1 implying a more accurate model.

A scoring model must be monitored on a regular basis due to underlying changes in the population as well as potential product changes.

Tradeoff Between Creditworthiness and Profitability

LO 23.g: Describe the customer relationship cycle and discuss the trade-off between creditworthiness and profitability.

Entities in the business of loaning money do not focus entirely on risk and creditworthiness; they also have to evaluate customers from the perspective of profitability. If a credit card is issued to a customer with a very high FICO score who pays their bill in full every month, the bank will not earn any interest from that customer on borrowed funds. At the same time, issuing that same credit card to a customer with a low FICO score is a greater risk because the customer may be unable to pay back loaned funds. Along with credit default scoring, lenders are using product and customer profit scoring measures to evaluate the potential profitability of a specific product and the potential profitability of a specific customer.

In utilizing scorecards to evaluate customers, there are several variations beyond just the credit bureau (FICO) scores. These additional scorecards can be used to evaluate both creditworthiness and profitability. They include:

- *Revenue scores*: used to evaluate existing customers on potential profitability.
- *Application scores*: used to support the decision to extend credit to a new applicant.
- *Response scores*: assign a probability to whether a customer is likely to respond to an offer.
- *Insurance scores*: assign a probability to potential claims by the insured.
- *Behavior scores*: assess existing customer credit usage and historical delinquencies.
- *Tax authority scores*: predict where potential audits may be needed for revenue collection.
- *Attrition scores*: assign a probability to the reduction or elimination of outstanding debt by existing customers.

The **customer relationship cycle** involves the process a lender goes through to market its products/services, screen applications from customers, manage customer accounts, and then cross-sell to those customers. Marketing efforts will focus on selling new or tailoring existing products to meet the needs of both new and existing customers. Applicant screening involves the acceptance or rejection of an application based on scorecards noted previously, as well as ultimately determining the appropriate price to charge for accepted applicants. Managing the customer account will primarily involve product pricing, credit line authorizations, modifications, renewals, and principal or interest collections. Cross-selling efforts will target existing customers by offering other lender products to meet their needs.

Risk-Based Pricing

LO 23.h: Discuss the benefits of risk-based pricing of financial services.

Recognizing that charging a single price for a product to all customers regardless of risk levels may lead to adverse selection (i.e., high-risk customers attracted to a relatively low price relative to their risk profile and low-risk customers pushed away by the higher price relative to their risk profile), lenders have been moving toward **risk-based pricing (RBP)**. RBP involves lenders charging different customers different prices based on their associated risks. Although RBP is still in the early stages of implementation in the financial retail sector, it has been utilized more frequently in credit card, home mortgage, and auto loan lines.

Key external and internal factors which account for risk and play into the interest rates and prices charged by lenders include:

- The probability of **take-up** (i.e., acceptance by the customer of the offered product).
- The probability of default (PD).
- The loss given default (LGD).
- The exposure at default (EAD).
- The cost of equity capital to the lender.
- Capital allocated to the transaction.
- Operating expenses of the lender.

Prices may be set on a tiered level based on score bands allocating risks from high to low. The lender can then map pricing strategies to metrics such as profit/loss, revenue, market share, and risk-adjusted return at the various score bands. Utilizing RBP effectively allows management to evaluate the inevitable tradeoffs among profitability, market share, and risk with the short and long-term goal of increasing shareholder value.

MODULE QUIZ 23.2

1. In assessing the key variables associated with a potential mortgage loan, a bank will charge a higher interest rate if the borrower has a relatively:
   A. high FICO score.
   B. high loan-to-value ratio.
   C. low debt-to-assets ratio.
   D. low debt-to-income ratio.

2. By implementing risk-based pricing on its mortgage products, a bank will likely charge a:
   A. higher interest rate to a customer with a higher FICO score.
   B. lower interest rate to a customer with a lower credit bureau score.
   C. higher interest rate to a customer with a higher probability of default.
   D. lower interest rate to a customer positioned on a lower relative score band.

KEY CONCEPTS

LO 23.a

Retail banking involves the acceptance of deposits and lending of money to customers. Credit risk (the probability that a borrower will default on debt obligations) represents the biggest risk in retail banking. Other risks include operational risks, business risks, reputation risks, interest-rate risks, and asset valuation risk.

LO 23.b

Retail credit risk differs from corporate credit risk in the following significant ways:

- Retail exposures are relatively small such that one default has minimal impact, whereas commercial exposures are much larger and single defaults can have a significant impact.
- Losses exceeding expected thresholds can have a much greater impact for corporate portfolios than retail portfolios.
- Lenders can take preemptive actions to reduce retail credit risks, whereas commercial portfolios often send warning signals after it is too late.

LO 23.c

The "dark side" of retail credit risk occurs when a large scale risk factor causes a decline in asset values coupled with an increase in default rates. The end result is losses which exceed an estimated threshold. Lenders offering mortgage loans must evaluate customers' ability to pay as well as determining whether a mortgage is "qualified." In addition, banks must segment their portfolios and set aside risk capital as well as assess exposures and probabilities of default along with potential losses.

LO 23.d

A credit risk scoring model assigns (to each credit applicant) a score which serves as a measure of borrower risk; the higher (lower) the score, the lower (higher) the risk that the borrower won't be able to pay the debt obligation. Models include credit bureau scores, pooled models, and custom models which all use applicant data and weight them based on their historical relationship to potential defaults.

LO 23.e

Key variables associated with mortgage credit applications include FICO scores, loan-to-value ratios, debt-to-income ratios, payment types, and documentation types. Cutoff scores are thresholds set by lenders which dictate whether credit will or will not be extended, as well as terms associated with the loans. Probability of default and loss given default metrics are critical to assessing the risk associated with various lender portfolios.

LO 23.f

The cumulative accuracy profile (CAP) and the accuracy ratio (AR) are used to assess the performance of a credit scorecard. The closer the accuracy ratio is to 1, the more accurate the CAP model is at predicting the distribution of defaults relative to the risk levels of the associated population.

LO 23.g

For new and existing credit applicants, lenders may use a variety of scorecards to evaluate both creditworthiness and potential profitability. The customer relationship cycle involves marketing products, screening applicants, managing customer accounts, and eventual cross-selling to an existing customer base.

LO 23.h

Risk-based pricing (RBP) involves charging different prices for the same product such that higher (lower) prices can be charged to higher (lower) risk customers. Several external and internal factors are used to determine the prices charged, which are then evaluated in conjunction with various key performance metrics at each score (risk) band in order to maximize the tradeoff between risk and profitability.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 23.1

1. **D** Refinancing a mortgage is considered a prepayment risk to the lender, which is a component of asset valuation risk. When rates decrease, borrowers are more likely to refinance their existing (higher rate) mortgage into a lower rate obligation. The lender then earns less in interest on the debt obligation than they would have previously. Reputation risk is primarily a concern for the lender. Business risk relates to strategic risks tied to new products and volume, while credit risk is the risk that the borrower (rather than the lender) will default. (LO 23.a)

2. **A** Capital must be set aside to protect banks in the event of default, but this is a response to the dark side of retail credit risk rather than a perpetuating factor. A process flaw which grants credit to high risk individuals, a new product which doesn't have historical loss data, and the social "acceptance" of failing to meet debt payments are all considered perpetuating factors of retail credit risk. (LO 23.c)

3. **C** An individual's credit file will show a history of credit requests, with multiple requests causing an applicant's credit score to decline. A credit bureau score model (rather than pooled model) will result in scores ranging from 300 to 850. A custom model is more expensive to implement than credit bureau scores. "Gross salary with current employer" is an example of a characteristic, with the actual salary number itself representing an attribute. (LO 23.d)

Module Quiz 23.2

1. **B** The loan-to-value ratio represents the amount of the mortgage versus the appraised value of the property. The higher this ratio is for a property and an associated borrower, the more risk there is to the lender. In order to protect their position, a lender will charge a higher interest rate. Each of the other scenarios will result in a lower interest rate. (LO 23.e)

2. **C** The more likely it is that a customer will default, the higher the interest rate the bank will charge. A customer with a higher (lower) FICO/credit bureau score will be offered a lower (higher) interest rate. A customer positioned on a lower relative score band will be offered a higher interest rate. (LO 23.h)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Damodaran.

READING 24

COUNTRY RISK: DETERMINANTS, MEASURES, AND IMPLICATIONS

Study Session 4

EXAM FOCUS

Sovereign risks vary across countries. Factors such as political risk, legal risk, position in the economic growth cycle, and economic diversity impact a country's risks as well as investor appetite for their bonds. Rating agencies rate sovereign risks, which is an important component of assessing a country's credit risk. In the last three decades, countries have relied less on bank debt in favor of issuing public bonds. This has given rise to analyzing country risk by using sovereign default spreads. For the exam, be able to compare and contrast the advantages of sovereign credit ratings and sovereign default spreads. Understand the issues that can impede the value of ratings, such as herd behavior, timeliness, and rating biases. Also, be able to identify sources of sovereign risk and describe the consequences that result from sovereign defaults, and be able to describe sovereign defaults across time. Finally, ensure that you understand and are able to contrast credit ratings, sovereign default spreads, and credit default swap (CDS) spreads.

MODULE 24.1: COUNTRY RISK

LO 24.a: Identify and explain the different sources of country risk.

Investors are increasingly exposed to **country risk**, both directly and indirectly. First, innovations in financial markets have made investing in nondomestic companies easier for investors. Individuals now have a range of investment options including mutual funds that invest in foreign markets, exchange-traded funds (ETFs), and shares of foreign stock that are traded on domestic exchanges (including American depositary receipts [ADRs] in the United States and global depositary receipts [GDRs] in European markets). Second, companies are increasingly global, and many firms rely on growth in foreign markets to drive profits and returns to shareholders. Risk exposures vary across countries, with government actions directly influencing country risk and therefore the attractiveness of an investment.

Key sources of country risk include: (1) where the country is in the economic growth life cycle, (2) political risks, (3) the legal systems of countries, including both the structure and the efficiency of legal systems, and (4) the disproportionate reliance of a country on one product/commodity or service.

Economic Growth Life Cycle

More mature markets and companies within those markets are less risky than those firms and countries in the early stages of growth. Young growth companies rely on a stable macro environment and often have limited resources to withstand setbacks relative to mature companies. Similarly, early growth and emerging market countries are more vulnerable and are thus exposed to higher levels of risk than mature countries. Early growth countries generally have higher growth rates in recoveries and larger declines in GDP growth in downturns than their more mature counterparts. Equity markets are impacted as well. For example, in 2008, U.S. and European equity markets dropped by 25% to 30% while emerging markets fell by 50% or more. This is evidence that there is greater risk in early growth global investing even in countries that have sound legal systems and are well governed.

Political Risk

Political risk is broad and includes everything from whether a country is a democracy or a dictatorship to the smoothness with which a country transfers political power (e.g., military coups vs. democratic elections). There are four main components of political risk:

1. **Continuous versus discontinuous risk.** While it may be surprising to those living in a democracy, some investors prefer the stability of investing in companies in countries with autocratic leadership (i.e., one leader controls decision-making). The notion is that government policies are locked in and predictable and changes can be made more easily compared to a democracy where an election can change government policies fairly significantly. One of the risks of an authoritarian state is that significant costs of corruption and lack of legal systems can outweigh the benefits.

   Risks in democracies are continuous but generally low. In contrast, risks in dictatorships are discontinuous. Policies change much less frequently, but changes are often severe and difficult to protect against (i.e., discontinuous risk). For example, wars can adversely impact investment in businesses, including the disappearance of most international investors in Russian businesses following Russia's invasion of Ukraine in 2022. Studies are mixed regarding which system, authoritarian or democratic, results in higher economic growth. A 2021 report by *The Economist* indicates that only 6.4% of the world's population lives in democracies, while nearly one-third lives in authoritarian countries.

2. **Corruption.** There are costs associated with government corruption. Corruption costs can be likened to an implicit tax, directly reducing company profits and returns and indirectly reducing investor returns. Because the "tax" is not explicit and may also result in legal sanctions against the firm operating in the corrupt system (e.g., if

a firm is caught bribing an official), it increases risk. Using survey data, Transparency International ranks countries based on the level of corruption that is present. According to its 2021 survey, Denmark, Finland, and New Zealand are recognized as the least corrupt countries, while South Sudan is perceived as the most corrupt, followed by Somalia and Syria.

3. **Physical violence.** There are economic costs (e.g., insurance and security costs) and physical costs (e.g., possible physical harm to employees or investors) associated with countries in conflict. The Institute for Economics and Peace publishes the annual Global Peace Index, which maps countries by their level of peace/violence.
4. **Nationalization and expropriation risk.** Firms that perform well may see their profits expropriated via arbitrary taxation by governments. A firm may be nationalized, in which case the owners will receive much less than the true value of the company. Risks are greater in countries where nationalization and/or expropriation are possible, especially impacting natural resource companies.

Legal Risk

The protection of property rights (i.e., the structure of the legal system) and the speed with which disputes are settled (i.e., the efficiency of the legal system) affect risk. If disputes cannot be settled in a timely fashion, it is in essence the same as a system that does not protect property rights at all. Overall, investor and business protection is key in determining legal risk. For example, the ability of insiders to issue new shares to themselves at below-market prices in a particular country could lead to investors paying less for shares given these risks.

Several nongovernment organizations have created an international property rights index. Australia and North America are ranked the highest, while Latin America and Africa are ranked the lowest in terms of protection of property rights.

Economic Structure

A disproportionate reliance on a single commodity or service in an economy increases a country's risk exposure. For example, the economies of some countries rely almost exclusively on oil production. Other countries have the bulk of their economic output tied to banking or insurance while still others depend on tourism. A downturn in the demand (and/or price) for the good or service on which the country is dependent can devastate the economy, increasing risks for businesses and investors. It is not only the affected industry but also all of the businesses in the country/region that can be affected by a downturn. Diversification in these countries, especially in smaller economies, can be difficult because maintaining a concentration in resources can create significant wealth for those who control these resources, and therefore they have no incentive to diversify away from the key resource.

The United National Conference on Trade and Development (UNCTAD) measures the degree of dependence on commodities in emerging markets. The study indicates that Africa and Latin America are especially dependent on commodity exports, resulting in economies that are highly sensitive to changes in commodity prices, which increases country risk.

PROFESSOR'S NOTE
This phenomenon can happen in regions, countries, and even neighborhoods. For example, when much of the trading was shut down in the futures pits in Chicago in July 2015, there were many reports that described the economic impact on the "region," restaurants, retailers, and barbershops, all facing imminent demise as a result of the death of the key economic engine in the neighborhood.

Evaluating Country Risk

LO 24.b: Evaluate the methods for measuring country risk and discuss the limitations of using those methods.

There are various risk services that evaluate country risk, including the following:

- **Political Risk Services (PRS).** This for-profit firm evaluates more than one hundred countries on the key areas of country risk. Political, economic, and financial risk dimensions are evaluated using 22 variables to measure risks. The firm provides a composite score as well as a score on each of the three dimensions. In its 2022 update, Lebanon was ranked lowest (i.e., the riskiest country) and Norway was ranked highest (i.e., the least risky country) in terms of aggregated country risks.
- ***Euromoney.*** This magazine surveys 400 economists who assess country risk factors and rank countries from 0 to 100 with higher numbers indicating lower risk.
- ***The Economist.*** Currency risk, sovereign debt risk, and banking risks are assessed by this magazine to develop country risk scores.
- **The World Bank.** The World Bank compiles risk measures from several sources on 215 countries. Risk measures assess six areas, including the level of corruption, government effectiveness, political stability, rule of law, voice and accountability, and regulatory quality. Positive numbers imply less risk and negative numbers imply more risk.

For investors and businesses to use country risk ratings, they must be meaningful to their specific concerns. There are many limitations associated with these risk services that may diminish their value to businesses and investors, including the following:

1. The methodologies used to generate scores are often developed by nonbusinesses and may have more relevance to economists and policymakers than to businesses and investors.
2. There is no standardization across the information providers. The World Bank's ratings are scaled around zero. Higher scores indicate lower risk in the *Euromoney* and PRS rankings while lower scores indicate lower risk in *The Economist*'s rankings. This makes it difficult to compare country risk assessments across providers.
3. The scores are better used as rankings than as true scores. For example, a country with a ranking of 70 in *The Economist* rankings can be interpreted as riskier than a country with a ranking of 35. However, one cannot conclude, using the rankings, that a country with a score of 70 is twice as risky as a country with a score of 35.

MODULE QUIZ 24.1

1. A portfolio manager for a hospital foundation is considering the inclusion of sovereign bonds in the fixed income portion of the foundation's portfolio. Much to the surprise of his colleagues, the manager plans to purchase the bonds of a country that has long been under authoritarian rule. He cites "lower political risk" when asked about his investment decision. Which of the following statements is most likely what the manager meant by his assertion of lower risk?
   A. Authoritarian regimes are more likely to control corruption in government agencies.
   B. Government policies that may affect debt repayment are often more stable under an authoritarian regime.
   C. Relative to a democracy, risks are greater on a day-to-day basis, but the effects are less detrimental overall.
   D. In most authoritarian countries, property rights are protected and property disputes are settled quickly.

2. In an attempt to understand country risk, an analyst at Global Funds examines multiple sources of information to determine the truest measure of risk. She considers sovereign risk ratings, default risk spreads, and composite measures of risk. Which of the following sources relies on surveys of several hundred economists to measure sovereign risk?
   A. Political Risk Services.
   B. *The Economist*.
   C. The World Bank.
   D. *Euromoney*.

MODULE 24.2: SOVEREIGN DEFAULT RISK

LO 24.c: Compare and contrast foreign currency defaults and local currency defaults.

Sovereign default risk refers to the risk that holders of government-issued debt fail to receive the full amount of promised interest and principal payments during the specified time period. Sovereign default risk can be used as a proxy for country risk. Sovereign default categories include foreign currency defaults and local currency defaults.

Foreign Currency Defaults

Throughout history, some governments have relied on debt borrowed from other countries or banks in those countries. The debt, denominated in the foreign currency, is called **foreign currency debt**. Countries default on foreign currency debt from time to time because they are without the foreign currency to meet the obligation and are unable to print money in a foreign currency to repay the debt. A large proportion of sovereign defaults are foreign currency defaults. Between 1983 and 2021 there have been more than 30 foreign currency defaults. Overall, the largest sovereign default during the period was Greece, which defaulted on more than $264 billion in 2012.

According to a 2007 study, defaults over the previous 200 years occurred primarily during seven distinct time periods: 1824–1834, 1867–1882, 1890–1900, 1911–1921,

1931–1940, 1976–1989, and 1998–2003. In Europe, the greatest number of defaults occurred from 1931 to 1940. The same period also saw a number of foreign currency defaults in Latin America. The early 1980s was another key period for Latin American defaults with 16 countries defaulting, 15 of which occurred between 1980 and 1983. There have also been defaults in Asia and Africa over the last 50 years.

The following general conclusions can be drawn based on previous studies on sovereign defaults:

1. Countries were more likely to default on funds borrowed from banks than on sovereign bond issues.
2. Latin America accounts for a large proportion of sovereign defaults in the last 50 years (measured in dollar value terms).

Except for the 1990s, in each of the last five decades, Latin America has accounted for at least 60% of the foreign currency defaults. In fact, Latin America has been at the center of sovereign defaults for the last 200 years. To put it in historical perspective, in the 19th century, Latin America attracted capital from France, Britain, and Spain because of abundant natural resources. Latin American countries did not have significant domestic savings and thus borrowed heavily in gold and in foreign currency during this period. Maturities were long, usually greater than 20 years. Military coups and conflicts were the primary triggers of default on this debt. Between 1820 and 1919, 58 of the 77 sovereign defaults worldwide occurred in Latin America. Between 1825 and 1940, Uruguay spent 12% of the time in default (the shortest period) while Honduras spent 79% of the time in default, and Latin American countries collectively spent 38% of the time in default.

Local Currency Defaults

Many of the countries that defaulted on foreign currency debt over the last several decades were simultaneously defaulting on **local (i.e., domestic) country debt**. An S&P study of local currency defaults since 1975 indicates that 23 issuers have defaulted. Defaulting countries range from Russia in 1998–1999 to Argentina in 2002–2004. The largest local currency default in dollar terms occurred in 1990 when Brazil defaulted on $62 billion of debt. Russia defaulted on $39 billion of ruble debt in 1998–1999.

A study by Moody's finds that countries are increasingly defaulting on both foreign and local currency debts concurrently as shown in Figure 24.1.

Figure 24.1: Changes in Concurrent Default in Foreign and Local Currency Debt

Default Type	1960–1996	1997–2007
Foreign currency only	57%	29%
Local currency only	38%	29%
Both foreign and local	5%	42%

From Figure 24.1, you can see that the instance of concurrent defaults (i.e., local currency and foreign currency simultaneous defaults) has increased from 5% in the 1960 to 1996 period to 42% in the 1997 to 2007 period. Foreign currency only and local currency only defaults fell during the same period.

In a 2021 study, the Bank of Canada assessed sovereign defaults by types of creditors between 1976 and 2020. The study found that defaulted debt shifted toward local currency debt and nonbank borrowings, from foreign currency debt and bank loans. A separate 2022 study found that countries have moved more toward defaults on local currency debt under domestic law, given the greater efficiency and faster time of restructurings compared to foreign currency debt defaults.

It is difficult to explain why countries default on local currency debt. It would seem that countries would simply print more money to meet their obligations. However, there are three reasons that help explain local currency defaults:

1. **Gold standard.** Before 1971, some countries followed the gold standard. This means the country was required to have gold reserves to back its currency. The gold standard thus limited the amount of currency a country could print, reducing its flexibility in terms of printing currency to repay debt.
2. **Shared currency.** The euro is an example of a shared currency. The advantage of a shared currency is convenience for businesses, tourists, and so on. It eliminates the costs of converting currencies and increases transparency. However, a shared currency limits the abilities of individual countries to print money. For example, during the 2015 Greek debt crisis, as a member of the European Union (EU), Greece was not able to print currency to pay off its debt.
3. **The tradeoff.** There are costs associated with printing money. Printing money may devalue and debase the currency. It also leads to higher inflation, sometimes exponentially higher inflation. There are costs associated with default and costs associated with printing money leading to currency devaluation. Countries have to decide which of these tradeoffs is the less costly option for them. A key factor in these decisions is whether and/or to what extent foreign currency debt is funding local currency assets. The greater the extent, the more negative the consequences of choosing to print more local currency because it increases inflation and devalues local currency, reducing asset values while leaving liabilities unchanged.

Consequences of Sovereign Default

LO 24.d: Explain the consequences of a country's default.

Historically, defaults were often followed by military actions. For example, when Egypt defaulted in the 1880s, Britain used military force to take over the government, as did Britain and France around the same time when Turkey defaulted. In the 20th century, countries that defaulted suffered a loss of reputation, making it more difficult and more expensive to borrow in the future. Countries also experienced turmoil in stock and bond markets, faced a decrease in real output, and had to deal with political instability as a result of default.

Default typically does mean a complete nonpayment or elimination of debt. Defaults usually involve complex negotiations leading to some form of exchange or restructuring of debt. An examination of research on sovereign defaults leads to the following conclusions:

- **GDP decline.** Gross domestic product (GDP) typically declines between 0.5% and 2.0% following a sovereign default. However, the decline is short-lived, usually within the first year following the default.
- **Lower sovereign ratings and borrowing costs.** One study finds that ratings of countries that have defaulted at least once since 1970 are one to two notches lower than the ratings of similar countries that have not defaulted. Also, borrowing costs are 0.5%–1.0% higher. The effects lessen over time.
- **Trade retaliation.** Sovereign default can cause trade retaliation. Export businesses are most affected. One study finds an average 8% drop in bilateral trade following a default. The study also finds the effects can last up to 15 years.
- **More fragile banking systems.** A study of 149 countries between 1975 and 2000 finds that there is a 14% probability of a banking crisis following a sovereign default, which is 11% higher than for nondefaulting countries.
- **Increased likelihood of political change.** Sharp currency devaluations often follow defaults. Countries that default on debt are more likely to see a change in the president or prime minister (a 45% increase in the probability of a change) and the finance minister or head of the central bank (a 64% increase in the probability of a change).

Factors Influencing Sovereign Default Risk

LO 24.e: Discuss measures of sovereign default risk and describe components of a sovereign rating.

Individuals, companies, and governments default for many of the same reasons. Primarily, they each borrow more than they can afford in good times and find themselves unable to repay the borrowed funds during downturns. Several factors influence a country's sovereign default risk. They are as follows:

1. **The country's level of indebtedness.** The level of indebtedness is the most fundamental factor used to determine the risk of default. One must consider not only the country's debts to foreign banks and investors, but also the amount the country owes its own citizens (e.g., for social safety nets such as welfare and universal health care). For comparison purposes, debt is typically scaled to a country's GDP. Some of the most indebted countries when looking at sovereign debt as a percent of GDP include the U.S., Japan, and France, which are considered highly creditworthy by financial markets and credit rating agencies. Debt levels in the U.S. have increased since 2008 and have further increased due to the COVID-19 pandemic. But this means that the level of indebtedness is not the only factor determining the risk of default. Other countries, such as Egypt and El Salvador, have high default risk as well.

Figure 24.2: Government Debt as a Percentage of GDP (as of 2020)

Country	Government Debt as a Percentage of GDP	
	2020	2010
Venezuela	304.12%	25.00%
Japan	254.13%	205.69%
Greece	211.21%	147.49%
Italy	155.82%	119.20%
Portugal	135.19%	100.21%
United States	133.92%	95.14%
Spain	119.92%	60.52%
Cyprus	119.14%	56.43%

Source: IMF Datasets, https://www.imf.org/external/datamapper/GG_DEBT_GDP@GDD/SWE

2. **Pension funds and social services.** Countries with greater pension commitments and health care commitments have higher default risk. (As these commitments increase as the population ages, countries with older populations face greater risks.)
3. **Tax receipts.** The greater the tax receipts, the more able a country is to make debt payments. A larger tax base should increase a country's revenues (i.e., tax receipts) and therefore lower default risk.
4. **Stability of tax receipts.** Governments must pay debt obligations in both good and bad economic times. This means the revenue stream must be stable to meet these fixed obligations. Countries with more diversified economies are more likely to have stable tax receipts. Countries like Jamaica, which depends on tourism, and Peru, which depends on silver and copper production, have more sovereign default risk than the governments of larger, more diversified economies such as India. Also, sales and value added tax systems are generally more stable than income tax systems.
5. **Political risk.** Autocracies may be more likely to default than democracies because defaults put pressure on, and may cause a change in, the leadership of the country. There may be less pressure on the leaders of dictatorships if the country defaults. Also, the more independent the central bank, the more difficult it may be for a country to print money.
6. **Backing from other countries/entities.** Rating agencies and other market participants reassessed and decreased the estimated default risk of Spain, Greece, and Portugal when those countries joined the EU. It is assumed that stronger economies like Germany, France, and the Nordic countries would protect weaker economies against default. However, there is no guarantee, and this is an implicit, not an explicit, backing.

Sovereign default risk is multifaceted and must be analyzed from many perspectives. The country's level of indebtedness, obligations to its citizens for things like pensions and medical care, and its tax systems are all relevant to assessing the risk of default. In addition, the trustworthiness of the government and the nature of the economy must also be considered when evaluating sovereign default risk.

Rating Agencies and Default Risk

Rating agencies have distinct advantages when it comes to assessing sovereign default risk. First, they have been assessing the default risk of corporations for more than a century, with these skills easily transferable to sovereign risk evaluation. Additionally, investors are accustomed to the ratings used for companies, so understanding the credit ratings of sovereign nations is intuitive.

By 1929, Moody's, one of the big three credit rating agencies, rated nearly 50 governments. Interest in government bonds waned following the Great Depression and World War II, however, it picked up slowly in the 1970s and has continued to grow. By 1994, there were 49 rated countries, rated from AAA/Aaa to BBB–/Baa3.

The market for sovereign bonds has since increased dramatically. In 2022, Moody's, S&P, and Fitch each rated more than 100 countries. Moody's and S&P currently provide two ratings for each country, a local currency rating for domestic currency bonds and a foreign currency rating for borrowings in a foreign currency. Figure 24.3 provides some examples of Moody's sovereign bond ratings.

Figure 24.3: Sample of Moody's Latin American Sovereign Ratings

Country	Foreign Currency Rating	Foreign Currency Outlook	Local Currency Rating	Local Currency Outlook
Argentina	Ca	STA (Stable)	Ca	STA (Stable)
Brazil	Ba2	STA (Stable)	Ba2	STA (Stable)
Chile	A1	NEG (Negative)	A1	NEG (Negative)
Mexico	Baa1	NEG (Negative)	Baa1	NEG (Negative)
Venezuela	C	STA (Stable)	C	STA (Stable)

Source: Moody's (July 2022)

The outlook column indicates the outlook on the ratings. For Moody's ratings, NEG indicates that there is a possibility of a downgrade, STA indicates a stable rating, and POS indicates the possibility of a ratings upgrade.

Generally, the local currency rating is at least as high as the foreign currency rating, because countries can print money in local currency to repay debt. There are some exceptions where the local currency rating is lower, as was the case of India in March 2010, which was assigned a local currency rating of Ba2 and a foreign currency rating of Baa3. Sovereign ratings are often similar across rating agencies but may diverge given their different views on idiosyncratic and macroeconomic risks, including their assessment of political risk.

Sovereign ratings could change over time, but usually change less frequently than corporate bond ratings. Rating changes are typically analyzed using rating transition tables, which indicate the rating changes of sovereigns over one-year periods. For example, using S&P's one-year rating transition data between 1975 and 2021, the calculated probability of a AAA-rated sovereign remaining AAA was 96.82%. A BBB-rated sovereign had a 90.25% probability of remaining unchanged, a 4.48% chance of being upgraded, and a 5.26% chance of being downgraded. When using 15-year

transition data, ratings change more frequently, with only 67.14% of AAA-rated and 25.25% of BBB-rated sovereigns holding their rating. A criticism of rating agencies is regional bias, meaning that agencies tend to underrate sovereigns in entire regions, for example in Latin America.

How Rating Agencies Measure Risk

The three main rating agencies use similar processes to determine sovereign ratings. The ratings process includes the following:

- **Ratings measure.** The goal is to determine the creditworthiness of a country. The agencies focus on the default risk faced by banks and private bondholders, not official creditors such as the World Bank and the International Monetary Fund (IMF). What is being measured also differs: S&P's ratings measure the probability of default while Moody's ratings measure both the probability of default and the severity of the default (i.e., the rating looks at the expected recovery rate in the event of default). Default is defined as the failure to pay principal or interest payments on the due date (i.e., outright default) or rescheduling or restructuring the debt (i.e., restructuring default).
- **Evaluating factors that may contribute to default.** These factors are related to the economic, political, and institutional detail of the country with respect to its ability to repay debt. Regarding the sovereign ratings methodology profile, S&P uses variables classified based on political risk, economic structure, economic growth prospects, fiscal flexibility, general government debt burden, offshore and contingent liabilities, monetary flexibility, external liquidity, and the country's external debt burden. Fitch and Moody's use similar frameworks for analyzing the risk of default.
- **Ratings process.** An analyst prepares a draft report, recommending the rating. A committee, typically composed of 5 to 10 people, debates the rating categories (i.e., political risk, economic structure, and so on) and other relevant information presented, and then votes on the final rating following closing arguments.
- **Local vs. foreign currency ratings.** Sovereign ratings typically include both a local currency rating and a foreign currency rating. The difference between the two is largely based on the country's monetary policy independence. Countries that have given up monetary policy independence (e.g., EU countries) will see their foreign and local currency ratings converge. Countries with floating rate exchange regimes that fund borrowing through sound domestic markets will have the greatest difference between the two ratings. Agencies use two general approaches to arrive at the foreign versus local currency ratings:
  - *Notch-up approach*. The foreign currency rating is the key indicator of sovereign default risk, and the local currency rating is "notched up" based on the domestic debt market and other domestic factors.
  - *Notch-down approach*. The local currency rating is the key indicator of sovereign default risk, and the foreign currency rating is "notched down" based on foreign exchange issues and constraints.
- **Ratings review process.** Ratings are reviewed on a periodic basis. In addition, news can trigger a ratings review. For example, an economic disaster in one country may

trigger a review not only of that country, but also of neighboring countries that may be subject to a contagion effect.

Sovereign Ratings vs. Default Risk

Rating agencies argue that despite some errors, there is a high correlation between sovereign ratings and sovereign default. S&P's cumulative default rate study from 1975 to 2021 supports this premise. For example, AAA-rated countries have not defaulted within a 15-month period following the rating assignment, while BBB-rated countries had a 4.78% probability of default over the same period, and CCC-rated countries had an 84.80% probability of default.

Shortcomings of the Sovereign Rating System

LO 24.f: Describe the shortcomings of the sovereign rating systems of rating agencies.

Rating agencies have been criticized on a number of counts. These include the following:

- **Ratings are biased upward.** Rating agencies are often viewed as too optimistic when it comes to rating sovereigns and corporations. The conflict of interest that is often cited as a problem in corporate ratings (i.e., the company pays the rating agency for the rating) is much less of a problem with sovereign ratings because any revenue received is small, while the reputational damage of overrating sovereigns is very high.
- **Herd behavior.** When one agency upgrades or downgrades a country, the other agencies tend to follow suit. This perceived lack of independence reduces the benefit of having three rating agencies.
- **Rating changes are not timely enough.** Investors need rating agencies to update ratings in a timely fashion. Some market participants feel that the agencies take too long to change ratings, leaving investors unprotected in the event of a crisis.
- **Overreaction leads to a vicious cycle.** Rating agencies have been criticized for overreacting to crises, lowering ratings too much in response to a crisis, which then worsens the crisis.
- **Ratings failures.** One study examined multiple ratings changes in a short period of time by S&P and Moody's. It can be argued that multiple rating changes within a single year imply that agencies were incorrect in their initial assessments of the country. The study offered several possible explanations for the ratings failures:
  - *Bad information*. Agencies rely on information from governments. There is significant variability in the amount and quality of the data that agencies receive across countries. If governments hide the truth and reveal only positive information, ratings will be incorrect. This may also help explain the upward bias discussed previously.
  - *Limited resources*. Rating sovereigns generate limited revenues for agencies. Analysts can be spread thin and often rate multiple countries. This may lead

analysts to rely on common information available in the market rather than doing their own research and may also contribute to herd behavior.

- *Revenue bias*. Rating agencies generally do not charge users for sovereign ratings. This means revenues must come either from the issuers or from other businesses. Rating agencies generate significant revenues from subsovereigns (e.g., states, provinces, counties, and cities). If some agency revenues arise from sovereigns or subsovereigns, a criticism is that there could be an incentive for the agency to refrain from issuing harsh judgments against a country. A sovereign ratings downgrade is usually followed by a series of subsovereign downgrades, and subsovereigns could fight the sovereign downgrade, contributing to the upward bias of sovereign ratings.
- *Other conflicts of interest*. There are off-shoot businesses from the core sovereign ratings business. These businesses may generate enough revenue to influence ratings. Typically, rating agency employees do not go work for the sovereigns they rate, thus, this issue does not generally pose a conflict of interest.

PROFESSOR'S NOTE

While some of the criticisms against rating agencies are valid, agencies have put in place significant guardrails against bias over the last several decades. The most significant of these guardrails is the separation—both physical and conceptual—between the commercial side of the ratings business (the relationship managers discussing products and fees with clients) and the analytical side (the rating analysts and supporting teams, which are prohibited from discussing commercial products with clients). This is meant to ensure the independence of the ratings and minimize any bias.

Credit Ratings, Default Spreads, and CDS Spreads

LO 24.g: Compare the use of credit ratings, market-based credit default spreads, and CDS spreads in predicting default.

Credit Ratings and the Sovereign Default Spread

There has been significant growth in the sovereign bond market, beginning in the 1980s. More countries have been avoiding bank debt in favor of issuing bonds. A key measure of sovereign default risk when comparing different sovereign bonds is the **sovereign default spread**. The sovereign default spread is the difference between the yield of a riskier sovereign bond yield and a riskless sovereign bond yield (e.g., U.S. Treasury bond yield). For example, the yield on a 10-year dollar-denominated Brazilian bond was 6.00% on July 1, 2022, while the yield on the 10-year U.S. Treasury bond was 3.02%. Because the U.S. Treasury bond is assumed to be risk free, the 2.98% difference between the two bond yields is the sovereign default spread and it reflects the market's assessment of the default risk of the Brazilian bond.

Figure 24.4 shows the comparison of default spreads and Moody's credit ratings for a sample of dollar-denominated bonds in July 2022.

Figure 24.4: Default Spreads on Dollar-Denominated Latin American Bonds

Country	Yield on Dollar-Denominated 10-Year Sovereign Bond	Yield on 10-Year U.S. Treasury Bond	Sovereign Default Spread	Moody's Sovereign Bond Rating
Brazil	6.19%	3.02%	3.17%	Ba2
Colombia	5.58%	3.02%	2.56%	Baa2
Indonesia	4.38%	3.02%	1.36%	Baa2
Turkey	10.92%	3.02%	7.90%	B2

Source: Bloomberg

Interestingly, Moody's sovereign ratings of both Columbia and Indonesia are the same (Baa2); however, the market demands a higher rate of return for the Colombian bonds (higher risk). In July 2022, the difference in default spreads between the two countries was 120 basis points (i.e., 2.56% – 1.36%).

The key advantages of the sovereign default spread are as follows:

1. **Market-based spreads are more dynamic than ratings.** As bonds trade and bond yields rise and fall, default spreads change, revealing information about the market's perception of risk. For example, in late 2005 the default spreads of Brazil and Venezuela were close, 3.18% and 3.09%, respectively. Spreads started to diverge between 2006 and 2009. By December 2010, the default spread had widened to 10.26% on the Venezuelan bonds and narrowed to 1.32% on the Brazilian bonds. It is clear that the market's perception of the relative risk of the two countries was changing, as evidenced by the diverging spreads.
2. **Spreads adjust quicker to new information.** Similar to the advantage that changes occur in real time, yield spreads adjust more quickly to new information regarding the sovereign relative to bond ratings. This means investors receive an earlier signal of potential threats and they can therefore adjust portfolios more quickly.

The key disadvantages of the sovereign default spread are as follows:

1. **Comparing local currency bond yields is less meaningful.** Local currency bonds do not have a risk-free security with which to compare. It is not practical to compare local currency bonds because differences in yields may reflect differences in expected inflation across countries rather than true risk. Also, even with dollar-denominated bonds, it is the assumption that U.S. Treasury bonds are default risk free that makes calculating a yield spread meaningful.
2. **Greater volatility in default spreads.** Default spreads are volatile and changes in spreads may be affected by variables that are unrelated to the default risk of the sovereign. For example, investor demand for the bonds and changes in liquidity can affect spreads but often have nothing to do with default risk.

Studies that have examined default spreads and ratings generally conclude the following:

- *Default spreads are positively correlated with ratings and with default.* In other words, low rated sovereign bonds are more likely to trade at higher yields (and yield spreads) and are more likely to default.
- *Default spreads are "leading indicators."* The spreads widen before a rating downgrade and narrow before a rating upgrade.
- *A rating change provides information to the market, despite the longer lag time relative to default spreads.* Rating agencies use market information to make decisions about rating changes. The market reacts to ratings and rating changes when pricing bonds. This means that both ratings and default spreads are useful to market participants when evaluating and understanding sovereign default risk.

CDS Spreads

Credit default swaps (CDSs) have gained significant prominence over the last decade, growing to nearly $60 trillion by the end of 2007 before declining in value after the great financial crisis. The overall number of players in this space is small, which exposes the CDS market to potential volatility if a few large entities fail. Although the sovereign CDS market remains a small portion of the overall market, its popularity has grown in recent years. By July 2022, there were CDS trading on 85 sovereigns.

A CDS is similar to buying insurance against a bond default. One party (the CDS buyer) pays periodic payments as a percentage of the bond notional value in each period to another party (the CDS seller). The CDS seller provides protection that in a predefined credit event, which could include a bond default or a restructuring, the seller makes the buyer "whole" against a loss on the bond. Settlement can be cash-based where the seller pays the difference between the bond's notional value and its market price, or physical where the buyer delivers the bond to the seller and receives the notional value in return.

For example, an investor may own $20 million notional value five-year Peruvian government bonds. If the investor is worried about a potential credit event over the next five years, it could buy a five-year CDS and pay 170 basis points (1.70%), or $340,000, annually to a CDS seller. The percentage (i.e., number of basis points) of notional value is known as the **CDS spread**. If a credit event such as a default or restructuring occurs over the next five years, the CDS buyer will either deliver the bond to the seller and receive $20 million, or receive from the seller the difference between the notional value and market value of the bond.

Note that there are two key risks in buying CDS protection. First, a credit event must occur in order for the protection buyer to receive a payment. If the bond value declines due to other reasons, no credit event occurred and the CDS buyer would not receive a payment. Second, the buyer is exposed to the credit risk that the seller may not be willing or able to make a payment even if a credit event occurs.

CDS spreads adjust more quickly to market information than ratings do, making them valuable to investors and institutional players. This difference in the speed of adjustment to new risks was especially evident during the 2009–2010 period, when CDS spreads of several European sovereigns including Greece changed much more

frequently than the sovereign ratings. Analyzing the change in CDS spreads leads to several important conclusions:

1. Changes in CDS spreads influence sovereign bond yields and ratings.
2. CDS spreads are better predictors of sovereign risk than credit ratings.
3. Increased macroeconomic risks result in increases in CDS spreads, which are also often accompanied by currency depreciations.
4. Contagion (or clustering) leads CDS spreads of groups of countries to move together.

There are two key limitations to relying on CDS spreads. First, CDS spreads reflect a range of risks including credit, market, and liquidity risks. As a result, using CDS spreads to assess only credit risk could overestimate the risks. Second, CDSs are often illiquid, which distorts their prices. Despite these limitations, CDSs contain significant information about default risk, although they have not been proven to be more beneficial in assessing default risk than market-based default spreads.

MODULE QUIZ 24.2

1. Which of the following statements regarding foreign currency defaults is true?
   A. African countries are responsible for the greatest number of defaults in the last 50 years.
   B. Before the 20th century, no country had ever defaulted on funds borrowed in a foreign currency.
   C. Latin America accounted for more than 60% of foreign currency defaults in the 1990s.
   D. Countries are more likely to default on funds borrowed from foreign banks than on sovereign bond issues.

2. Following a sovereign default:
   A. borrowing costs rise 1.0% to 2.5%, and the country is more likely to suffer a banking crisis.
   B. countries are more likely to suffer banking crises and GDP growth generally falls 2.0% to 3.5%, but the drop is usually short-lived.
   C. ratings fall one to two notches, and GDP growth falls between 0.5% to 2.0%.
   D. trade retaliations are likely to occur but are short-lived, lasting only six months to one year.

KEY CONCEPTS

LO 24.a

Key sources of country risk include where the country is in the economic growth life cycle, political risks, the legal systems of countries, including both the structure and the efficiency of legal systems, and the disproportionate reliance of a country on one commodity or service.

Regarding economic life cycle, more mature markets and companies within those markets are less risky than those firms and countries in the early stages of growth.

Regarding political risk, there are at least four components of political risk, including the level of corruption in the country, the occurrences of physical violence due to wars

or civil unrest, the possibility of nationalization and expropriations, and the continuity and severity of risks versus discontinuous risks.

Regarding legal risks, the protection of property rights and the speed with which disputes are settled affect default risk.

Regarding economic structure, a disproportionate reliance on a single commodity or service in an economy increases a country's risk exposure.

LO 24.b

Entities including Political Risk Services (PRS), *The Economist*, *Euromoney*, and the World Bank evaluate countries on key areas of country risk. Some are critical of these composite risk measures because they are not readily comparable with each other due to a lack of standardization across the information providers. Also, the methodologies used to generate scores are often developed by nonbusiness entities and may have more relevance to economists and policymakers than to businesses and investors. Finally, the scores are better used as rankings than as a way to interpret the relative risk of countries.

LO 24.c

There are many causes of sovereign defaults. It is easier to understand foreign currency defaults than local currency defaults. Countries are often without the foreign currency to meet the debt obligation and are unable to print money to repay the debt. This makes up a large proportion of sovereign defaults.

Many of the countries that defaulted on foreign currency debt over the last several decades were simultaneously defaulting on local country debt. Three reasons may explain local currency defaults: (1) the use of the gold standard before 1971 made it more difficult for some countries to print money, (2) shared currencies, such as the euro, make it impossible for countries to control their own monetary policy, and (3) some counties may conclude based on a tradeoff theory that the costs of currency debasement and potentially higher inflation are greater than the costs of default.

LO 24.d

Historically, defaults were often followed by military actions. Research suggests the following additional consequences of sovereign defaults:

- GDP growth falls following a sovereign default (between 0.5% and 2.0%).
- Borrowing costs are higher following default (by 0.5% to 1.0%).
- Sovereign default can cause trade retaliation (bilateral trade can drop by 8%).
- Sovereign defaults are often followed by banking crises (14% probability of a banking crisis following a sovereign default).
- Sovereign defaults often lead to political change in top leadership and top finance positions.

LO 24.e

Several factors determine a country's sovereign default risk. The country's level of indebtedness, obligations such as pension commitments and social service commitments, the country's level of and stability of tax receipts, political risks, and backing from other countries or entities all impact a country's likelihood of defaulting on sovereign debt.

Rating agencies consider several factors when evaluating default risk. These factors are related to the economic, political, and institutional characteristics of a country with respect to its ability to repay debt. The ratings process includes an analyst preparing a draft report and recommending a rating. A committee votes on the recommendation including the final rating. Ratings are reviewed periodically and may also be reviewed following a significant market event that impacts the likelihood of default.

LO 24.f

Rating agencies have been criticized on numerous counts, including:

- ratings are often viewed as upward biased;
- the ratings established by major rating agencies (i.e., S&P, Moody's, and Fitch) tend to mirror each other;
- sovereign rating changes are slow to change;
- rating agencies could overreact to news impacting a sovereign; and
- ratings are simply incorrect in some cases.

LO 24.g

Advantages of sovereign default spreads relative to sovereign bond ratings are that changes occur in real time, risk premiums adjust to new information more quickly, and there is more granularity in default spreads than in risk ratings.

Disadvantages of sovereign default spreads include more volatility, spreads include more information than just default risk (e.g., changes in liquidity and investor demand), and local currency bonds cannot be compared with each other because differences may reflect differences in expected inflation rather than differences in default risk.

Credit default swaps (CDSs) have gained prominence over the last decade. A CDS is similar to buying insurance against a bond default. Key risks in buying CDS protection include the need for a specified credit event to occur in order for a payout to happen, and the exposure by the CDS buyer to the credit risk of the seller.

Relying on CDS spreads is useful because they adjust more quickly to market information than ratings. Changes in CDS spreads influence sovereign bond yields and ratings. Increases in CDS spreads are also often accompanied by currency depreciations. However, CDS spreads of a group of countries may cluster (i.e., change together). The usefulness of CDSs is limited because their spreads reflect not only credit

risk but also liquidity risk and investor demand. CDSs are also often illiquid, which distorts their prices.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 24.1

1. **B** Some investors prefer the stability of investing in countries with autocratic governments because government policies are locked in and generally more predictable compared to democratic countries where an election can significantly change government policies. Risks in a democracy are continuous, but usually low. In contrast, risks in a dictatorship are discontinuous. Policies change much less frequently, but changes are often severe and difficult to protect against. The portfolio manager is willing to accept the bigger, discontinuous risk as a tradeoff for the more frequent, but less damaging, continuous risk. (LO 24.a)

2. **D** Numerous services attempt to evaluate country risk in its entirety. They include Political Risk Services (PRS), *The Economist*, *Euromoney*, and the World Bank. *Euromoney* surveys 400 economists who assess country risk factors and rank countries from 0 to 100, with higher numbers indicating lower risk. (LO 24.b)

Module Quiz 24.2

1. **D** Historically, countries have been more likely to default on foreign bank debt than on sovereign bonds. Latin America is responsible for the greatest number of foreign currency defaults over the last five decades with more than 60% of defaults in each decade with the exception of the 1990s. Over the last 200 years there are many instances of default. The defaults primarily occurred in seven distinct time periods: 1824–1834, 1867–1882, 1890–1900, 1911–1921, 1931–1940, 1976–1989, and 1998–2003. Thus, countries did borrow and default in the 19th century. (LO 24.c)

2. **C** Examining sovereign defaults leads to the following conclusions: (1) gross domestic product (GDP) growth falls between 0.5% and 2.0% following a sovereign default, and the decline is short lived, (2) ratings of countries that have defaulted at least once since 1970 are one to two grades lower than the ratings of similar countries that have not defaulted and borrowing costs are 0.5% to 1.0% higher with the effects lessening over time, (3) sovereign default can cause trade retaliation, lasting up to 15 years, with export businesses most sharply affected, (4) a banking crisis is more likely to follow a default, and (5) sharp currency devaluations and changes in a country's leadership often follow defaults. (LO 24.d)

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP assigned reading—Hull, Chapter 17.

READING 25

ESTIMATING DEFAULT PROBABILITIES

Study Session 5

EXAM FOCUS

This topic discusses various approaches for estimating default probabilities, which are used to calculate credit value at risk and determine counterparty credit risk. For the exam, understand how to interpret a rating migration matrix. Also, be aware of the relationships between borrowing ratings and the probability of default as well as recovery rates and default rates. Knowledge of credit default swap (CDS) mechanics and the associated calculations (e.g., up-front premium, hazard rates) is important, as is being able to calculate default probabilities from historical data and credit spreads. In addition, understand the differences between risk-neutral and real-world default probability estimates. Finally, be able to apply the Merton model to compute equity value, loss on debt, distance to default, and probability of default.

MODULE 25.1: CREDIT RATINGS AND DEFAULT PROBABILITIES

LO 25.a: Compare agencies' ratings to internal credit rating systems.

Credit ratings of larger publicly traded bond issuers are provided to the general public by rating agencies. Credit ratings are indicative of credit quality; therefore, they may change based on information (positive or negative) released to the market. When assigning ratings, rating agencies aim to provide stability to avoid frequent rating changes that might occur based on short-term information received. In other words, rating agencies take a long-term view of an issuer's creditworthiness. In that regard, "rating through an economic cycle" is an important consideration as information may be received that suggests a higher likelihood of default within the next year, but an overall insignificant change in default probability over the subsequent four years, for example.

Rating agencies typically do not provide credit ratings for small- and mid-sized firms. As a result, many lenders have developed their own internally developed credit rating systems for such firms. Internal credit rating systems usually look at profitability (e.g., return on equity), liquidity (e.g., quick ratio), and solvency (e.g., debt-to-equity ratio) measures. Because borrowings must be repaid with cash, lenders will use company-prepared financial statements as a starting point and make the required adjustments to create a cash flow statement. A cash flow statement is especially useful when determining a firm's repayment ability.

Altman's Z-Score

LO 25.b: Describe linear discriminant analysis (LDA), define the Altman's Z-score and its usage, and apply LDA to classify a sample of firms by credit quality.

Linear discriminant analysis (LDA) is a popular statistical method used for developing credit scoring models. A scoring function is a linear function of variables produced by an LDA. The variables are chosen based on their estimated contribution to the likelihood of default and come from an extensive pool of qualitative features and financial ratios.

Altman's Z-score is an application of LDA that is used to predict defaults. The following five financial ratios are used:

1. X_1: working capital / total assets
2. X_2: retained earnings / total assets
3. X_3: earnings before interest and taxes (EBIT) / total assets
4. X_4: market value of equity / book value of total liabilities
5. X_5: sales / total assets

Altman's Z-score was initially devised for publicly traded manufacturing firms. Its reliability has been confirmed by testing out-of-sample data and discovering few Type I and Type II errors. The Z-score model has also been adapted and refined for use with private manufacturing firms.

The contributions (weights) of each of the five financial ratios to the overall score are as follows:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

In applying LDA to classify firms based on credit quality, the guidelines regarding the computed Z-score are as follows:

- > 3: no default is likely
- 2.7–3: potential default
- 1.8–2.7: reasonable probability of default
- < 1.8: high likelihood of default

Thus, the higher (lower) the score, the more likely the firm will be classified as a solvent (insolvent).

Historical Default Probabilities

LO 25.d: Describe a rating migration matrix and calculate the probability of default, cumulative probability of default, and marginal probability of default.

S&P borrowing ratings range from AAA (highest rating; best quality) to D (lowest rating; in payment default). With this rating scale, *investment-grade bonds* are rated from AAA to BBB (best quality to good quality), while *non-investment-grade* (junk) bonds are rated from BB to D.

Figure 25.1 provides an example of a **rating migration matrix** developed by S&P covering a period of almost 40 years. This matrix quantifies the average default rates of issuers based on initial credit rating.

Figure 25.1: Cumulative Issuer-Weighted Default Rates for 1981–2020 (in %)

	Time Horizon (in Years)							
Rating	1	2	3	4	5	7	10	15
AAA	0.00	0.03	0.13	0.24	0.34	0.51	0.70	0.90
AA	0.02	0.06	0.11	0.21	0.30	0.49	0.70	0.99
A	0.05	0.13	0.22	0.33	0.46	0.76	1.20	1.84
BBB	0.16	0.43	0.75	1.14	1.54	2.27	3.24	4.54
BB	0.63	1.93	3.46	4.99	6.43	8.89	11.64	14.65
B	3.34	7.80	11.75	14.89	17.35	20.99	24.62	28.24
CCC/C	28.30	38.33	43.42	46.36	48.58	50.75	52.76	54.76

Source: S&P Global Ratings Research

From the data in Figure 25.1, a bond that begins with an AA rating has a 0.02% chance of defaulting by the end of the first year, a 0.06% chance of defaulting by the end of the second year, a 0.11% chance of defaulting by the end of the third year, and so on. These values represent the **cumulative probability of default**. Based on those amounts, it is possible to calculate the **marginal probability of default**. For example, the probability of an AA-rated bond defaulting *within* its third year is computed as 0.11% (cumulative probability to the end of Year 3) minus 0.06% (cumulative probability to the end of Year 2), which is equal to 0.05%.

Borrowing Rating vs. Probability of Default

LO 25.c: Describe the relationship between borrower rating and probability of default.

In general, the lower (higher) the borrowing rating, the greater (lower) the probability of default. Furthermore, for investment-grade bonds and some non-investment-grade

bonds, the (marginal) probability of default in a given year will increase with time during the initial years. In other words, the assumption is that the issuer is assumed to be stable at the outset, but with time, there is an increased potential of financial deterioration. Using the example of a bond with a baseline rating of AA, the marginal probabilities of default in Years 1, 2, and 3 are 0.02%, 0.04%, and 0.05%, respectively.

Additionally, for other categories of non-investment-grade bonds, it may be the case that the marginal probabilities of default start to decrease with time because the initial years are critical in terms of survival. The assumption is that if the issuer has not defaulted within the initial years, then its financial state may not be as precarious as initially assessed. Using the example of a bond with a baseline rating of B, the marginal probabilities of default in Years 1, 2, 3, and 4 are 3.34%, 4.46%, 3.95%, and 3.24%, respectively. Notice that after an increase in default probability in Year 2 over Year 1, the default probabilities in subsequent years show a consistent declining trend.

Hazard Rates

LO 25.e: Define the hazard rate and use it to define probability functions for default time as well as to calculate conditional and unconditional default probabilities.

Using the rating migration matrix information in Figure 25.1, we have already shown the calculation for an **unconditional default probability**, which is the marginal probability of default. For example, using the BB-rated bond, the unconditional default probability in Year 4 (from a Year 0 perspective) is 1.53% (= 4.99% − 3.46%).

The probability that the BB-rated bond remains in place until the end of Year 3 is 96.54% (= 100% − 3.46%). The **conditional default probability** (i.e., probability that the bond will default in Year 4, conditional on no earlier default) is then computed as 1.58% (= 1.53% / 96.54%). If one year is considered a sufficiently short amount of time, then the conditional default probability could also be called the **hazard rate** of default intensity.

The following equation can be used to calculate the **probability of default** by time *t*:

$$Q(t) = 1 - e^{-\bar{\lambda}(t)\times t}$$

where:

$\bar{\lambda}(t)$ = average hazard rate between time 0 and time *t*

Note that only the average hazard rate between t = 0 and *t* is required, because it is assumed to be a constant parameter. Assuming a constant hazard rate of 2% per year, the probability of default by the end of Year 1 (t = 1) is 1.98% (= $1 - e^{-0.02\times 1}$) and by the end of Year 2 (t = 2) is 3.92% (= $1 - e^{-0.02\times 2}$).

Continuing the computations just listed with the assumed 2% hazard rate, the probability of default by the end of Year 4 is 7.69% (= $1 - e^{-0.02\times 4}$). By the end of Year 5, it is 9.52% (= $1 - e^{-0.02\times 5}$). This means that the unconditional probability of default during Year 5 is equal to 1.83% (= 9.52% − 7.69%). It also means that the conditional

probability of default in Year 5, conditional on no earlier default, is 1.98% [= 1.83% / (1 – 7.69%)].

Recovery Rates

LO 25.f: Describe recovery rates and their dependencies on default rates.

When an issuer files for bankruptcy, most of the creditors are unlikely to receive full repayment of the face value of their claims. In that regard, a bond's **recovery rate** becomes relevant and can be thought of as the trading price as a fraction of the face value approximately a month following default. Figure 25.2 provides a summary of average recovery rates for six classes of bonds.

Figure 25.2: Recovery Rates on Corporate Bonds for 1983–2021

Class	Average Recovery Rate (%)
First lien bond	54.6
Second lien bond	44.3
Senior unsecured bond	38.0
Senior subordinated bond	31.0
Subordinated bond	32.1
Junior subordinated bond	22.3

Source: Moody's

From this information, it makes sense that the more senior bonds (e.g., first lien bond at 54.6% average recovery rate) have higher recovery rates than more junior bonds (e.g., subordinated bond at 32.1% average recovery rate).

Note that the recovery rate on debt is negatively correlated to the default rate. This can clearly be demonstrated with mortgages in that higher mortgage default rates mean more foreclosures and more properties for sale in the market. That, in turn, will lead to a decline in property prices, which reduces the lender's recovery rate.

Similarly with bonds, a strong economy usually has few bond defaults—but when there is a default, the average recovery rate is relatively high because of the strong economy. However, a weak economy usually has more bond defaults—and when there is a default, the average recovery rate is relatively low because of the weak economy. In short, the negative correlation between default rates and recovery rates is especially challenging for lenders in a weak economy because there are both high default rates and low recovery rates.

MODULE QUIZ 25.1

1. The following financial data pertains to Nielsen Corp. (Nielsen):

Working capital	$525,000
Total assets	$5,100,000
Book value of liabilities	$1,850,000
Market value of liabilities	$2,050,000
Retained earnings	$1,120,000
Market value of equity	$4,215,000
Earnings before interest and taxes	$480,000
Sales	$1,760,000

The Altman's Z-score equation is as follows:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

When using the computed Z-score to classify Nielsen based on credit quality, what is the most appropriate classification for the company?

A. No default is likely.
B. Potential default.
C. Reasonable probability of default.
D. High likelihood of default.

2. The following information is an excerpt from a rating migration matrix for a B-rated bond:

Time Horizon	Average Cumulative Default Rate
1 year	3.34%
2 years	7.80%
3 years	11.75%
4 years	14.89%
5 years	17.35%

What is the probability that the bond will default during the fourth year conditional on no earlier default?

A. 2.46%.
B. 2.79%.
C. 3.14%.
D. 3.56%.

MODULE 25.2: CREDIT DEFAULT SWAPS

LO 25.g: Define a credit default swap (CDS) and explain its mechanics including the obligations of both the default protection buyer and the default protection seller.

A **credit default swap (CDS)** is a credit derivative that is similar to a typical swap in that one party makes payments to another party. The purchaser of the CDS seeks credit protection (i.e., default protection buyer) and will usually make fixed quarterly payments (in advance) to the seller of the CDS (i.e., default protection seller) for the life

of the swap, or until a credit event occurs. Credit events may include nontimely payment, debt restructuring, or bankruptcy.

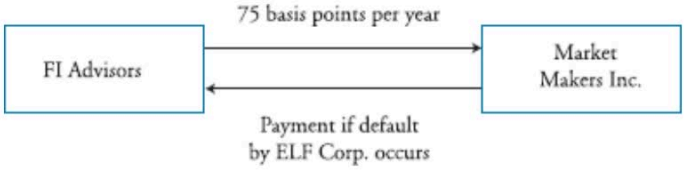
The underlying "reference entity" in a CDS is the firm in question, and the CDS protection buyer is concerned that a credit event (e.g., default) may occur at this firm. If default occurs, and assuming the terms of the swap agreement dictate settlement by physical delivery, the swap will be settled with the seller of the CDS paying the buyer the face value (notional principal) of the bonds and receiving the bonds in exchange.

To illustrate, suppose FI Advisors (FIA) owns fixed-income securities issued by ELF Corp. (ELF, the reference entity) with a par value of $200 million. FIA would like to protect its position against credit risk by using a CDS and is able to purchase such protection from Market Makers, Inc. (MM) for 75 basis points (i.e., the CDS spread) of a notional principal of $200 million on September 20, 2024. The life of the CDS is five years, which will require FIA to pay the equivalent of $1.5 million (= $200 million × 0.0075) to MM every year (or about $375,000 per quarter on the standard maturity dates of March 20, June 20, September 20, and December 20).

If ELF does not default, then FIA receives nothing from the swap. If ELF does default on July 20, 2028, for example, MM will pay FIA the notional principal of $200 million and receive the ELF bonds (assuming physical settlement). Note that the CDS contract likely provides a list of acceptable bonds (e.g., same seniority) that may be delivered upon default but sell at different percentages of face value. This introduces the possibility of the **cheapest-to-deliver (CTD) bond**. In the case of cash settlement, dealers are surveyed a specified number of days following a credit event to determine the midpoint between bid and ask prices of the CTD bond. If that CTD bond is valued at $0.40 to the dollar, then the cash payoff would be 60% or $120 million in this example.

Additionally, the protection buyer pays the ongoing premium to the protection seller only until there is a credit event. Using the example of a July 20, 2028 default and payment in arrears, then a final accrual payment of about $125,000 (= $375,000 per quarter × 1/3) to cover the one-month period from June 20, 2028 to July 20, 2028 would be required—and then the premium payments would cease. The example CDS transaction structure is illustrated in Figure 25.3.

Figure 25.3: Credit Default Swap Example



CDS contracts are traded with maturities that include 1, 2, 3, 5, 7, and 10 years, with 5 years being the most common in practice. Because of the standard maturity dates mentioned earlier, the actual life of the CDS contract might not be identical to the stated life, but it will be close. For example, a CDS that is initiated on August 10, 2025 would probably mature on September 20, 2030. The first payment due on September 20, 2025 would be an accrual amount to account for the period from August 10 to

September 20, 2025. Subsequently (e.g., December 20, 2025 payments and onward), the quarterly payments would be fixed.

Credit Indices

There are two well-known investment-grade **credit indices**, each with 125 reference entities. Maturities of 3, 5, 7, and 10 years (likely on June 20 and December 20) are most common. Those two portfolios are:

- CDX NA IG (North America) and
- iTraxx Europe.

To demonstrate the payment structure for credit indices, suppose that the 5-year iTraxx Europe index is quoted as bid 150 bps and ask 152 bps. Assuming an investor seeks €500,000 of protection per firm, using the ask price, it means that there are total annual payments of 0.0152 × €500,000 × 125 = €950,000. Or, if an investor sells €500,000 of protection per firm, using the bid price, it means total annual receipts of 0.015 × €500,000 × 125 = €937,500.

Assuming the purchase of protection, if one reference entity defaults, then the investor receives the appropriate payoff as well as a future reduction in the annual payments of €7,600 (= €950,000 / 125).

Fixed Coupons

In a CDS contract, the protection buyer pays a regular fixed coupon to the protection seller periodically over the life of the contract in return for a payment upon a prespecified credit event or reference entity (or entities). The fixed coupon paid periodically by the protection buyer is standardized to 100 basis points (i.e., 1%) for investment-grade issuers, because standardizing the regular payment in this way makes settlement and clearing of contracts more straightforward. It is important to note that this standardized coupon is not the fair premium (i.e., CDS spread) that should be regularly paid by the protection buyer.

In terms of an **up-front premium**, there are three possibilities:

1. CDS spread = fixed coupon; no up-front premium
2. CDS spread > fixed coupon; up-front premium equal to the present value of (CDS spread - fixed coupon) paid by protection buyer to protection seller
3. CDS spread < fixed coupon; up-front premium equal to the present value of (CDS spread - fixed coupon) paid by protection seller to protection buyer

CDS Spreads

LO 25.h: Describe CDS spreads and explain how CDS spreads can be used to estimate hazard rates.

The **CDS spread** is essentially the price of the CDS expressed in basis points. Using the previous example of 150 basis points (i.e., 1.5%), credit protection on the specific

underlying debt requires a payment of 1.5% from the protection buyer. In other words, the credit protection seller requires a 1.5% premium for bearing the credit risk.

The **bond yield spread** is the excess of the corporate bond yield over a comparable risk-free bond. In theory, the CDS spread and the bond yield spread should be equal. Putting everything together, a 5-year corporate bond with a 6% annual yield might be hedged using a 5-year CDS with a CDS spread of 1.5%. Netting the two positions produces a riskless position in the form of a net return of 4.5% (implied risk-free rate) to the investor, assuming no bond default. Or, in the event of a bond default, the return is 4.5% until default. After default, the investor will receive the face value of the bond, which can be invested at the risk-free rate until the end of the five years:

- If CDS spread < bond yield spread, buy the corporate bond, and buy CDS protection to earn more than the risk-free rate
- If CDS spread > bond yield spread, sell the corporate bond, and sell CDS protection to borrow at less than the risk-free rate

Note that this discussion has focused on fixed rates. Asset swaps and asset swap spreads provide direct estimates of the bond yield increment above a market reference rate (e.g., floating rate).

CDS-Bond Basis

LO 25.i: Define and explain CDS-bond basis.

The **CDS-bond basis** is defined as:

CDS-bond basis = CDS spread - bond yield spread

As mentioned, the CDS-bond basis should be zero (or near zero) because the CDS spread and the bond yield spread should be equal. In practice, however, this is not always the case due to the following reasons:

- Bonds may sell for much higher than par (negative basis) or lower than par (positive basis).
- CDSs are subject to counterparty risk (negative basis).
- A CTD possibility exists with a CDS (positive basis).
- CDS payoffs exclude accrued interest (negative basis).
- CDS contracts may contain a restructuring clause that allows for payoffs, even without default (positive basis).
- The bond yield spread is computed using a risk-free rate that is dissimilar to the one used by the market.

Due to constraints caused by illiquidity, arbitrage opportunities involving bonds and CDSs cannot always be fully exploited, which may also result in a nonzero basis. The CDS-bond basis was generally positive before the financial crisis of 2007–2009 but has since become negative and less significant.

Approximating Hazard Rates

An approximate calculation of the hazard rate based on credit spreads (e.g., CDS spreads or bond yield spreads), which can be used for a bond that trades near par value, is expressed as:

$$\bar{\lambda} = \frac{s(T)}{1 - RR}$$

where:

$\bar{\lambda}$ = average hazard rate
$s(T)$ = credit spread for maturity T
RR = recovery rate

For example, assume that an issuer's 5-year credit spread is 210 basis points (i.e., annual expected loss of 2.1%) and that the expected recovery rate in the event of a default is 30%. This suggests that the average probability of a default per year over five years, conditional on no earlier default, is equal to 0.021 / (1 − 0.3) = 3%.

This example could be expanded to bootstrap a term structure of the hazard rate using credit spreads for numerous maturities. Assume that spreads for 3-, 5-, and 10-year CDSs are 80, 90, and 110 bps, respectively, and that the expected recovery rate is 65%:

- Average hazard rate over 3 years: 0.008 / (1 − 0.65) = 0.0229 (or 2.29%)
- Average hazard rate over 5 years: 0.009 / (1 − 0.65) = 0.0257 (or 2.57%)
- Average hazard rate over 10 years: 0.011 / (1 − 0.65) = 0.0314 (or 3.14%)

Therefore, the average hazard rate between Year 3 and Year 5 is:

[(5 × 0.0257) − (3 × 0.0229)] / 2 = 0.0299, or 2.99%

The average hazard rate between Year 5 and Year 10 is:

[(10 × 0.0314) − (5 × 0.0257)] / 5 = 0.0371, or 3.71%

More Precise Hazard Rates

A more precise calculation of hazard rates should be used for a bond that does not trade near its par value. Assume a 5-year risky corporate bond with 6% annual coupons paid semiannually that yields 7% annually on a continually compounded basis. The risky bond would be priced at 95.34 (assuming 100 face value). Also, assume a risk-free government bond with 6% annual coupons paid semiannually that yields 5% annually on a continually compounded basis. The risk-free bond would be priced at 104.09. As a result, the price difference of 8.75 (risk-free vs. risky bond) is the expected loss from default over the bond term.

Figure 25.4 provides a summary of calculations of the present value of the expected loss on the assumption that defaults can only occur at five specified times that are immediately before coupon payment dates. The recovery rate is assumed to be 40%, and the default probability is denoted as Q and assumed to be constant for the entire life of the bond.

Figure 25.4:Expected Loss From Bond Default ($100 Notional)

Time (Years)	Default Probability	Recovery Amount ($)	Default-Free Value ($)	Loss ($)	Discount Factor	PV of Expected Loss ($)
0.5	Q	40	106.73	66.73	0.9753	65.08Q
1.5	Q	40	105.97	65.97	0.9277	61.20Q
2.5	Q	40	105.17	65.17	0.8825	57.52Q
3.5	Q	40	104.34	64.34	0.8395	54.01Q
4.5	Q	40	103.46	63.46	0.7985	50.67Q
Total						**288.48Q**

The expected value of a default-free bond at t = 4.5 years is 103.46, which is calculated as:

$$3 + 103e^{-0.05\times0.5} = 103.46$$

Based on a recovery rate of 40% of the face value, the loss given default (LGD) is 63.46 (= 103.46 − 40), and on a discounted basis using a 5% annual discount rate, the loss is 50.67 and the expected loss is 50.67Q.

Based on the information in Figure 25.4, the total expected loss for all the periods is 288.48Q. Given the 8.75 amount computed earlier, the default probability (Q) = 8.75 / 288.48 = 3.03%.

Using a sample of bonds maturing in 2, 5, and 7 years, a bootstrapping process could be used for estimating the term structure of default probabilities. The 2-year bond could be used to estimate the default probability for the first two years, the 5-year bond to estimate the annual default probabilities for Years 4 and 5, and the 7-year bond to estimate the annual default probabilities for Years 6 and 7.

MODULE QUIZ 25.2

1. The five-year CDX NA IG Index (125 companies) is quoted as bid 141 bps and ask 143 bps. An investor plans to sell $1 million of protection on each company. At the beginning of the third year before the annual protection payment, one of the companies defaults. Assuming no other defaults, the investor's cash flow for the third year is closest to:
   A. $748,400 inflow.
   B. $748,400 outflow.
   C. $773,200 inflow.
   D. $773,200 outflow.

2. In the context of arbitrage trades, if the CDS spread is significantly greater than the bond yield spread, what is the most appropriate action by the investor?
   A. Buy the corporate bond and buy CDS protection.
   B. Buy the corporate bond and sell CDS protection.
   C. Sell the corporate bond and buy CDS protection.

D. Sell the corporate bond and sell CDS protection.

MODULE 25.3: COMPARING DEFAULT PROBABILITIES

LO 25.j: Compare default probabilities calculated from historical data with those calculated from credit yield spreads.

Figure 25.5 provides the 7-year average cumulative default probabilities for issuers of varying credit ratings (from S&P in Column 1) and the 7-year credit spread for varying ratings (from Merrill Lynch in Column 2).

Figure 25.5: Comparing Cumulative Default Probabilities With Credit Spreads

Rating	Cumulative 7-Year Default Probabilities (%): 1981–2020	7-Year Credit Spread (bps): 1996–2007
AAA	0.51	35.74
AA	0.49	43.67
A	0.76	68.68
BBB	2.27	127.53
BB	8.89	280.28
B	20.99	481.04
CCC/C	50.75	1,103.70

The amounts in Figure 25.5 can be used to estimate the average 7-year hazard rates. There are two methods for making these calculations. The first method applies the following equations:

$$Q = 1 - e^{-7\bar{\lambda}}$$

$$\bar{\lambda}(7) = -\frac{1}{7}\ln(1 - Q)$$

Method 1: A BBB-rated issuer has a cumulative 7-year default rate of 2.27%, so the average hazard rate is calculated as:

$$\bar{\lambda}(7) = -\frac{1}{7}\ln(1 - 0.0227) = 0.00328 = 0.328\%$$

Method 2: Using the same BBB-rated issuer as Method 1, but using the 7-year credit spread and a 40% recovery rate assumption, the average 7-year hazard rate is:

$$\bar{\lambda}(7) = 0.012753 \,/\, (1 - 0.4) = 0.02126 = 2.126\%$$

A summary of the average 7-year hazard rates is provided in Figure 25.6 for the full range of credit ratings using the same methodologies just discussed.

Figure 25.6: Hazard Rates Differences

Rating	Rate From Default Probabilities (%)	Rate Implied by Credit Spreads (%)	Difference
AAA	0.073	0.596	0.523
AA	0.070	0.728	0.658
A	0.109	1.145	1.036
BBB	0.328	2.126	1.798
BB	1.330	4.671	3.341
B	3.366	8.017	4.651
CCC/C	10.118	18.395	8.207

Based on the rate comparisons in Figure 25.6, the hazard rates computed from credit spreads are significantly greater than those computed from historical data. As credit quality improves (worsens), the differences are less (greater). A simple conclusion here is that the return for bearing credit risk is more than sufficient because the return is greater than the expected cost of the defaults. Such excess return is even more prevalent when transacting in lower-credit-quality instruments and during periods of economic turmoil with high credit spreads.

The key explanation for these hazard rate calculations is that bond defaults are not independent and often depend on economic conditions; on a stand-alone basis, a good (bad) economy decreases (increases) default probabilities. Historical data suggests a wide range of default rates from year to year, and this variation results in *systematic risk* (risk that is nondiversifiable) whereby the market provides investors with a return premium for taking on the additional risk. With lower-quality debt being analogous to equity, it makes sense for the expected return (and consequently, excess return) on such debt to increase.

Additionally, it may not be possible or even practical to fully diversify away all the unsystematic risk in a bond portfolio (e.g., compared to an equity portfolio). In fact, taking on the unsystematic risk may provide additional return to the investor. Finally, a relatively minor point to consider for higher credit spreads is that corporate bonds are less liquid, which suggests a liquidity premium for bearing such risk.

Real-World vs. Risk-Neutral Default Probabilities

LO 25.k: Describe the difference between real-world and risk-neutral default probabilities and determine which one to use in the analysis of credit risk.

Under risk neutrality, the risk-free rate is assumed to be the expected growth rate of the firm's assets, whereas in the real world, the expected growth rate is the risk-free rate plus a market risk premium. Therefore, all things being equal, the computed asset values will be higher in the real world compared to risk neutrality.

The result is that assuming the same face value of debt, the **real-world default probability** will be lower than the **risk-neutral default probability** due to the higher asset valuation of real-world probabilities. For valuation purposes, risk-neutral

estimates (e.g., estimates of default rates from credit spreads) make sense because no additional premium is required for bearing risk. In contrast, for scenario analysis, real-world estimates (e.g., default estimates from historical data) would be more appropriate.

Default Probability Estimation With the Merton Model

LO 25.l: Using the Merton model, calculate the value of a firm's debt and equity, the volatility of firm value, and the volatility of firm equity.

LO 25.m: Using the Merton model, calculate distance to default and default probability.

The **Merton model** views firm equity value as a long call option on the firm's assets. For this model, we assume that there is debt outstanding that needs to be repaid at time *T*. Terminology for the Merton model is expressed as follows:

- V_0 = current value of the firm's assets
- V_T = value of the firm's assets at time *T*
- D = total debt to be repaid at time *T*
- E_0 = current value of the firm's equity
- E_T = value of the firm's equity at time *T* (i.e., V_T - D)
- σ_V = asset volatility
- σ_E = equity volatility

In the Merton model, the strike price of the call option is the total debt to be repaid (D). Therefore, $E_T = \max(V_T - D, 0)$ because if $V_T > D$, it makes sense to continue to service the debt and retain the positive value of E_T. From a purely quantitative perspective, if $V_T < D$, then it makes sense to default on the debt because the value of the equity is zero.

On the assumption of no dividends, the Black-Scholes-Merton equity value today is stated as:

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}}$$

$$d_2 = d_1 - \sigma_v \sqrt{T}$$

N = cumulative normal distribution function

When the option is not exercised, there is a default by the firm, and the probability of occurrence is denoted as $N(-d_2)$. Calculating that probability must be done indirectly and requires values for V_0 and σ_V. Stochastic calculus can be applied such that:

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_V V_0$$

The delta of equity is equal to $\partial E / \partial V$, so $N(d_1)$ can be used as a substitute:

$$\sigma_E E_0 = N(d_1)\sigma_V V_0$$

Once V_0 and σ_V are solved, the remaining items of d_1, d_2, $N(d_1)$, and $N(-d_2)$ can be solved.

Putting it all together, assume a firm's current equity value is \$3 million ($E_0 = 3$) and its equity volatility is 80% ($\sigma_E = 0.80$). There is debt outstanding of \$10 million ($D = 10$) that needs to be repaid in one year ($T = 1$). The annual risk-free rate is assumed to be 5% ($r = 0.05$).

Assuming the appropriate calculations have been performed, it is determined that $V_0 = 12.40$ and $\sigma_V = 0.2123$. It is now possible to use the previous equations to determine the remaining values:

- Using the equation $\sigma_E E_0 = N(d_1)\sigma_V V_0$, with five variables and four known variables, it is possible to solve for the unknown variable of $N(d_1)$ as 0.9117.
- From there, apply the Black-Scholes-Merton equation and solve for unknown variable $N(d_2)$ as 0.873.
  - The (risk-neutral) *probability of default* is $N(-d_2) = 1 - N(d_2) = 1 - 0.873 = 0.127$.
  - The *distance to default* is d_2, which first requires the calculation of d_1; in this example, $d_1 = 1.3547$ and $d_2 = 1.1408$.
- The current market value of debt is $V_0 - E_0 = 12.40 - 3 = 9.40$. The discounted value of the debt repayment is $10e^{-0.05\times1} = 9.51$. This leads to the expected loss on the debt equal to $(9.51 - 9.40) / 9.51 = 1.2\%$.

Note that one of the key outputs from the Merton model is the **distance to default**, which is the number of standard deviations that the asset price must fall to lead to a default at time *T*. The higher (lower) the distance to default, the lower (higher) the probability of default. The distance to default, which is equal to d_2, can be computed directly as:

$$d_2 = \frac{\ln(V_0) - \ln(D) + (r - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}$$

Merton Model Performance

LO 25.n: Assess the quality of the default probabilities produced by the Merton model, the Moody's KMV model, and the Kamakura model.

Empirical evidence supports that the Merton model, Moody's-KMV model, and the Kamakura model provide reliable and reasonably accurate rankings of default probabilities. The Merton model's output is a risk-neutral default probability because