The Thermodynamics of Number Theory:

An Analysis of Kaprekar's Routine as a Self-Organizing Critical System

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Abstract

ABSTRACT: This study investigates the behavior of Kaprekar’s Routine through the lens of discrete dynamical systems. While the decimal constant 6174 is well-known, this paper extends the analysis to higher digit lengths and non-decimal bases. We confirm that the routine acts as a self-organizing criticality with 'annealing' properties, where local entropy increases facilitate global ordering. We further validate Ludington's Bound, showing that constant emergence is a finite geometric feature.

1. INTRODUCTION In 1949, D.R. Kaprekar discovered that the number 6174 acts as a fixed point for a specific arithmetic process. This paper classifies the nature of this convergence, asking if it shares properties with physical decay or chaotic attractors. 2. THE THOUGHT EXPERIMENT We proposed two analogies: A) The Mandelbrot Analogy: Mapping 'stopping time' to reveal fractal basins. B) The Carbon Decay Analogy: Comparing numerical shedding to exponential decay.

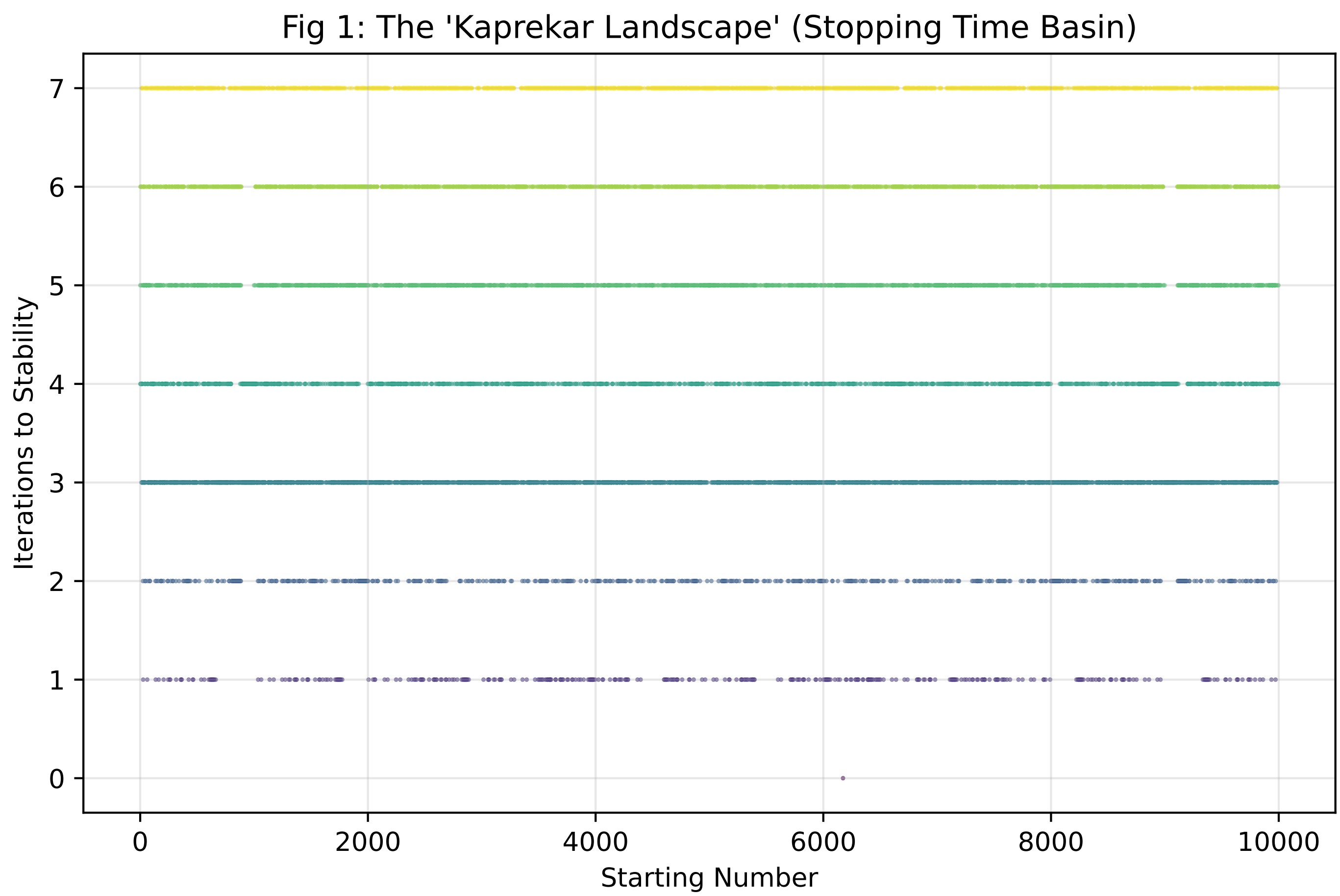

Fig 1: The 'Kaprekar Landscape' (Stopping Time Basin)
Iterations to Stability
Starting Number

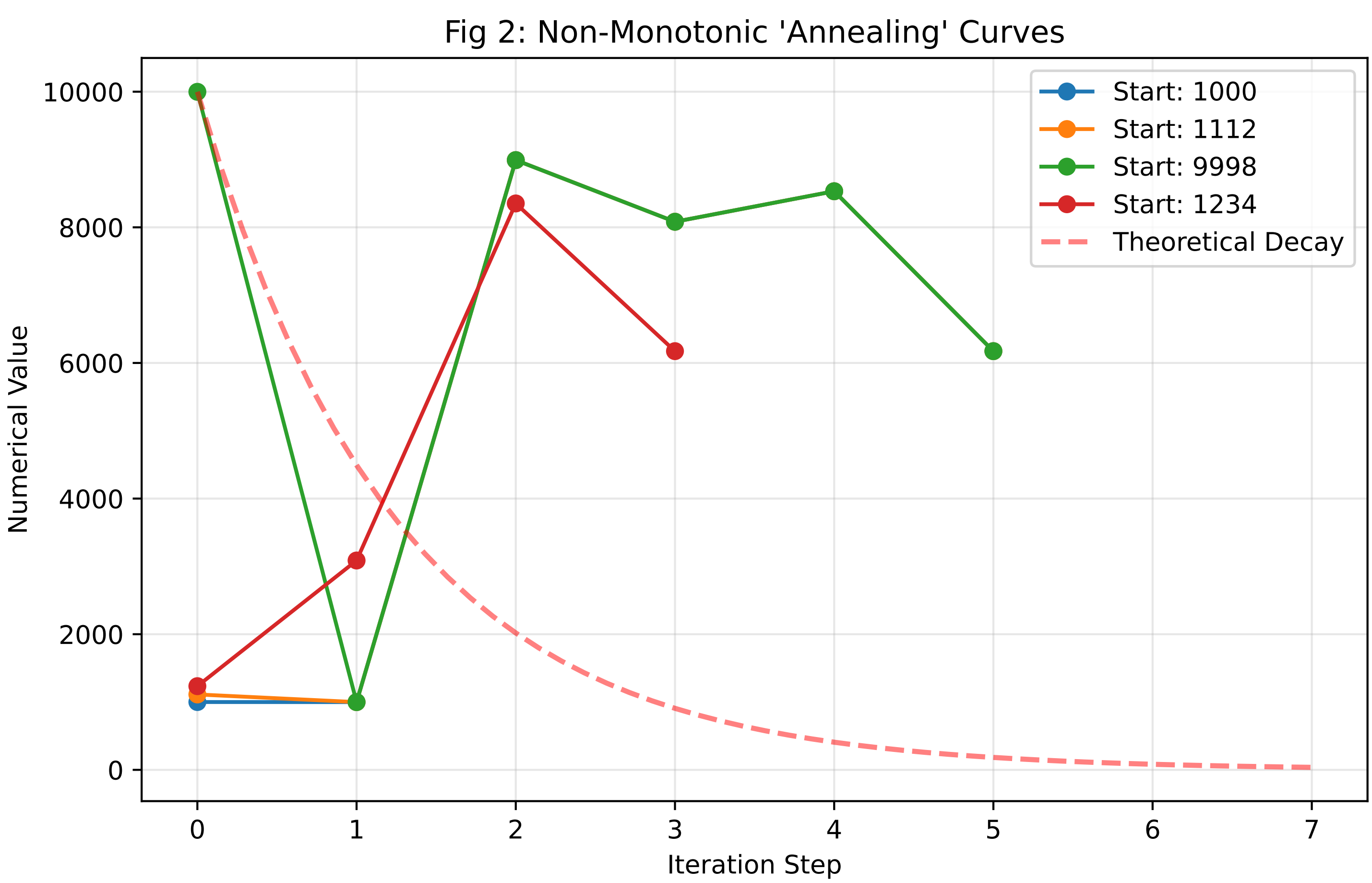

Fig 2: Non-Monotonic 'Annealing' Curves
Start: 1000
Start: 1112
Start: 9998
Start: 1234
Theoretical Decay
Numerical Value
Iteration Step

3. RESULTS & DISCUSSION

3.1 Ludington's Bound

We confirmed that while Base 10 allows an 'Expansion Property' (generating larger constants like 63317664), this is not universal. In Base 5, the constant curve 'crashes' into chaos as digits increase, validating Young's theorem (1979).

3.2 The Annealing Verdict

As seen in Fig 2, the routine is NOT monotonic decay. Numbers like 1000 spike in value before collapsing. This suggests the routine functions as Simulated Annealing—injecting energy to escape local minima.

4. CONCLUSION

Kaprekar's routine is a Negentropic System. It organizes random inputs into a singular ground state (6174) through a process of ordered criticality.

REFERENCES

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