Route Optimization

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In this problem, we are managing the transportation of heaters and air-conditioners between two warehouses and five stores. Our goal is to minimize the distance traveled, while meeting all demand requirements.

From seven locations (two warehouses and five stores), we obtained a graph with seven vertices and 21 edges. In other words, to form a path between any two locations (of the seven), we compute "seven choose two", or $\binom{7}{2} = 21$. To consider both directions (going to and from all locations) we double this number to obtain 42 directional edges.

To obtain the objective function, we must minimize the distance traveled by delivery trucks to and from the determined locations. Let t_{ij} = number of trips from location i to location j. Every two locations, i and j, have distances between them, denoted d_{ij}. We will then need to multiply the distance of each path by the number of times taking that path.Thus the objective function is as follows:

$$\text{Minimize: } \sum_{i \neq j} d_{ij} t_{ij} \qquad (z)$$

To obtain constraints, we will consider the following physical properties:

- Each truck can only hold a certain amount of heaters/air-conditioners.
- A truckload should be smaller once dropping off the demanded goods.
- Trucks must travel along one of the 21 paths and cannot "teleport".
- The demanded load on each truck must be delivered in a countable number of trips.

For the first bullet point, we are given the volumes of the heaters and air-conditioners of $0.4m^3$ and $0.8m^3$, respectively. The total volume capacity per truck is 20 m^3. Computing the total capacity of heaters/air-conditioners per truck, we obtain:

$$0.4H \ + \ 0.8A \ \leq \ 20$$

We are given the demanded number of heaters and air-conditioners at each of the five stores, of which we will convert to truckloads. Since 1 truckload = 20 cubic meters, we divide both sides of the above equation by 20, giving us:

$$\tfrac{1}{50}H \ + \ \tfrac{1}{25}A \ \leq \ 1 \text{ truckload}$$

For example, if Store A demands 2 heaters and 4 air-conditioners, we will obtain:

$$\tfrac{2}{50} + \tfrac{4}{25} \ = \ .2 \text{ truckloads}$$

For the second bullet point, we realize that a truck will travel to a location, drop off the demanded goods, then travel to another location. In other words, the truckload amount going to j minus the demand at j equals the truckload amount leaving j. Letting x_{ij} = truckload amount from i to j, and r_j = required demand at j, we obtain the following constraint:

$$\textstyle\sum_{i \neq j} x_{ij} \ - \ r_j \ = \ \sum_{i \neq j} x_{ij} \qquad (1)$$

For the third bullet point, we realize that the number of trucks arriving at a location should be equal to the number of trucks leaving that location. In other words, the number of trips from i to j equals the number of trips from j to i. Recalling that t_{ij} = number of trips from i to j, we obtain the constraint:

$$\textstyle\sum_{i \neq j} t_{ij} \ = \ \sum_{i \neq j} t_{ji} \qquad (2)$$

For the fourth bullet point, we realize that a truck cannot travel, for example, $\frac{1}{2}$ of a trip. In other words, if Store A demands $\frac{1}{2}$ truckloads, this will require one full trip. This gives us the constraints:

$$x_{ij} \ \leq \ t_{ij} \qquad (3)$$

$$t_{ij} \ \epsilon \ Integers \qquad (4)$$

Lastly, we cannot have a negative number of trips or truckloads, which gives us the constraint:

$$x_{ij},\ t_{ij} \ \geq \ 0 \tag{5}$$

We now have the linear program:

$$\begin{aligned}
\text{Minimize: } & \textstyle\sum_{i \neq j} d_{ij} t_{ij} \\
\text{Subject to: } & \textstyle\sum_{i \neq j} x_{ij} \ - \ r_j \ = \ \sum_{i \neq j} x_{ij} \\
& \textstyle\sum_{i \neq j} t_{ij} \ = \ \sum_{i \neq j} t_{ji} \\
& x_{ij} \ \leq \ t_{ij} \\
& t_{ij} \ \epsilon \ Integers \\
& x_{ij},\ t_{ij} \ \geq \ 0
\end{aligned}$$

Solving this system with an optimization program, we obtain the following routes:

Truck 1

Leaves Warehouse 1 with .86 truckloads → Delivers .3 truckloads to Store 1

Leaves Store 1 with .56 truckloads → Delivers .06 truckloads to Store 3

Leaves Store 3 with .5 truckloads → Delivers .5 truckloads to Store 4

Leaves Store 4 with 0 truckloads → Returns to Warehouse 1

Truck 2

Leaves Warehouse 2 with .42 truckloads → Delivers .1 truckloads to Store 5

Leaves Store 5 with .32 truckloads → Delivers .32 truckloads to Store 2

Leaves Store 2 with 0 truckloads → Returns to Warehouse 2

This route requires a total distance traveled of 24 kilometers, which is the minimal distance required to satisfy the given demand.