Assignment - Computational Graph

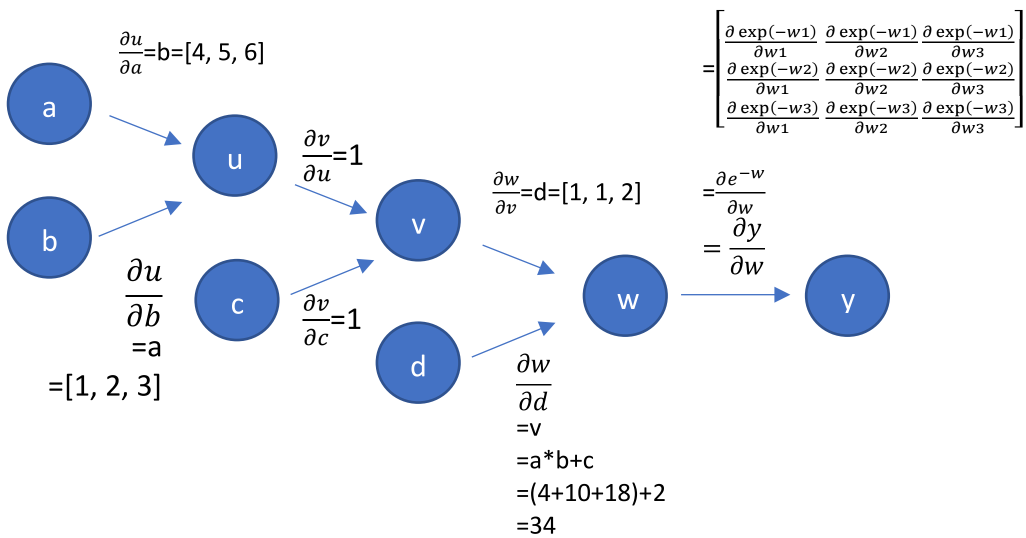
1. Back-propagation

*y = 1 / exp[(a * b + c) * d]*

where a=[1,2,3], b=[4,5,6], c=2, d=[1,1,2].

Let a*b =u, a*b+c=v, (a*b+c)xd=w

And a=[1 2 3], b=[4 5 6], c=2, d=[1 1 2], u=32, v=34, w=34[1 1 2]=[w1 w2 w3]



$$\frac{\partial y}{\partial a} = \frac{\partial y}{\partial a}\frac{\partial y}{\partial a}\frac{\partial y}{\partial a}\frac{\partial y}{\partial a}$$

$$= \begin{bmatrix} -e^{-w1} & 0 & 0 \\ 0 & -e^{-w2} & 0 \\ 0 & 0 & -e^{-w3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} * 1 * [4 \;\; 5 \;\; 6] = \begin{bmatrix} -e^{-w1} \\ -e^{-w2} \\ -2e^{-w3} \end{bmatrix} [4 \;\; 5 \;\; 6] = \begin{bmatrix} -4e^{-w1} & -5e^{-w1} & -6e^{-w1} \\ -4e^{-w2} & -5e^{-w2} & -6e^{-w2} \\ -8e^{-w3} & -10e^{-w3} & -12e^{-w3} \end{bmatrix}$$

$$\frac{\partial y}{\partial b} = \frac{\partial y}{\partial w}\frac{\partial w}{\partial v}\frac{\partial v}{\partial u}\frac{\partial u}{\partial b}$$

$$= \begin{bmatrix} -e^{-w1} & 0 & 0 \\ 0 & -e^{-w2} & 0 \\ 0 & 0 & -e^{-w3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} * 1 * [1 \;\; 2 \;\; 3] = \begin{bmatrix} -e^{-w1} \\ -e^{-w2} \\ -2e^{-w3} \end{bmatrix} [1 \;\; 2 \;\; 3] = \begin{bmatrix} -e^{-w1} & -2e^{-w1} & -3e^{-w1} \\ -e^{-w2} & -2e^{-w2} & -3e^{-w2} \\ -2e^{-w3} & -4e^{-w3} & -6e^{-w3} \end{bmatrix}$$

$$\frac{\partial y}{\partial c} = \frac{\partial y}{\partial w}\frac{\partial w}{\partial v}\frac{\partial v}{\partial c}$$

$$= \begin{bmatrix} -e^{-w1} & 0 & 0 \\ 0 & -e^{-w2} & 0 \\ 0 & 0 & -e^{-w3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} * 1 = \begin{bmatrix} -e^{-w1} \\ -e^{-w2} \\ -2e^{-w3} \end{bmatrix}$$

$$\frac{\partial y}{\partial c} = \frac{\partial y}{\partial w}\frac{\partial w}{\partial d}$$

$$= \begin{bmatrix} -e^{-w1} & 0 & 0 \\ 0 & -e^{-w2} & 0 \\ 0 & 0 & -e^{-w3} \end{bmatrix} 34$$

2. Programming